A CLASS OF COOPERATIVE COMMUNICATION PROTOCOLS FOR MULTI-NODE WIRELESS NETWORKS

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ABSTRACT

In this paper, a class of cooperative communication protocols with arbitrary N-relay nodes is proposed for wireless networks, in which each relay coherently combines the signals received from the source and m $(1 \leq m \leq N-1)$ previous relays. Exact symbol-error-rate (SER) expressions for an arbitrary N-node network employing MPSK or QAM modulation are derived. Also an approximate expression for the SER, that is tight at high SNR, is provided. Furthermore, an optimal power allocation problem based on minimizing the asymptotically tight SER expression is formulated. Closed form solutions for the optimal power allocation problem are provided for some network topologies.

1. INTRODUCTION

Spatial diversity has been studied intensively in the context of point-to-point communications, where it is introduced by utilizing multiple-input-multiple-output (MIMO) systems. On the other hand, in wireless networks, e.g., cellular and ad-hoc networks, it might not be feasible to have multiple antennas installed at the terminals due to space limitations. To overcome this problem, and to benefit from the performance enhancement introduced by MIMO systems, the concept of cooperative diversity in wireless networks has been recently introduced [1, 2, 3, 4, 5, 6].

In [1] and [2], Laneman *et al.* proposed different cooperative diversity protocols and analyzed their performance in terms of outage behavior. Terminologies other than cooperative diversity are also used in the research community to refer to the same concept of achieving spatial diversity via forming virtual antennas. User cooperation diversity was introduced by Sendonaris *et al.* in [3] and [4]. In this two-part series of papers, the authors implemented a two-user CDMA cooperative system, where both users are active and use orthogonal codes to avoid multiple access interference. In [7], Boyer *et al.* introduced the concept of multihop diversity, where each relay combines the signals received from all of the previous transmissions. This kind of spatial diversity is specially applicable in multihop ad-hoc networks. The authors in [7] assumed that an error at any intermediate relay results in an error at the final destination, and through this assumption they derived upper bounds on the probability of outage and error performance of the system. These calculations were done only for systems with binary phase-shift keying (BPSK) modulation.

In this paper, we propose a class of cooperative decode-and-forward protocols for arbitrary N-relay wireless networks, in which each relay can combine the signal received from the source along with one or more of the signals transmitted by previous relays. We use cyclic-redundancy-check (CRC) [8] codes to encode the transmitted information such that each receiving node can judge whether it decodes the information correctly or not which is different from [7]. We refer to the cooperative protocol in which each relay combines the signals received from the previous m relays along with that from the source as $\mathcal{C}(m)$, where $1 \leq m \leq N-1$. Note that the multihop diversity scheme introduced in [7] is similar to the scheme $\mathcal{C}(N-1)$ we are considering if without CRC at each relaying node. First, we analyze the performance of general cooperation scenario $\mathcal{C}(m)$, $1 \leq m \leq N-1$, and provide exact symbol-error-rate (SER) expressions for both MPSK and QAM signalling. We also provide an approximate expression for the SER at high SNR. Our theoretical analysis reveals a very interesting result: The class of proposed cooperative protocols $\{\mathcal{C}(m)\}_{m=1}^{N-1}$ shares the same asymptotic performance at high enough SNR. Hence, a simple cooperative protocol for a multi-node network in which each relay combines the signals received from the source and the previous relay, namely $\mathcal{C}(1)$, has the same asymptotic performance as a much more complicated scenario, in which each relay combines the signals received from the source and all of the previous relays, namely $\mathcal{C}(N-1)$. Finally, we study optimal power allocation for the proposed class of cooperative diversity schemes. We show that the optimal power allocation follows a certain ordering in which the source is allocated the largest amount of power and the first relay has the least power allocation ratio. Closed form solutions of optimal power allocation for some network topologies of practical interest are provided.

2. SYSTEM MODEL AND PROTOCOL DESCRIPTION

We consider an arbitrary N-relay wireless network, where information is to be transmitted from a source to a destination. Due to the broadcast nature of the wireless channel, some relays can overhear the transmitted information and thus can cooperate with the source to send its data. The wireless link between any two nodes in the network is modeled as a Rayleigh fading narrowband channel with additive white Gaussian noise (AWGN) with variance $\mathcal{N}_o$. The channel fades for different links are assumed to be statistically independent. For medium access, the relays are assumed to transmit over orthogonal channels.

The cooperation strategy we are considering employs a decode-and-forward protocol at the relaying nodes. In each phase of the cooperation protocol, if the node decodes correctly, it retransmits the information to the destination, otherwise it remains idle. We assume that an ideal CRC [8] code is applied over the information transmitted in any phase of the protocol, such that the receiver can judge whether it can correctly decode the information or not. This is different from the cooperation scheme proposed in [7], which

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assumes that a decoding error at any intermediate terminal results in an error at the destination. Various scenarios for the cooperation among the relays can be implemented. A general cooperation scenario, denoted as $\mathcal{C}(m)$ $(1 \le m \le N-1)$, can be implemented in which each relay combines the signals received from the m previous relays along with that received from the source.

For a general scheme $\mathcal{C}(m)$, $1 \le m \le N-1$, each relay decodes the information after combining the signals received from the source and the previous m relays. The cooperation protocol has $(N+1)$ Phases. In Phase 1, the source transmits the information, and the received signal at the destination and the i-th relay can be modeled respectively as

$$y_{s,d} = \sqrt{P_0}h_{s,d}x + n_{s,d}, \tag{1}$$

$$y_{s,r_i} = \sqrt{P_0}h_{s,r_i}x + n_{s,r_i}, \quad 1 \le i \le N, \tag{2}$$

where P_0 is the power transmitted at the source, x is the transmitted symbol with unit power, $h_{s,d} \sim CN(0, \sigma^2_{s,d})$ and $h_{s,r_i} \sim CN(0, \sigma^2_{s,r_i})$ are the channel fading coefficients between the source and the destination, and i-th relay, respectively, and $CN(\alpha, \sigma^2)$ denotes a circularly symmetric complex Gaussian random variable with mean α and variance σ^2. The terms $n_{s,d}$ and n_{s,r_i} denote the AWGN. In Phase 2, if the first relay correctly decodes, it forwards the decoded symbol with power P_1 to the destination, otherwise it remains idle. Generally in Phase l, $2 \le l \le N$, the l-th relay combines the received signals from the source and the previous $\min\{m, l-1\}$ relays using a maximal-ratio-combiner (MRC) as

$$y_{r_l} = \sqrt{P_0}h^*_{s,r_l}y_{s,r_l} + \sum_{i=\max(1,l-m)}^{l-1} \sqrt{\hat{P}_i}h^*_{r_i,r_l}y_{r_i,r_l}, \tag{3}$$

where $h_{r_i,r_l} \sim CN(0, \sigma^2_{r_i,r_l})$ is the channel gain between the i-th and the l-th relays. In (3), y_{r_i,r_l} denotes the signal received at the l-th relay from the i-th relay, and can be modeled as

$$y_{r_i,r_l} = \sqrt{\hat{P}_i}h_{r_i,r_l}x + n_{r_i,r_l}, \tag{4}$$

where $\hat{P}_i$ is the power transmitted at relay i in Phase $(i+1)$, and $\hat{P}_i = P_i$ if relay i correctly decodes the transmitted symbol, otherwise $\hat{P}_i = 0$. The l-th relay uses y_{r_l} in (3) as the detection statistics. If relay l decodes correctly it transmits with power $\hat{P}_l = P_l$ in Phase $(l+1)$, otherwise it remains idle. Finally, in Phase $(N+1)$, the destination coherently combines all of the received signals using an MRC as follows

$$y_d = \sqrt{P_0}h^*_{s,d}y_{s,d} + \sum_{i=1}^{N} \sqrt{\hat{P}_i}h^*_{r_i,d}y_{r_i,d}. \tag{5}$$

In all the cooperation scenarios considered, the total transmitted power is fixed as $P_0 + \sum_{i=1}^{N} P_i = P$.

3. EXACT SER PERFORMANCE ANALYSIS

In this section, we present SER performance analysis for a general cooperative scheme $\mathcal{C}(m)$ for any $1 \le m \le N-1$. First, we introduce some terminologies that will be used throughout the paper. For a given transmission, each relay can be in one of two states: either it decoded correctly or not. Let us define a $1 \times n$, $1 \le n \le N$, vector $\mathbf{S}_n$ to represent the states of the first n relays for a given transmission. The k-th entry of the vector $\mathbf{S}_n$ denotes the state of the k-th relay, $1 \le k \le n$, as follows

$$S_n[k] = \begin{cases} 1 & \text{if relay } k \text{ correctly decodes,} \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Since the decimal value of the binary vector $\mathbf{S}_n$ can take on values from 0 to $2^n - 1$, for convenience we denote the state of the network by an integer decimal number. Let $\mathbf{B}_{x,n}$ be the $1 \times n$ binary representation of a decimal number x, with $B_{x,n}[1]$ being the most significant bit. So, $\mathbf{S}_N = \mathbf{B}_{x,N}$ indicates that the k-th relay, $1 \le k \le N$, is in state $S_N[k] = B_{x,N}[k]$.

We consider a general cooperation scheme $\mathcal{C}(m)$, $1 \le m \le N-1$, in which the k-th $(1 \le k \le N)$ relay coherently combines the signals received from the source along with the signals received from the previous $\min\{m, k-1\}$ relays. The state of each relay in this scheme depends on the states of the previous m relays, i.e., whether these relays decoded correctly or not. This is due to the fact that the number of signals received at each relay depends on the number of relays that decoded correctly from the previous m relays. Hence, the joint probability of the states is given by

$$\begin{aligned} P(\mathbf{S}_N) = &P(S_N[1]) \cdot P(S_N[2] \mid S_N[1]) \cdots \\ &\cdot P(S_N[N] \mid S_N[N-1], \cdots, S_N[N-m]). \end{aligned} \tag{7}$$

Conditioning on the network state, which can take on 2^N values, the probability of error at the destination given the channel state information (CSI) can be calculated using the law of total probability as follows

$$P_{e|CSI} = \sum_{i=0}^{2^N-1} Pr(e \mid \mathbf{S}_N = \mathbf{B}_{i,N}) Pr(\mathbf{S}_N = \mathbf{B}_{i,N}), \tag{8}$$

where e denotes the event that the destination decoded in error, and the above summation is over all possible network states.

Now, let us compute the terms in (8). The destination collects the copies of the signal transmitted in the previous phases using a MRC (5). The resulting SNR at the destination can be computed as

$$SNR_d = \frac{P_0 \mid h_{s,d} \mid^2 + \sum_{j=1}^{N} P_j B_{i,N}[j] \mid h_{r_j,d} \mid^2}{\mathcal{N}_o}, \tag{9}$$

where $B_{i,N}[j]$ takes value 1 or 0 and determines whether the j-th relay has decoded correctly or not. The k-th relay coherently combines the signals received from the source and the previous m relays. The resulting SNR can be calculated as

$$SNR^m_{r_k} = \frac{P_0 \mid h_{s,r_k} \mid^2 + \sum_{j=max(1,k-m)}^{k-1} P_j B_{i,N}[j] \mid h_{r_j,r_k} \mid^2}{\mathcal{N}_o}. \tag{10}$$

If M-PSK modulation is used in the system, with instantaneous SNR γ, the SER given the channel state information is given by [9]

$$P^{PSK}_{CSI} = \Psi_{PSK}(\gamma) \triangleq \frac{1}{\pi}\int_0^{(M-1)\pi/M} \exp\left(-\frac{b_{PSK}\gamma}{\sin^2(\theta)}\right) d\theta, \tag{11}$$

where $b_{PSK} = \sin^2(\pi/M)$. If M-QAM ($M = 2^k$ with k even) modulation is used in the system, the corresponding conditional SER can be expressed as [9]

$$P^{QAM}_{CSI} = \Psi_{QAM}(\gamma) \triangleq 4CQ(\sqrt{b_{QAM}\gamma}) - 4C^2Q^2(\sqrt{b_{QAM}\gamma}), \tag{12}$$

in which $C = 1 - \frac{1}{\sqrt{M}}$, $b_{QAM} = 3/(M-1)$, and $Q(x)$ is the complementary distribution function (CDF) of the Gaussian distribution, and is defined as $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp(-\frac{t^2}{2})dt$.

Let us focus on computing the SER in the case of M-PSK modulation, and the same procedure is applicable for the case of M-QAM modulation. From (9), and for a given network state $\mathbf{S}_N = \mathbf{B}_{i,N}$, the conditional SER at the destination can be computed as

$$Pr(e|\mathbf{S}_N = \mathbf{B}_{i,N}) = \Psi_{PSK}(SNR_d). \tag{13}$$

Denote the conditional probability that the k-th relay is in state $B_{i,N}[k]$ given the states of the previous m relays by $P^m_{k,i}$. From (10), this probability can be computed as follows

$$P^m_{k,i} = \begin{cases} \Psi_{PSK}(SNR^m_{r_k}), & \text{if } B_{i,N}[k] = 0, \\ 1 - \Psi_{PSK}(SNR^m_{r_k}), & \text{if } B_{i,N}[k] = 1. \end{cases} \tag{14}$$

To compute the average SER, we need to average the probability in (8) over all channel realizations, i.e., $P_{SER}(m) = E_{CSI}\left[P_{e|CSI}\right]$. Using (7), (13), and (14), $P_{SER}(m)$ can be expanded as follows

$$P_{SER}(m) = \sum_{i=0}^{2^N-1} E_{CSI}\left[\Psi_{PSK}(SNR_d) \prod_{k=1}^{N} P^m_{k,i}\right]. \tag{15}$$

Since the channel fades between different pairs of nodes in the network are statistically independent by the virtue that different nodes are not co-located, the quantities inside the expectation operator in the above equation are functions of independent random variables, and thus can be further decomposed as

$$P_{SER}(m) = \sum_{i=0}^{2^N-1} \left(E_{CSI}\left[\Psi_{PSK}(SNR_d)\right] \prod_{k=1}^{N} E_{CSI}\left[P^m_{k,i}\right] \right). \tag{16}$$

The above analysis is applicable to the M-QAM case by changing the function $\Psi_{PSK}(\cdot)$ into $\Psi_{QAM}(\cdot)$. The exact SER can be determined in the following theorem [10] and the proof is omitted for lack of space.

Theorem 1 *The SER of an N-relay decode-and-forward cooperative diversity network utilizing protocol $C(m)$, $1 \leq m \leq N-1$, and M-PSK or M-QAM modulation is given by*

$$P_{SER}(m) = \sum_{i=0}^{2^N-1} F_q\left(\left[1 + \frac{b_q P_0 \sigma^2_{s,d}}{N_o \sin^2(\theta)}\right] \prod_{j=1}^{N} \left[1 + \frac{b_q B_{i,N}[j] P_j \sigma^2_{r_j,d}}{N_o \sin^2(\theta)}\right] \right) \prod_{k=1}^{N} G^m_k(B_{i,N}[k]), \tag{17}$$

where $q = 1$ and $q = 2$ correspond to M-PSK and M-QAM, respectively.

The constants $b_1 = b_{psk}$, $b_2 = \frac{b_{QAM}}{2}$, and the function $F_q(\cdot)$ is defined as

$$F_1(x(\theta)) = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \frac{1}{x(\theta)} d\theta,$$
$$F_2(x(\theta)) = \frac{4C}{\pi}\int_0^{\pi/2} \frac{1}{x(\theta)} d\theta - \frac{4C^2}{\pi}\int_0^{\pi/4} \frac{1}{x(\theta)} d\theta. \tag{18}$$

In (17), $G^m_k(1) = 1 - G^m_k(0)$ and $G^m_k(0) = F_q\left(\left[1 + \frac{b_q P_0 \sigma^2_{s,r_k}}{N_o \sin^2(\theta)}\right] \times \prod_{j=\max(1,k-m)}^{k-1} \left[1 + \frac{b_q B_{i,N}[j] P_j \sigma^2_{r_j,r_k}}{N_o \sin^2(\theta)}\right]\right)$.

We illustrate with some simulation experiments to quantify the theoretical results we obtained. We focused on the cooperative protocol $C(1)$. The network size was taken to be $N = 1, 2, 3$ relays in addition to the source and the destination. The simulation environment is assumed as follows: The channel gain between any two links in the network is considered unity, and the AWGN at all nodes has variance $N_o = 1$. The total transmitted power in each case is considered fixed to P. The performance of direct transmission without relaying is plotted as a benchmark. Figure 1 depicts the SER vs. P/N_o performance with QPSK modulation. It is clear that the analytical SER expression exactly matches the simulation results, which confirms our theoretical analysis.

4. OPTIMAL POWER ALLOCATION

In this section, we try to find the optimal power allocation strategy for the multi-node cooperative scenarios considered in the previous sections. The exact SER expression in (17) is complicated, hence we provide an approximate expression for the SER [10] and the proof is omitted for lack of space.

Theorem 2 *At enough high SNR, the SER of an N relay decode-and-forward cooperative diversity network employing cooperation scheme $C(m)$ and utilizing M-PSK or M-QAM modulation can be approximated by*

$$P_{SER}(m) \simeq \frac{(N_o/P)^{N+1}}{b_q^{N+1}\sigma^2_{s,d}} \sum_{j=1}^{N+1} \frac{g_q(N-j+2)g_q^{j-1}(1)}{a_0^j \prod_{i=j}^{N} a_i \sigma^2_{r_i,d} \prod_{l=1}^{j-1} \sigma^2_{s,r_l}}. \tag{19}$$

where a's are the ratios of the total available power P as follows, $P_0 = a_0 P$ and $P_i = a_i P$, $1 \leq i \leq N$. The function $g_q(\cdot)$ is defined as

$$g_q(x) = \begin{cases} \frac{1}{\pi}\int_0^{(M-1)\pi/M} \sin^{2x}(\theta)d\theta, & q = 1 \\ \frac{4C}{\pi}\left(\int_0^{\pi/2} \sin^{2x}(\theta)d\theta - C\int_0^{\pi/4} \sin^{2x}(\theta)d\theta\right), & q = 2. \end{cases} \tag{20}$$

An interesting point to notice is that the approximate SER expression is independent of the class parameter m. We can show that the approximation in (19) is tight at high enough SNR, thus we use it to determine the asymptotic optimum power allocation, also we drop the parameter m as the asymptotic SER performance is independent of it.

The nonlinear optimization problem can be formulated as follows

$$\mathbf{a}_{opt} = \arg\min_{\mathbf{a}} P_{SER} \tag{21}$$
$$\textbf{subject to} \quad a_i \geq 0 \ (0 \leq i \leq N), \quad \sum_{i=0}^{N} a_i = 1,$$

where $\mathbf{a} = [a_0, a_1, \cdots, a_N]$ is the power allocating vector. Solving the optimality conditions, we can prove the following relations between the powers allocated at different nodes

$$P_0 \geq P_N \geq P_{N-1} \geq \cdots \geq P_1. \tag{22}$$

The above set of inequalities demonstrates an important concept: Power is allocated at different nodes according to the received signal quality at these node. We refer to the quality of the signal

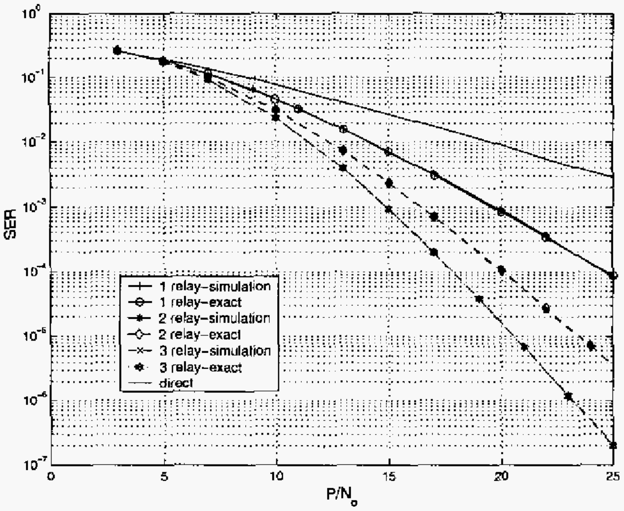

Fig. 1. Comparison between the exact and simulated SER.

copy at a node as the reliability of the node, thus the more reliable the node the more power allocated to this node. To further illustrate this concept, in a virtual array the antenna elements constituting the array (the cooperating nodes) are not allocated at the same place and the channels among them are noisy. The source is the most reliable node as it has the original copy of the signal. According to the cooperation protocol described in Section 2, each relay combines the signal received from the source and the previous relays. As a result, each relay is more reliable than the previous relay, and hence the N-th relay is the most reliable node and the 1-st relay is the least reliable node.

There are a few special cases of practical interest that permits a closed-form solution for the optimization problem in (21), and they are discussed in the sequel.

4.1. Networks with linear topologies

We will take the effect of the geometry on the channel qualities. The channel attenuation between any two nodes $\sigma_{i,j}^2$ depends on the distance between these two nodes $d_{i,j}$ as follows: $\sigma_{i,j}^2 \propto d^{-2\alpha}$, where α is the propagation constant. For a linear network topology, the most significant channel gains are for the channels between the source and the first relay σ_{s,r_1}^2, and that between the last relay and the destination $\sigma_{r_N,d}^2$, the other channel gains are considerably smaller than these two channels. In the SER expression in (19), these two terms appear multiplicated together in all the terms except the first and the last terms. Hence these two terms dominate the SER expression, and we can further approximate the SER in this case as follows

$$P_{SER} \simeq \frac{(N_o/P)^{N+1}}{b_q^{N+1}\sigma_{s,d}^2}\left[\frac{g_q(N+1)}{a_0\prod_{i=1}^{N} a_i\sigma_{r_i,d}^2}+\frac{g_q^{N+1}(1)}{a_0^{N+1}\prod_{i=1}^{N}\sigma_{s,r_i}^2}\right]. \tag{23}$$

The optimal power allocation for a linear network can be found in the following theorem.

Theorem 3 *The optimal power allocation for a linear network that minimizes the SER expression in (23) is as follows*

$$P_0 = \frac{1+\kappa}{1+\kappa+N}P, \qquad P_i = \frac{1}{1+\kappa+N}P, \quad 1 \le i \le N, \tag{24}$$

where κ is found through solving the equation $\kappa(1+\kappa)^N = A$, in which A is a constant given by $(N+1)\frac{g^{N+1}(1)\prod_{i=1}^{N}\sigma_{r_i,d}^2}{g(N+1)\prod_{i=1}^{N}\sigma_{s,r_i}^2}$.

The proof is omitted for space limitations. Theorem 3 agrees with optimality conditions we found for the general problem in (22). Also, it shows an interesting property that in linear network topologies equal power allocation at the relays is asymptotically optimal.

4.2. Relays located near the source or the destination

The cooperating relays can be chosen to be closer to the source than to the destination, in order for the $N+1$ cooperating nodes to mimic a multi-input-single-output (MISO) transmit antenna diversity system. This case is of special interest as it was shown in [11] that decode-and-forward relaying can be a capacity achieving scheme when the relays are taken to be closer to the source and it has the best performance compared to amplify-and-forward and compress-and-forward relaying in this case. In order to model this scenario in our SER formulation, we will consider the channel gains from the source to the relays have higher gains than those from the relays to the destination, i.e., $\sigma_{s,r_i}^2 \gg \sigma_{r_i,d}^2$ for $1 \le i \le N$. Taking this into account, the approximate SER expression in (19) can be further approximated as

$$P_{SER} \simeq \frac{N_o^{N+1} g_q(N+1)}{b_q^{N+1}\sigma_{s,d}^2 P^{N+1} a_0 \prod_{i=1}^{N} a_i\sigma_{r_i,d}^2}. \tag{25}$$

It is clear from the above equation that the SER depends equally on the power allocated to all nodes including the source, and thus the optimal power allocation strategy for this case is simply given by

$$P_0 = P_i = \frac{P}{N+1}, \qquad 1 \le i \le N. \tag{26}$$

This result is intuitively meaningful as all the relays are located near to the source and thus they all have high reliability and are allocated equal power as if they form a conventional antenna array.

Now we consider the opposite scenario in which all the relays are located near the destination. In this case the channels between the relays and the destination are of a higher quality, higher gain, than those between the source and the relays, i.e., $\sigma_{r_i,d}^2 \gg \sigma_{s,r_i}^2$ for $1 \le i \le N$. In this case the SER can be approximated as

$$P_{SER} \simeq \frac{N_o^{N+1} g(1)^{N+1}}{b_q^{N+1}\sigma_{s,d}^2 P_0^{N+1}\prod_{k=1}^{N}\sigma_{s,r_k}^2}. \tag{27}$$

The SER in the above equation is not a function of the power allocated at the cooperating relays, and thus the optimal power allocation in this case is simply $P_0 = P$, i.e., allocating all the available power at the source. This result is interesting as it reveals a very important concepts: If the relays are located closer to the destination than to the transmitter then direct transmission can lead better performance than decode-and-forward relaying. This is also consistent with the results in [11] which show that the performance of the decode-and-forward strategy degrades significantly when the

Exhaustive Search	Analytical Results
$P_0 = 0.421P$	$P_0 = 0.42P$
$P_1 = 0.286P$	$P_1 = 0.29P$
$P_2 = 0.293P$	$P_2 = 0.29P$

Table 1. $N = 2$ relays, uniform network topology.

Exhaustive Search	Analytical Results
$P_0 = 0.3353P$	$P_0 = 0.333P$
$P_1 = 0.331P$	$P_1 = 0.333P$
$P_2 = 0.333P$	$P_2 = 0.333P$

Table 2. $N = 2$ relays, all relays near the source.

relays get closer to the destination . This result can be intuitively interpreted as follows: The farther the relays from the source the more noisy the channels between them and the less reliable the signals received by those relays to the extent that we can not rely on them on forwarding copies of the signal to the destination.

4.3. Numerical Examples

In this subsection, we present some numerical results to verify the analytical results for the optimal power allocation problem for the considered network topologies. The propagation constant α is determined by the environment and is taken equal to 2 throughout our simulations. We provide comparisons between the optimal power allocation via exhaustive search to minimize the SER expression in (19), and optimal power allocation provided by the closed form expressions provided in this section, for $N = 2$ relays.

Table 1 demonstrates the results for a linear network topology, where $d_{s,r_1} = d_{r_1,r_2} = \cdots = d_{r_N,d}$. The variance of the direct link between the source and the destination is taken to be $\sigma_{s,d}^2 = 1$. Second, for the case when all the relays are near the source, the channel links are taken to be: $\sigma_{s,r_i}^2 = \sigma_{r_i,r_j} = 10$, while $\sigma_{s,d}^2 = \sigma_{r_i,d}^2 = 0.1$. Table 2 illustrates the results for this case. Finally, for the case when all of the relays are near the destination, the channel link qualities are taken to be: $\sigma_{s,d}^2 = \sigma_{s,r_i}^2 = 0.1$, while $\sigma_{r_i,r_j}^2 = \sigma_{r_i,d}^2 = 10$. Table 3 illustrates the results for this case. In all of the provided numerical examples it is clear that the optimal power allocations obtained via exhaustive search agree with that via analytical results for all the considered scenarios.

5. CONCLUSION

In this paper, we propose a class of cooperative diversity protocols for multi-node wireless networks employing decode-and forward relaying. We derive exact expressions for the SER of a general cooperation scheme for both MPSK and MQAM modulation. Also, we provide approximations for the SER which are shown to be tight at high enough SNR. Our theoretical analysis reveals a very interesting result: This class of cooperative protocols shares the same asymptotic performance at high enough SNR. Moreover, we formulate the optimal power allocation problem, and show that the optimum powers allocated at the nodes for an arbitrary network follow a certain ordering. Furthermore, we provide closed form solutions for the optimal power allocation for some network topologies of practical interest, and we show through numerical examples that the simulation results match with our theoretical analysis.

Exhaustive Search	Analytical Results
$P_0 = 0.8923P$	$P_0 = P$
$P_1 = 0.029P$	$P_1 = 0$
$P_2 = 0.07P$	$P_2 = 0$

Table 3. $N = 2$ relays, all relays near the destination.

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