On the Robustness of Space-Time Coding for Spatially and Temporally Correlated Wireless Channels

Larry T. Younkins
Johns Hopkins University
Applied Physics Laboratory,
Laurel, MD 20723, USA
Email: larry.younkins@jhuapl.edu

Weifeng Su
Department of ECE
University of Maryland,
College Park, MD 20742, USA
Email: weifeng@eng.umd.edu

K. J. Ray Liu
Department of ECE
University of Maryland,
College Park, MD 20742, USA
Email: kjrliu@eng.umd.edu

***Abstract*— The robustness of space-time coding techniques for wireless channels that exhibit both temporal and spatial correlation is investigated. A general space-time covariance model is developed and employed to evaluate the exact pairwise error probability for several space-time codes. A significant degradation in the performance of space-time coding techniques is observed for cases where the scatterers are located in close proximity to the mobile and the spacing between transmit antennas is a fraction of a wavelength. The conditions for which the commonly used assumption of independent transmission paths is valid are investigated as a function of the scattering radius and the spacing of the transmit and receive antennas.**

I. Introduction

Wireless systems employing multiple transmit and receive antennas have the potential for tremendous gains in channel capacity through exploitation of independent transmission paths due to scattering. Transmit diversity, achieved through the use of space-time coding techniques at the base station is a recent innovation motivated by the need for higher throughput in the wireless channel. A simple two-branch transmit diversity scheme was first proposed by Alamouti [1]. It was demonstrated that this scheme provides the same diversity order as a wireless system employing a single transmit antenna and two receive antennas and utilizing maximal-ratio combining (i.e. classical receive diversity). The bit-error-rate (BER) performance of the proposed scheme was evaluated assuming that the path from each transmit antenna to each receive antenna experiences mutually uncorrelated Rayleigh amplitude fading.

Tarokh et al. [2] proposed additional space-time block codes utilizing three and four transmit antennas. These codes are based upon complex-valued orthogonal designs [3] and have the feature that only linear processing is required at the receiver for decoding. In recently published work Wang et al. [4] derive the exact pairwise error probability for space-time coding over quasi-static or fast-fading Rayleigh channels in the presence of spatial fading correlation. For analytical tractability, the authors assume the channel matrix can be decomposed as a product of the square roots of the transmit and receive correlation matrices, respectively. The effects of spatial correlation on space-time coding performance are investigated for several scenarios but it is unclear how the parameters chosen relate to physical scattering parameters such as effective scattering radius, etc.

The majority of the research to date on space-time coding techniques has employed the assumption of uncorrelated transmission paths without regard for the conditions under which this assumption is justified. The degree of correlation between channel transmission paths from a transmit antenna to a receive antenna depends significantly on the scattering environment and on the antenna separation at the transmitter and receiver. For example, if the majority of the channel scatterers are located in close proximity to the mobile then the transmission paths will be highly correlated unless the transmit antennas are sufficiently separated in space. Early research that characterized the spatial and temporal characteristics of the mobile radio channel was performed by Jakes [5] and Clarke [6]. In these works a geometric scattering model was employed that places scatterers uniformly on a circular ring a fixed distance from the mobile. More recently, Chen et al. [7] extended this 'circular ring' scatterer model to include multiple antennas at the base station, a single antenna at the mobile and Doppler effects due to motion of the mobile. Shiu et al. [8] investigated the effects of fading correlation on the capacity of multiple-antenna wireless systems by employing the Jakes model to multiple antennas at the base station as well as the mobile. However, Doppler effects due to mobile motion were not considered. Abdi [9] developed at space-time correlation model for multiple antenna wireless systems by employing the 'circular ring' scattering geometry but allowing a non-uniform distribution of scatterers. Specifically, the von Mises density was used to describe the angle of arrival of the multipath with respect to the mobile. Doppler effects are included in this model. Independently, Safar [10] derived a special case of this model in which the angle of arrival was uniformly distributed.

In the work presented here we develop a general space-time covariance model based upon scatterer geometry, transmit and receive antenna geometry and a linear motion model for the mobile. The model is applicable to arbitrary scatterer geometry and includes Doppler effects due to mobile motion.

 0-7803-8344-3/04/$20.00

The model is evaluated for the special case of the 'circular ring' scattering geometry due to Jakes and is used to quantify the performance of space-time coding techniques for wireless channels that exhibit both temporal and spatial correlation. The conditions under which the transmission paths can be considered to be independent are quantified in terms of the required antenna spacing and scattering radius. Additionally, the worst-case error performance for several space-time codes is evaluated in terms of physical parameters such as transmit and receive antenna spacing, scattering radius and normalized Doppler frequency.

II. Channel Model and Pairwise Error Probability

Consider a wireless system employing M transmit antennas and N receive antennas. The signal received at the q^{th} antenna at time t is

$$y_q(t) = \sqrt{\frac{\rho}{M}} \sum_{p=1}^{M} h_{p,q}(t) c_p(t) + z_q(t) \tag{1}$$

where ρ denotes the signal-to-noise ratio per receive antenna, $h_{p,q}(t)$ is the complex path gain between the p^{th} transmit antenna and the q^{th} receive antenna at time t, $c_p(t)$ denotes the space-time code symbol transmitted by the p^{th} antenna at time slot t and $z_q(t)$ is independent complex Gaussian noise with zero mean and unit variance. Each space-time signal is described by a $T \times M$ matrix $\mathbf{C}$ with the columns corresponding to the space dimension and the rows corresponding to the time dimension Each entry in the code matrix $\mathbf{C}$ consists of linear combinations of the complex-valued signal constellation variables $x_1, x_2, \ldots, x_k$. These variables are determined by the type of modulation employed (e.g. M-QAM, M-PSK, etc.) and the specific data to be encoded. The space-time code symbol $c_p(t)$ is chosen as the entry in the code matrix corresponding to the p^{th} column and t^{th} row.

Equation (1) can be re-written in vector form as [12], [15]

$$\mathbf{Y} = \sqrt{\frac{\rho}{M}} \mathbf{D}\mathbf{H} + \mathbf{Z} \tag{2}$$

where the $NT \times MNT$ matrix $\mathbf{D}$ is constructed from the space-time signal matrix $\mathbf{C}$ as

$$\mathbf{D} = \mathbf{I}_N \otimes diag\left[D_1, D_2, \cdots, D_M\right] \tag{3}$$

$$D_i = diag\left(c_i(1), c_i(2), \cdots, c_i(T)\right), \quad i = 1, 2, \cdots, M. \tag{4}$$

and $\otimes$ denoting the tensor matrix product. The $MNT \times 1$ channel vector $\mathbf{H}$ is defined by

$$\mathbf{H} = \left(\mathbf{h}_{1,1}^{'}, \cdots, \mathbf{h}_{M,1}^{'}, \cdots, \mathbf{h}_{1,N}^{'}, \cdots, \mathbf{h}_{M,N}^{'},\right)^{'} \tag{5}$$

$$\mathbf{h}_{i,j} = \left(h_{i,j}(1), h_{i,j}(2), \cdots, h_{i,j}(T)\right)^{'} \tag{6}$$

The $NT \times 1$ received signal vector $\mathbf{Y}$ is defined by

$$\mathbf{Y} = \left(y_1(1), \cdots, y_1(T), \cdots, y_N(1), \cdots, y_N(T)\right)^{'} \tag{7}$$

and similarly for the $NT \times 1$ noise vector $\mathbf{Z}$.

The pairwise error probability given the channel vector $\mathbf{H}$ is

$$Pr\left(\mathbf{C} \rightarrow \tilde{\mathbf{C}} \mid \mathbf{H}\right) = Q\left(\sqrt{\frac{\rho}{2M} \left\| \left(\mathbf{D} - \tilde{\mathbf{D}}\right) \mathbf{H} \right\|^2}\right) \tag{8}$$

where $\|\mathbf{x}\|$ denotes the norm of the vector $\mathbf{x}$, i.e. $\|\mathbf{x}\|^2 = \mathbf{x}^\dagger \mathbf{x}$ and $Q(x)$ denotes the Gaussian Q function.

Using the form of the Gaussian Q function due to Craig [13] and a result from Turin [14],[15] regarding the characteristic function of a quadratic form of a complex Gaussian vector we have following expression for the pairwise error probability between $\mathbf{C}$ and $\tilde{\mathbf{C}}$ is

$$Pr\left(\mathbf{C} \rightarrow \tilde{\mathbf{C}}\right) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{i=1}^{K} \left(1 + \frac{\rho}{M} \frac{\lambda_i}{4 \sin^2 \theta}\right)^{-1} d\theta \tag{9}$$

with K corresponding to the rank of the matrix

$$\left(\mathbf{D} - \tilde{\mathbf{D}}\right) \mathbf{R} \left(\mathbf{D} - \tilde{\mathbf{D}}\right)^\dagger \tag{10}$$

$\{\lambda_i\}_{i=1}^K$ its eigenvalues and $\mathbf{R} = E\{\mathbf{H}\mathbf{H}^\dagger\}$.

Given space-time codes $\mathbf{C}$ and $\tilde{\mathbf{C}}$ and the channel (space-time) covariance matrix $\mathbf{R}$ the pairwise error probability can be calculated from (9). In the sequel, the worst-case pairwise error probability is determined from (9) for all pairs of space-time signals $\mathbf{C}$ and $\tilde{\mathbf{C}}$ and employing the union bound. The next section discusses the development of the space-time covariance model for the channel.

III. Space-Time Covariance Model

In this section we present a space-time covariance model that is applicable to arbitrary scatterer geometry, multiple antennas at the base station and the mobile, and includes Doppler effects due to mobile motion. The resulting model is then evaluated for the special case of the 'circular ring' scattering geometry.

The complex path gain between the p^{th} antenna at the mobile and the r^{th} antenna at the base station is denoted by $h_{p,r}(t)$. It consists of contributions from K discrete scatterers with the m^{th} scatterer characterized by its amplitude A_m, phase ψ_m and spatial location $\vec{x}_m$. All scatterers are assumed to be coplanar with the mobile and base station. The spatial locations of the array phase centers for the mobile and base are $\vec{x}_{mobile}$ and $\vec{x}_{base}$, respectively. The spatial location of the p^{th} antenna at mobile is denoted by $\vec{x}_{mobile}^{p}$ and the spatial location of the r^{th} antenna at the base station is denoted by $\vec{x}_{base}^{r}$. Figure 1 illustrates the geometry for the scattering model. Assuming a plane wave with frequency f_c is transmitted by the base, the expression for the complex path gain $h_{p,r}(t)$ is:

$$h_{p,r}(t) = \sum_{m=0}^{K-1} A_m \exp\left(j\psi_m\right) \exp\left[-j2\pi f_c \tau_m(t)\right] \times \exp\left[+j\vec{k}_{mobile}^{m} \cdot \vec{x}_{mobile}^{p} + j\vec{k}_{base}^{m} \cdot \vec{x}_{base}^{r}\right] \tag{11}$$

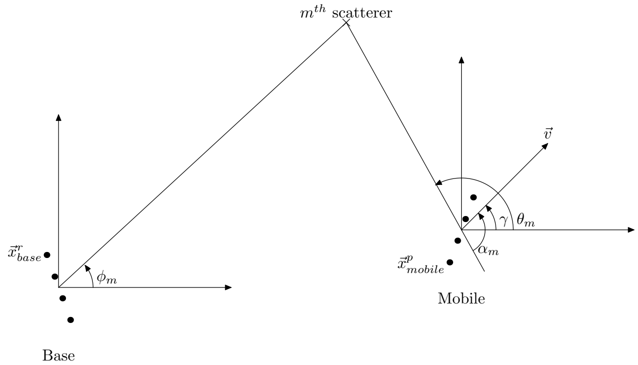

Fig. 1. Scattering Model Geometry

In the previous expression $\tau_m(t)$ denotes the path delay associated with the m^{th} scatterer and

$$\vec{k}^m_{mobile} = \frac{2\pi}{\lambda}(\cos\theta_m, \sin\theta_m, 0) \qquad (12)$$
$$\vec{k}^m_{base} = \frac{2\pi}{\lambda}(\cos\phi_m, \sin\phi_m, 0)$$

with λ denoting the transmitted wavelength. The angle θ_m corresponds to the angle of arrival at the mobile associated with the signal re-radiated from the m^{th} scatterer. The phases associated with the m^{th} and n^{th} scatterers, ψ_m and ψ_n, are assumed independent and uniformly distributed on $(-\pi, \pi)$ and independent of all other random quantities. With this assumption the expression for the correlation between the transmission paths associated with the signal received at the p^{th} element of the mobile array and transmitted from the r^{th} element of the base array and the signal received by the q^{th} element of the mobile array and transmitted from the s^{th} element of the base array at time lag Δt is

$$E\Big\{h_{p,r}(t)h^*_{q,s}(t+\Delta t)\Big\} = \qquad (13)$$
$$E\Big\{\sum_{m=0}^{K-1} A_m^2 \exp\left[j2\pi f_c\left(-\tau_m(t) + \tau_m(t+\Delta t)\right)\right]$$
$$\times \exp\left[+j\vec{k}^m_{mobile}\cdot(\vec{x}^p_{mobile} - \vec{x}^q_{mobile})\right]$$
$$\times \exp\left[+j\vec{k}^m_{base}\cdot(\vec{x}^r_{base} - \vec{x}^s_{base})\right]\Big\}$$

In order to specify the path delay associated with the m^{th} scatterer, $\tau_m(t)$, some assumptions about the motion of the mobile must be made. In what follows we employ a linear approximation for the path delay. Specifically,

$$\tau_m(t) \approx \tau_m^0 + \frac{|\vec{v}|t}{c}\cos\alpha_m \qquad (14)$$

where τ_m^0 corresponds to the static (time-invariant) portion of the path delay and α_m is the angle between the mobile velocity vector $\vec{v} = |\vec{v}|\cos(\gamma)$ and the line joining the initial mobile location and the location of the m^{th} scatterer. In this expression c denotes the speed of light and $|\vec{x}|$ denotes the norm of the vector $\vec{x}$.

Employing the linear approximation for the path delay, we have the following expression for the space-time correlation function

$$E\Big\{h_{p,r}(t)h^*_{q,s}(t+\Delta t)\Big\} = \exp(-j2\pi f_c T) \qquad (15)$$
$$\times E\Big\{\sum_{m=0}^{K-1} A_m^2 \exp\left[j2\pi f_c\left(\frac{|\vec{v}|\Delta t}{c}\cos\alpha_m\right)\right]$$
$$\times \exp\left[+j\vec{k}^m_{mobile}\cdot(\vec{x}^p_{mobile} - \vec{x}^q_{mobile})\right]$$
$$\times \exp\left[+j\vec{k}^m_{base}\cdot(\vec{x}^r_{base} - \vec{x}^s_{base})\right]\Big\}$$

Define

$$\vec{x}^r_{base} - \vec{x}^s_{base} = d^{rs}_{base}(\cos\xi^{rs}_{base}, \sin\xi^{rs}_{base}, 0) \qquad (16)$$
$$\vec{x}^p_{mobile} - \vec{x}^q_{mobile} = d^{pq}_{mobile}(\cos\xi^{pq}_{mobile}, \sin\xi^{pq}_{mobile}, 0)$$

The term d^{rs}_{base} corresponds to the distance between the r^{th} and s^{th} array elements at the base and ξ^{rs}_{base} corresponds to the angle between the line joining the array elements and the x-axis. Similarly, d^{pq}_{mobile} corresponds to the distance between the p^{th} and q^{th} array elements at the mobile and ξ^{pq}_{mobile} corresponds to the angle between the line joining the array elements and the x-axis.

Utilizing (16) and $\cos\alpha_m = -\cos(\gamma - \theta_m)$, (15) becomes

$$E\Big\{h_{p,r}(t)h^*_{q,s}(t+\Delta t)\Big\} = \exp(-j2\pi f_c \Delta t) \qquad (17)$$
$$E\Big\{\sum_{m=0}^{K-1} A_m^2 \exp\left[-j2\pi f_d \Delta t \cos(\theta_m - \gamma)\right]$$
$$\times \exp\left[+j\frac{2\pi}{\lambda}d^{rs}_{base}\cos(\phi_m - \xi^{rs}_{base})\right]$$
$$\times \exp\left[+j\frac{2\pi}{\lambda}d^{pq}_{mobile}\cos(\theta_m - \xi^{pq}_{mobile})\right]\Big\}$$

where $f_d = f_c\frac{|\vec{v}|}{c}$ corresponds to the maximum Doppler shift associated with the mobile. Given the array geometry at the mobile and the base station, the velocity vector associated with the mobile, and the joint probability density for A_m, ϕ_m, and θ_m, (17) can be used to compute the desired space-time correlation.

A special case of the previous result is also of interest. Consider the case for which most of the scatterers are in the vicinity of the mobile. From the perspective of the base station, the angular spread of the multipath is small. Define $d = |\vec{x}^0_{mobile} - \vec{x}_{base}|$ and $R_m = |\vec{x}^0_{mobile} - \vec{x}_m|$. d is the distance between the initial mobile location and the base and R_m corresponds to the scattering radius associated with the m^{th} scatterer. If $d \gg R_m$ then the angle $\phi_m \approx \frac{R_m}{d}\sin\theta_m$ and small angle approximations may be used for $\sin\phi_m$ and $\cos\phi_m$. Due to space restrictions the resulting expression for the space-time correlation is not presented for this case.

Equation (17) is now evaluated for the case of the 'circular ring' scattering model attributed to Jakes [5] and small angular spread. While there are other scattering models that are based upon measurements and are more realistic, see [16],[17], for example, the Jakes model yields a closed-form expression for the complex path correlation. For the Jakes model the radius of each scatterer is fixed, i.e. $R_m = R$, and the angle of arrival θ_m is independent for each scatterer and uniformly distributed on $(-\pi, \pi)$. It is further assumed that the scatterer amplitude is fixed, i.e. $A_m = A$. With these assumptions, evaluating the expectation in (17) yields

$$E\Big\{h_{p,r}(t)h^*_{q,s}(t+\Delta t)\Big\} = MA^2 \exp\left(-j2\pi f_c \Delta t\right) \tag{18}$$
$$\times \exp\left[j2\pi\left(\frac{d^{rs}_{base}}{\lambda}\cos\xi^{rs}_{base}\right)\right]$$
$$\times J_0\left(2\pi\left[\left(\frac{d^{rs}_{base}}{\lambda}\frac{R}{d}\sin\xi^{rs}_{base} + \frac{d^{pq}_{mobile}}{\lambda}\sin\xi^{pq}_{mobile}\right.\right.\right.$$
$$\left.\left.\left. - f_d\Delta t\sin\gamma\right)^2 + \left(\frac{d^{pq}_{mobile}}{\lambda}\cos\xi^{pq}_{mobile} - f_d\Delta t\cos\gamma\right)^2\right]^{1/2}\right)$$

where $J_0(\cdot)$ denotes the zeroth-order Bessel function. This result is in agreement with that derived earlier in [10] and [9] for the special case of isotropic scattering.

IV. Experimental Results

In this section we evaluate the worst-case block error probability using the Jakes 'circular ring' scattering model for several space-time codes employing two and four transmit antennas and up to three receive antennas. Linear array geometry was employed at the base and mobile for all results. Variations in both spatial and temporal correlation are considered and the results are compared to the case of an uncorrelated (space and time) channel. The scattering radius for the Jakes model was varied from R=10,50,200m and the distance between the mobile and base (array phase centers) was fixed at d=1000m. The smallest value for the scattering radius yields the ratio $R/d = 0.01$ and corresponds to angular spread due to multipath of approximately 1° from the perspective of the base station. The mobile location was broadside to the base antenna array and its velocity was chosen such that the maximum Doppler frequency was approximately f_d=78Hz corresponding to a carrier frequency of 850MHz and a maximum speed of 100km/hr. The space-time symbol period T_s was chosen such that the normalized Doppler frequency was f_dT_s=0.0033 corresponding to quasi-static fading with a symbol to fading ratio of approximately 300:1.

The space-time block codes investigated include the orthogonal code [1],[2],[3], the orthogonal code with sphere packing [12], the diagonal algebraic code [18],[19] and the cyclic code [20].

This section presents results for the quasi-static (f_dT_s=0.0033) and uncorrelated wireless channels with variations in spatial correlation due to transmit antenna spacing, receive antenna spacing and scattering radius for the Jakes model. Results for two transmit antennas are presented first followed by results for four transmit antennas.

A. 2 Transmit Antennas

For 2 transmit antennas the orthogonal code due to Alamouti [1] was used with a 16-QAM symbol constellation. For the diagonal algebraic code we also chose 16-QAM symbols and the unitary rotation matrix was chosen to be

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & e^{j\pi/4}\\ 1 & -e^{j\pi/4}\end{pmatrix} \tag{19}$$

For all space-time codes the spectral efficiency was 4 bits/s/Hz.

Figure 2 shows the worst-case block error probability versus signal to noise ratio and scattering radius for 2 transmit antennas ($\lambda/2$ spacing) and 1 receive antenna. The normalized Doppler frequency for this case was f_dT_s=0.0033, representing quasi-static fading. To achieve a block error probablity of 10^{-2} for the uncorrelated channel approximately 26.4dB signal to noise ratio is required for the diagonal algebraic code. The orthogonal code and orthogonal code with sphere packing realize performance improvements of 1.4dB and 1.8dB, respectively, over the diagonal algebraic code for the uncorrelated channel. For a scattering radius of R=10m, approximately 38.4dB signal to noise ratio is required to achieve a block error probability of 10^{-2} for the diagonal algebraic code. The orthogonal code and orthogonal code with sphere packing yield improvements of 0.4dB and 0.6dB, respectively, for this case. These results highlight the dependence of space-time coding performance on spatial correlation for the quasi-static channel. Fractional wavelength antenna spacing at the transmitter and small scattering radius yield transmission paths that are highly correlated and result in degraded performance relative to the uncorrelated channel. Increasing the spacing of the transmit antennas mitigates this effect to a certain extent. It was found that an antenna spacing of 30λ is required to achieve performance within 0.5dB of the uncorrelated channel for 10^{-2} block error probability. For a carrier frequency of 850MHz the transmitted wavelength is λ=0.35m.

Figure 3 shows the results for 2 transmit antennas (5λ spacing) and 2 receive antennas ($\lambda/2$ spacing) and f_dT_s=0.0033. A signal to noise ratio of 17.3dB is required to achieve a block error probability of 10^{-2} for the diagonal algebraic code and an uncorrelated channel. The orthogonal code and orthogonal code with sphere packing achieve gains of 0.6dB and 1.1dB, respectively, over the diagonal algebraic code for the uncorrelated channel. For the channel with scattering radius R=10m the required signal to noise ratios to achieve 10^{-2} block error probability are 21.2, 21.0 and 20.6dB, respectively, for the diagonal algebraic code, orthogonal code, and orthogonal code with sphere packing.

B. 4 Transmit Antennas

For the case of 4 transmit antennas we investigated three space-time codes having a spectral efficiency of 2 bits/s/Hz. These codes are: the orthogonal code with sphere packing [12], the cyclic code [20], and the diagonal algebraic code with unitary rotation matrix

$$\frac{1}{2}\begin{pmatrix}1 & e^{j\pi/8} & e^{j2\pi/8} & e^{j3\pi/8}\\ 1 & -e^{j\pi/8} & e^{j2\pi/8} & -e^{j3\pi/8}\\ 1 & je^{j\pi/8} & -e^{j2\pi/8} & -je^{j3\pi/8}\\ 1 & -je^{j\pi/8} & -e^{j2\pi/8} & je^{j3\pi/8}\end{pmatrix} \tag{20}$$

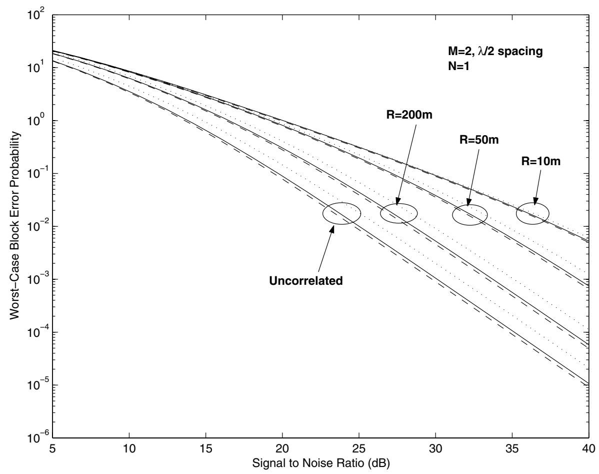

Fig. 2. Orthogonal code with 16-QAM symbols (solid curve), orthogonal code with sphere packing (dashed curve), diagonal algebraic code (dotted curve). Worst-case block error probability versus signal to noise ratio and scattering radius, 2 transmit antennas ($\lambda/2$ spacing), 1 receive antenna, f_dT_s = 0.0033.

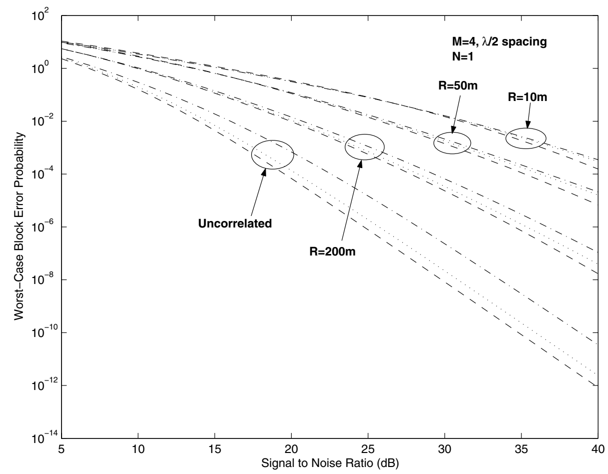

Fig. 4. Orthogonal code with sphere packing (dashed curve), diagonal algebraic code (dotted curve), cyclic code (dash-dotted curve). Worst-case block error probability versus signal to noise ratio and scattering radius, 4 transmit antennas ($\lambda/2$ spacing), 1 receive antenna, f_dT_s = 0.0033.

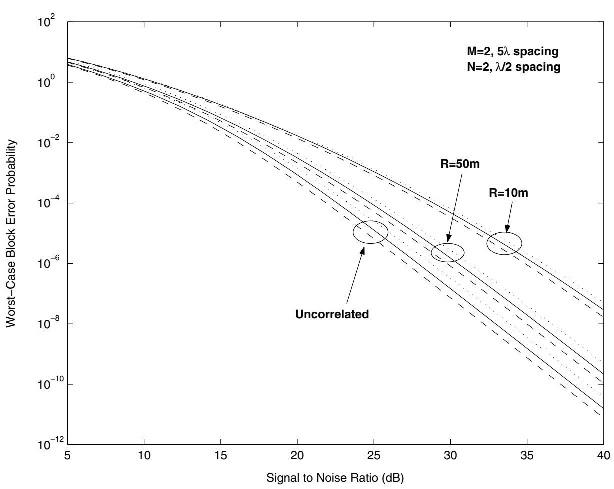

Fig. 3. Orthogonal code with 16-QAM symbols (solid curve), orthogonal code with sphere packing (dashed curve), diagonal algebraic code (dotted curve). Worst-case block error probability versus signal to noise ratio and scattering radius, 2 transmit antennas (5λ spacing), 2 receive antennas ($\lambda/2$ spacing), f_dT_s = 0.0033.

and QPSK signal constellation. Figure 4 shows the worst-case block error probability versus signal to noise ratio and scattering radius for 4 transmit antennas ($\lambda/2$ spacing) and 1 receive antenna. The normalized Doppler frequency for this case was f_dT_s=0.0033, representing quasi-static fading. To achieve a block error probablity of 10^{-4} for the uncorrelated channel a signal to noise ratio of approximately 22.6dB is required for the cyclic code. The diagonal algebraic and the orthogonal code with sphere packing realize performance improvements of 2.2dB and 3.0dB, respectively, over the cyclic code for the uncorrelated channel. For a scattering radius of R=10m, approximately 42.8dB signal to noise ratio is required to achieve a block error probability of 10^{-4} for the cyclic code. The diagonal algebraic and the orthogonal code with sphere packing yield improvements of 0.4dB and 2.0dB, respectively, for this case. Note that the differences in performance of the various space-time codes diminishes as the scattering radius becomes small. Also note that roughly 20dB additional signal to noise ratio is required to maintain a block error probability of 10^{-4} for a scattering radius of R=10m compared with the uncorrelated channel. Figure 5 shows the worst-case block error probability versus signal to noise ratio and transmit antenna spacing for scattering radius R=10m and normalized Doppler frequency f_dT_s=0.0033. It was found that a transmit antenna spacing of 40λ was required to achieve performance within 0.5dB of that for the uncorrelated channel at a block error probability of 10^{-4}.

Figure 6 shows the results for 2 receive antennas ($\lambda/2$ spacing) and 4 transmit antennas (5λ spacing) for f_dT_s=0.0033 and scattering radius R=10,50,200m and the uncorrelated channel. For the case of 2 receive antennas the cyclic code achieves a block error probability of 10^{-4} at a signal to noise ratio of 14.4dB for the uncorrelated channel. A performance improvement of 1.8dB and 2.1dB, respectively, is observed for the diagonal algebraic code and orthogonal code with sphere packing for the uncorrelated channel. For the case of 3 receive antennas (not shown) the cyclic code achieves a block error probability of 10^{-4} at a signal to noise ratio of 10.9dB for the uncorrelated channel. A performance improvement of 1.3dB and 1.4dB, respectively, is observed for the diagonal algebraic code and orthogonal code with sphere packing for the uncorrelated channel.

V. Conclusions

We have investigated the robustness of several space-time codes for wireless channels that exhibit both spatial and temporal correlation. The best-case wireless channel for all space-time codes was uncorrelated in space and time.

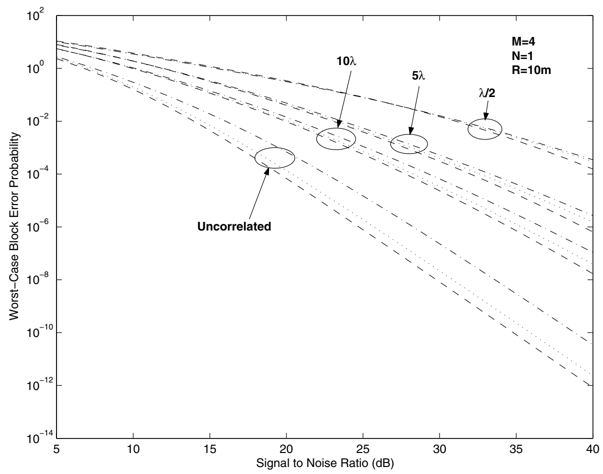

Fig. 5. Orthogonal code with sphere packing (dashed curve), diagonal algebraic code (dotted curve), cyclic code (dash-dotted curve). Worst-case block error probability versus signal to noise ratio and transmit antenna spacing, 4 transmit antennas ($\lambda/2$ spacing), 1 receive antenna, $f_d T_s$ = 0.0033, R=10m.

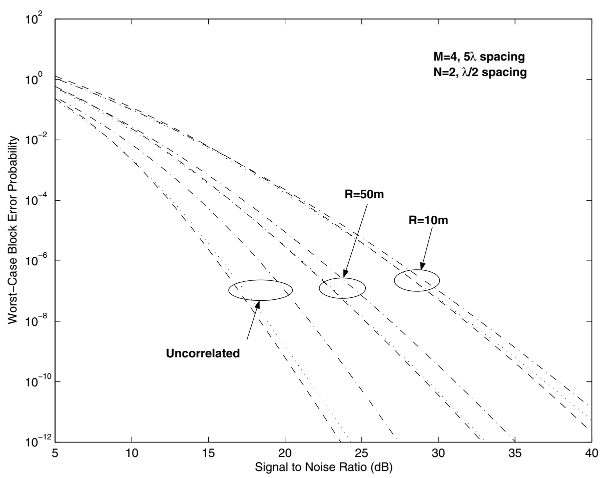

Fig. 6. Orthogonal code with sphere packing (dashed curve), diagonal algebraic code (dotted curve), cyclic code (dash-dotted curve). Worst-case block error probability versus signal to noise ratio and scattering radius, 4 transmit antennas (5λ spacing), 2 receive antennas ($\lambda/2$ spacing), $f_d T_s$ = 0.0033.

For the quasi-static wireless channel ($f_d T_s$ = 0.0033), spatial correlation caused by fractional wavelength spacing at the transmitter or scatterers located in close proximity to the mobile, resulted in significant performance degradation. For example, for the case of 2 transmit antennas there was a 13dB difference in signal-to-noise ratio required to achieve 10^{-2} block error probability for the uncorrelated channel compared to the channel with scattering radius R=10m for $\lambda/2$ transmit antenna spacing. It was found that increasing the spacing of transmit antennas to 30λ (10.5m) yielded performance within 0.5dB of that for the uncorrelated channel for all space-time codes. For the case of 4 transmit antennas there was a 21dB difference in signal-to-noise ratio required to achieve 10^{-4} block error probability for the uncorrelated channel compared to the channel with scattering radius R=10m for $\lambda/2$ transmit antenna spacing. For this case it was found that increasing the spacing of transmit antennas to 40λ (14.0m) yielded performance within 0.5dB of that for the uncorrelated channel for all space-time codes.

REFERENCES

[1] S. Alamouti, "A Simple Transmit Diversity Technique for Wireless Communications," *IEEE Journal on Selected Areas in Communications*, Vol. 16, No. 8, October 1998.
[2] V. Tarokh et al., "Space-Time Block Coding for Wireless Communications: Performance Results," *IEEE Journal on Selected Areas in Communications*, Vol. 17, No. 3, March 1999.
[3] V. Tarkokh et al., "Space-Time Block Codes from Orthogonal Designs," *IEEE Transactions on Information Theory*, Vol. 45, No. 5, July 1999.
[4] J. Wang, M.K. Simon, M.P. Fitz and K. Yao, "On the Performance of Space-Time Codes over Correlated Rayleigh Fading Channels," *IEEE Transactions on Communications*, to appear.
[5] W.C. Jakes, *Microwave Mobile Communications*, IEEE-Press, 1974.
[6] R.H. Clarke, "A Statistical Theory of Mobile-Radio Reception," *Bell System Technical Journal*, pp.957-1000, July-Aug. 1968.
[7] T.A. Chen et al. "A Space-Time Model for Frequency Nonselective Rayleigh Fading Channels with Applications to Space-Time Modems," *IEEE Journal on Selected Areas in Communications*, Vol. 18, No. 7, July 2000.
[8] D.S. Shiu et al. "Fading Correlation and Its Effect on the Capacity of Multielement Antenna Systems," *IEEE Transactions on Communications*, Vol. 48, No. 3, March 2000.
[9] A. Abdi, M. Kaveh "A Space-Time Correlation Model for Multielement Antenna Systems in Mobile Fading Channels," *IEEE Journal on Selected Areas in Communications*, Vol. 20, No. 3, April 2002.
[10] Z. Safar and K.J.R. Liu "Space-Time Correlation of MIMO Flat Rayleigh Fading Channels," in *EUSIPCO 2002*, Toulouse France, Sept. 2002.
[11] W. Su and X.-G. Xia, "On Space-Time Block Codes from Complex Orthogonal Designs," *Wireless Personal Communications*, Vol. 25, No. 1, pp. 1-26, 2003.
[12] W. Su, Z. Safar and K.J.R. Liu, "Space-Time Signal Design for Time-Correlated Rayleigh Fading Channels," in *Proceedings of the IEEE International Conference on Communications*, Vol. 5, pp. 3175-3179, May 2003.
[13] J.W. Craig, "A New, Simple and Exact Result for Calculating the Probability of Error for Two-Dimensional Signal Constellations," in *IEEE MILCOM Conf. Rec.*, Boston, MA, 1991.
[14] G.L. Turin, "The Characterisitic Function of Hermitian Quadratic Forms in Complex Normal Variables," *Biometrika*, pp. 199-201, 1960.
[15] S. Siwamogsatham, M.P. Fitz and J.H. Grimm, "A New View of Performance Analysis of Transmit Diversity Schemes in Correlated Rayleigh Fading," *IEEE Transactions on Information Theory*, Vol. 48, No. 4, Apr. 2002.
[16] K. Pedersen, P. Mognesen, and B. Fleury, "A Stochastic Model of the Temporal and Azimuthal Dispersion Seen at the Base Station in Outdoor Propagation Environments," *IEEE Transactions on Vehicular Technology*, Vol. 49, No. 2, pp. 437-447, Mar. 2000.
[17] R. Janaswamy, "Angle and Time of Arrival Statistics for the Gaussian Scatter Density Model," *IEEE Transactions on Wireless Communications*, Vol. 1, No. 3, July 2002.
[18] Y. Xin, Z. Wang and G. Giannakis, "Space-time Diversity Systems Based On Linear Constellation Precoding," *IEEE Transactions on Wireless Communications*, Vol. 2, No. 2, pp. 294-309, March 2003.
[19] M.O. Damen, K. Abed-Meraim and J.-C. Belfiore, "Diagonal Algebraic Space-Time Block Codes," *IEEE Transactions on Information Theory*, Vol. 48, No. 3, pp. 628-636, March 2002.
[20] B. M. Hochwald and W. Sweldens, "Differential Unitary Space-Time Modulation," *IEEE Transactions on Communications*, Vol. 48, No. 12, pp. 2041-2052, December 2000.