Matrix Rotation Based Signal Design for Differential Unitary Space-Time Modulation

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***Abstract*— In this paper, we consider the design of Matrix Rotation Based (MRB) space-time signals based on the design criterion of minimizing the union bound on pairwise block error probability ($PBEP$). We further propose to design the signal parameters via non-integer searching to get better signals. Superiority of our improved design over the previous design are demonstrated through numerical calculations and performance simulations. With our proposed design for two transmit antennas and one or two receive antennas, we achieve the coding gain of about 1 dB over that of the previous design.**

I. Introduction

Without knowledge of channel state information at neither transmitter nor receiver, differential unitary space-time (DUST) modulation scheme [6], [7] was proposed for a multiple input multiple output (MIMO) system under slow Rayleigh flat-fading channels. The technique is considered as an extension of the standard single-antenna differential phase shift keying (DPSK) system. The proposed design of unitary space-time signals in [6], [7] is based on minimizing pairwise block error probability ($PBEP$). In particular, it has been shown in [7] that, at asymptotically high signal-to-noise ratio (SNR), the $PBEP$ performance of a good DUST constellation is determined by the so-called diversity product.

Based on the design criterion of maximizing the diversity product, a large number of DUST codes have been proposed, for example, diagonal codes or cyclic group codes in [6], [7], generalized quaternion codes or dicyclic group codes [6], fixed-point-free group codes [9] using representation theory, and a non-group signal constellation called parametric codes [10]. The parametric codes was particularly designed for two transmit antenna systems. Recently, the matrix rotation based (MRB) space-time signals [13], with similar concept as [10], have been proposed for a MIMO system with even number of transmit antennas.

It was argued recently in [14] that the main target of the performance evaluation is block error probability (BEP), not the $PBEP$. The codes optimized over the worst case $PBEP$ do not guarantee the optimum performances in terms of the BEP. Thus, in [14], a code design criterion of minimizing the union bound on $PBEP$ was proposed, and some new cyclic codes were obtained.

In this paper, we design the MRB space-time signals by using the design criterion of minimizing the union bound on $PBEP$. Moreover, we propose to search non-integer parameters for the MRB signal scheme. The search method and computational complexity reduction are also presented. The merit of the proposed design is demonstrated by numerical calculations and performance simulations.

The rest of this paper is organized as follows: Section II outlines the channel model and the DUST modulation scheme. In Section III, we describe our design method and discuss the searching complexity. Section IV illustrates some simulation results. Finally, Section V concludes the paper.

II. Background

A. Channel Model

We consider a MIMO system with M_T transmit and M_R receive antennas. The channel coefficients are assumed constant over T channel coherent time intervals, and unknown to neither the transmitter nor the receiver. The baseband received signal y_t^j at antenna $j = 1, 2, \ldots, M_R$ during time $t = 1, 2, \ldots, T$ is given by

$$y_t^j = \sqrt{\rho} \sum_{i=1}^{M_T} h_{i,j} s_t^i + w_t^j, \tag{1}$$

where s_t^i is the baseband transmitted signal at the i^{th} transmitted antenna at time t, $h_{i,j}$ is the complex fading channel coefficient from the i^{th} transmit antenna to the j^{th} receive antenna, and w_t^j is an additive complex white Gaussian noise at the j^{th} receive antenna at time t. We assume that $h_{i,j}$ and w_t^j are independent, and they are complex Gaussian random variables with zero mean and unit variance, $\mathcal{CN}(0, 1)$. The transmitted signal is normalized to have unit energy during one transmission period to ensure that ρ is the averaged SNR per receiver, i.e.,

$$\mathrm{E}\left[\sum_{i=1}^{M_T} \left|s_t^i\right|^2\right] = 1, \tag{2}$$

where E represents the expectation operator. Considering T successive time slots together, we can remodel the received signal in (1) to a more convenient matrix form as:

$$\mathbf{Y}_\tau = \sqrt{\rho}\mathbf{S}_\tau\mathbf{H}_\tau + \mathbf{W}_\tau, \qquad \tau = 0, 1, \ldots, \tag{3}$$

where τ is the time index of block transmissions, $\mathbf{Y}_\tau$ is the $T \times M_R$ received signal matrix, $\mathbf{S}_\tau$ is the $T \times M_T$ transmitted signal matrix, $\mathbf{H}_\tau$ is the $M_T \times M_R$ fading-coefficient matrix, and $\mathbf{W}_\tau$ is the $T \times M_R$ noise matrix.

B. DUST Modulation Scheme and Exact $PBEP$

We briefly review the DUST modulation system [6], [7] as follows. The transmission process initiates by transmitting a square size transmission matrix $S_0 = I_{M_T \times M_T}$, where $I_{M_T \times M_T}$ is an $M_T \times M_T$ identity matrix. The subsequent transmitted signal stream follows the fundamental differential transmitter equation [7],

$$\mathbf{S}_\tau = \mathbf{\Phi}_{z_\tau}\mathbf{S}_{\tau-1}, \qquad \tau = 1, 2, \ldots, \tag{4}$$

where $z_\tau \in \{0, 1, ..., L-1\}$ denotes an integer index of a distinct unitary matrix signal $\mathbf{\Phi}_{z_\tau}$ drawn from a signal constellation $\mathcal{V}$ of size $L = 2^{RM_T}$, with R representing the information rate in $b/s/Hz$. Observe that the transmitted signal matrix $\mathbf{S}_\tau$ in (4) is also a unitary matrix for $\tau \geq 0$. However, it generally does not belong to $\mathcal{V}$, unless the constellation preserve a group under matrix multiplication.

By (3), the two consecutive received signal matrices at the receiver are

$$\mathbf{Y}_{\tau-1} = \sqrt{\rho}\mathbf{S}_{\tau-1}\mathbf{H}_{\tau-1} + \mathbf{W}_{\tau-1}, \tag{5}$$

$$\mathbf{Y}_\tau = \sqrt{\rho}\mathbf{S}_\tau\mathbf{H}_\tau + \mathbf{W}_\tau. \tag{6}$$

Substituting (4) into (6) and assuming that the channel coefficients are almost constant over two consecutive blocks, i.e., $\mathbf{H}_\tau \approx \mathbf{H}_{\tau-1}$, (5) and (6) are combined into an expression

$$\mathbf{Y}_\tau = \mathbf{\Phi}_{z_\tau}\mathbf{Y}_{\tau-1} + \sqrt{2}\mathbf{W}'_\tau, \tag{7}$$

where $\mathbf{W}'_\tau = \frac{1}{\sqrt{2}}\left(\mathbf{W}_\tau - \mathbf{\Phi}_{z_\tau}\mathbf{W}_{\tau-1}\right)$ is an $M_T \times M_R$ additive independent noise matrix each component is $\mathcal{CN}(0,1)$ distributed. The decoder performs maximum likelihood decoding and the decision rule can be expressed as [7]:

$$\hat{z}_\tau^{ML} = \arg\min_{l\,\in\,\mathbb{Z}_l} \left\|\mathbf{Y}_\tau - \mathbf{\Phi}_l\mathbf{Y}_{\tau-1}\right\|_F, \tag{8}$$

where $\mathbb{Z}_l = \{0, 1, ..., L-1\}$ and $\|\mathbf{A}\|_F = \sqrt{Tr\left(\mathbf{A}^\dagger\mathbf{A}\right)} = \sqrt{Tr\left(\mathbf{A}\mathbf{A}^\dagger\right)}$ is the Frobenius norm. $Tr\left(\cdot\right)$ denotes the trace operator and the superscript $(\cdot)^\dagger$ represents complex conjugate and transpose of a matrix.

The exact expression of the $PBEP$ of mistaking $\mathbf{\Phi}_l$ for $\mathbf{\Phi}_{l'}$ is [14]

$$P\left(\mathbf{\Phi}_l \to \mathbf{\Phi}_{l'}\right) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{m=1}^{\Delta_H}\left(1 + \frac{\gamma\lambda_m}{4\sin^2\theta}\right)^{-M_R} d\theta, \tag{9}$$

where $\gamma = \frac{\rho^2}{(1+2\rho)}$, λ_m is the m^{th} eigenvalue of the matrix $\mathbf{C}_S = \left(\mathbf{\Phi}_l - \mathbf{\Phi}_{l'}\right)\left(\mathbf{\Phi}_l - \mathbf{\Phi}_{l'}\right)^\dagger$, and Δ_H is the rank of $\mathbf{C}_S$. Since the DUST signal matrices always have full rank of M_T, Δ_H can be replaced by M_T.

III. Design Method

A. The MRB Space-Time Signal Scheme

For asymptotically high SNR, the Chernoff bound of (9) depends on the product of non-zero eigenvalues of $\mathbf{C}_s$; $\Delta_P = \prod_{m=1}^{M_T}\lambda_m$. This leads to a design criterion that aims to maximize the following diversity product [7],

$$\zeta = \frac{1}{2}\min_{l \neq l' \in\ \mathbb{Z}_l} \left|\det\left(\mathbf{\Phi}_l - \mathbf{\Phi}_{l'}\right)\right|^{1/M_T}. \tag{10}$$

Many DUST signal constellations, such as [6]-[11] and [13], were designed based on the performance measure in (10). Recently in [13], the MRB space-time signal scheme was introduced particularly for communication systems with even number of transmit antennas. It is a non-group signal structure with combination of a diagonal cyclic code and a full-rotation matrix.

Assume an even number of transmit antennas, M_T, and a unitary signal constellation of size L. A set of MRB space-time signals is defined as [13]:

$$\mathcal{V} = \left\{\mathbf{\Phi}_l\left(\mathbf{K}\right)\ :\ l = 0, 1, ..., L-1\right\}, \tag{11}$$

where each $\mathbf{\Phi}_l$ is a unitary matrix depending on the parameters $\mathbf{K}$ =$\{k_{11}, ..., k_{1M_T}; k_2\}$ whose elements are integer numbers from $\mathbb{Z}_l$. Specifically, denote $\mathbf{j} = \sqrt{-1}$ and $\theta_L = \frac{2\pi}{L}$, then for any $l = 0, 1, ..., L-1$, $\mathbf{\Phi}_l$ is given by:

$$\mathbf{\Phi}_l\left(\mathbf{K}\right) = \mathbf{\Lambda}^l \cdot \left[\mathbf{I}_N \otimes \mathbf{\Psi}\left(k_2\theta_L\right)\right]^l, \tag{12}$$

where

$$\mathbf{\Lambda} = diag\left(e^{\mathbf{j}\theta_L k_{11}}, \ldots, e^{\mathbf{j}\theta_L k_{1M_T}}\right),$$

$$\mathbf{\Psi}(\theta) = \begin{bmatrix} cos(\theta) & sin(\theta) \\ -sin(\theta) & cos(\theta) \end{bmatrix},$$

$\mathbf{I}_N$ is the identity matrix of size $N \times N$ with $N = \frac{M_T}{2}$, and $\otimes$ represents the tensor product.

B. Improved MRB Space-Time Signal Design

It has been argued in [14] that constellation designs that achieve maximum diversity product ζ may not be appropriate, especially at medium range of SNRs. In deed, a good signal constellation fails the criterion (10) if one of its corresponding eigenvalues is very small. The smallest eigenvalue dominates the criterion, and an optimum signal

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TABLE I: Comparison of constellation parameters and union bounds for the MRB space-time signal design with M_T = 2 transmit antennas.

M_R	L	New Parameters		Original Parameters	
		$[\tilde{k}11, \tilde{k}12, \tilde{k}2]$	PUB	$[k11, k12, k2]$	PUB
1	4	[0.389, 3.611, 1.338] @ 20dB	$5.1786e^{-4}$	[1, 1, 0]@ 20dB	$6.4076e^{-4}$
2	4	[1.616, 2.384, 0.692] @ 20dB	$3.5227e^{-7}$	[1, 1, 0]@ 20dB	$6.4036e^{-7}$
1	8	[2, 6, 5] @ 30dB	$1.8647e^{-5}$	[3, 5, 2]@ 30dB	$1.8647e^{-5}$
2	8	[2, 6, 5] @ 20dB	$1.8795e^{-6}$	[3, 5, 2]@ 20dB	$1.8795e^{-6}$
1	16	[1.5, 14.5, 7.5] @ 30dB	$6.9510e^{-5}$	[3, 9, 4]@ 30dB	$7.7840e^{-5}$
2	16	[2, 10, 9] @ 20dB	$1.3671e^{-5}$	[3, 9, 4]@ 20dB	$1.3671e^{-5}$
1	32	[4, 28, 15] @ 30dB	$2.3474e^{-4}$	[3, 5, 8]@ 30dB	$2.6699e^{-4}$
2	32	[9.1, 30.7, 8.2] @ 20dB	$1.1025e^{-4}$	[3, 5, 8]@ 20dB	$1.2863e^{-4}$
1	64	[2.5, 51.5, 8.5]@ 30dB	$7.9933e^{-4}$	[3, 21, 2]@ 30dB	$1.1197e^{-3}$
2	64	[26.5, 59.5, 0.5]@ 20dB	$5.4148e^{-4}$	[3, 21, 2]@ 20dB	$1.2310e^{-3}$

parameter can not be reached. Hence, it was suggested in [14] that DUST codes should be designed based on the design criterion of minimizing the union bound on $PBEP$.

Specifically, assuming that all the L space-time signals, $\mathbf{\Phi}_l$, are equally likely transmitted, the performance measure of BEP is approximated by the union bound of the exact $PBEP$,

$$BEP \le \frac{2}{L} \sum_{l=0}^{L-2} \sum_{l'=l+1}^{L-1} P\left(\mathbf{\Phi}_l \rightarrow \mathbf{\Phi}_{l'}\right) \triangleq PUB, \qquad (13)$$

where $P\left(\mathbf{\Phi}_l \rightarrow \mathbf{\Phi}_{l'}\right)$ is specified in (9).

We now consider designing the MRB space-time signals using the design criterion of minimizing the union bound in (13). For demonstration purpose, we focus on a constellation design method for two transmit antennas. Similar procedures can be applied to constellation design for higher number of transmit antennas. In case of $M_T = 2$, the code structure of the MRB signal scheme is

$$\mathbf{\Phi}_l = \begin{bmatrix} e^{\mathbf{j}\theta_L l \tilde{k}_{11}} & 0 \\ 0 & e^{\mathbf{j}\theta_L l \tilde{k}_{12}} \end{bmatrix} \cdot \mathbf{\Psi}\left(l\tilde{k}_2\theta_L\right), \qquad (14)$$

where $l = 0, 1, ..., L-1$. The MRB space-time signals in (14) are determined by three parameters $\tilde{\mathbf{K}} = \{\tilde{k}_{11}, \tilde{k}_{12}; \tilde{k}_2\}$. Our design goal is to find a set of parameters $\tilde{\mathbf{K}}$ that minimize the PUB in (13).

Moreover, in our design, we relax the set of parameter $\tilde{\mathbf{K}}$ in (14) to be the set of non-integer numbers, i.e., $\tilde{\mathbf{K}} = \{\tilde{k}_{11}, \tilde{k}_{12}; \tilde{k}_2 \mid 0 < \tilde{k}_{11}, \tilde{k}_{12}, \tilde{k}_2 < L\}$. With such extension, we increase the set of search parameters which allow us to have more chance to obtain better signals. Note that in all of previous constellation designs in [6]-[11] and [13]-[14], the set of signal parameters is confined to the set of integer numbers. Actually, such requirement is not necessary in DUST modulation scheme.

C. Search Method

For any number of constellation size $L \ge 2$, and given values of M_T, M_R, and SNR (ρ) of interest, we perform exhaustive computer search for the best set of non-integer parameters $\tilde{\mathbf{K}}$ that minimize the PUB. We target constellation performances in the range of 10^{-4} to 10^{-7}, with the operating SNRs from 20 to 30 dB depending on L, M_T, and M_R.

With symmetrical property of the full-rotation matrix $\Psi(\theta)$, we found that the summation of the best parameters $\tilde{k}_{11}$ and $\tilde{k}_{12}$ is approximately L. Simplifying the search algorithm below is sufficient to find the best set of parameters, $\tilde{\mathbf{K}}$, and reduce complexity dramatically:

$$\begin{aligned} &a)\ 0 < \tilde{k}_{11} \le L/2, \\ &b)\ \left(L/2 + \tilde{k}_{11}\right) \le \tilde{k}_{12} < L, \\ &c)\ 0 < \tilde{k}_2 \le L/2. \end{aligned}$$

For signal constellation of small size, i.e., L = 4, we use step size 0.001 for each parameters in $\tilde{\mathbf{K}}$. For other constellation sizes, due to the complexity of the search space, we limit our search to a searching step of 0.1.

For large signal constellation sizes, we further reduce computational complexity by using an approximated $PBEP$ [14],

$$P\left(\mathbf{\Phi}_l \rightarrow \mathbf{\Phi}_{l'}\right) \lesssim \frac{1}{2}\left\{1 - \Gamma\left(k, \alpha_1\right)\right\}, \qquad (15)$$

where the expression of $\Gamma\left(k, \alpha_1\right)$ is

$$\Gamma\left(k, \alpha_1\right) = \alpha_1 \sum_{k=0}^{M_T M_R - 1} \begin{pmatrix} 2k \\ k \end{pmatrix} \left(\frac{1-\alpha_1^2}{4}\right)^k, \qquad (16)$$

with $\lambda_{gm} = \left(\Delta_P\right)^{1/M_T}$ and $\alpha_1 = \sqrt{\frac{\gamma\lambda_{gm}}{4+\gamma\lambda_{gm}}}$. Explicitly, for M_T = 2 and M_R = 1, we have

$$PUB \approx \frac{2}{L} \sum_{l=0}^{L-2} \sum_{l'=l+1}^{L-1} \left(0.5 - 0.75\alpha_1 + 0.25\alpha_1^3\right). \qquad (17)$$

Similarly, in case of M_T = 2 and M_R = 2,

$$PUB \approx \frac{2}{L} \sum_{l=0}^{L-2} \sum_{l'=l+1}^{L-1} \mathbf{\Upsilon}(\alpha_1), \qquad (18)$$

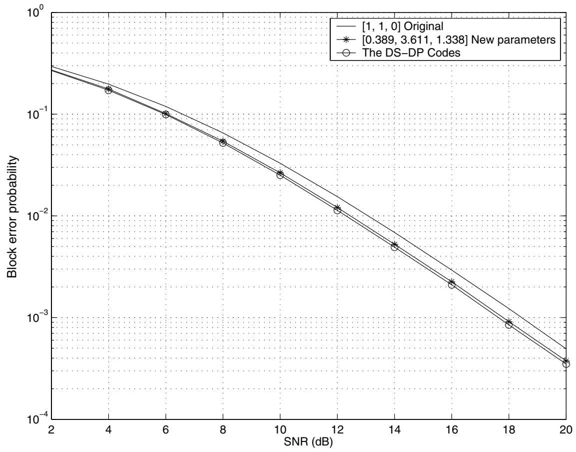

Fig. 1: Performance for L = 4, M_T = 2, and M_R = 1.

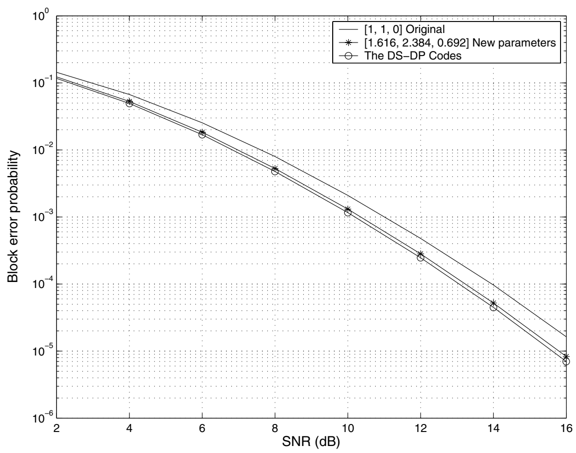

Fig. 2: Performance for L = 4, M_T = 2, and M_R = 2.

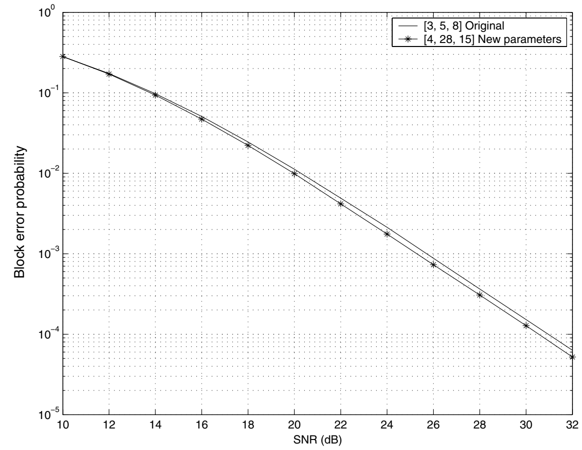

Fig. 3: Performance for L = 32, M_T = 2, and M_R = 1.

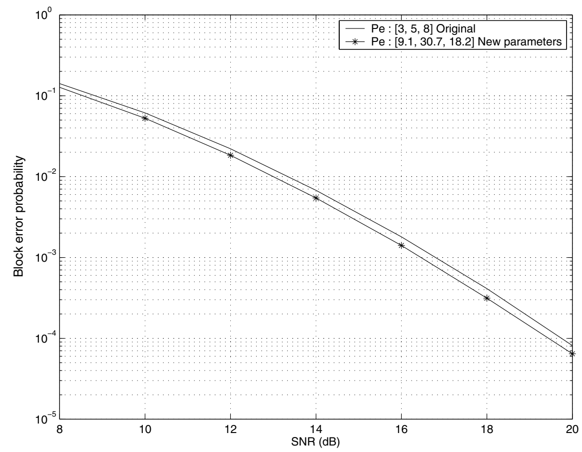

Fig. 4: Performance for L = 32, M_T = 2, and M_R = 2.

where

$$\boldsymbol{\Upsilon}(\alpha_1) = 0.5 - 1.09375\alpha_1 + 1.09375\alpha_1^3 - 0.65625\alpha_1^5 + 0.15625\alpha_1^7.$$

Instead of performing numerical integrations, the approximated PUB in (17) and (18) require only algebraic computations which reduce execution times considerably. Table I shows our parameter search results for signal constellation size L = 4, 8, 16, 32, and 64, where @ $\mathcal{X}\ dB$ means an operating SNR at $\mathcal{X}\ dB$. To illustrate coding advantages of our design, we list PUB of the codes designed in [13] comparing with PUB from our design. We observe that for L = 4, 32, and 64, the union bounds of the new designs are smaller than that of the original designs. Note also that depending on a predetermined operating SNR, the constellation parameters can be different for system with one or two receive antennas. In case of L = 8 and 16, although the obtained parameters are different from those in [13], the union bounds of them are almost the same.

IV. Simulation Results

We simulated the DUST modulation schemes for two transmit and one or two receive antennas. The channel fading coefficients are assumed to be independent between antennas, but time correlated according to Jakes' model [15] with autocorrelation $J_0(2\pi f_D T_s \tau)$, where f_D is the maximum Doppler frequency of 75 Hz, T_s is the sampling period with normalized fading parameter $f_D T_s$ = 0.0025, and $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind. The constellation sizes range from L = 4 to 64 which are equivalent to transmission rate (R) of 1 to 3 $b/s/Hz$, respectively. We used the MRB signal structure in (14) and presented constellation performances in terms of BEP versus SNR curves.

Figures 1 and 2 show the performances of the MRB signals with constellation size L = 4, i.e., R = 1 $b/s/Hz$. We observe that the codes with new parameters have coding advantages at BEP range $10^{-3} - 10^{-4}$ about 0.75 - 1 dB over the design in [13]. Moreover, we compare our signal performances to those of a code with optimum

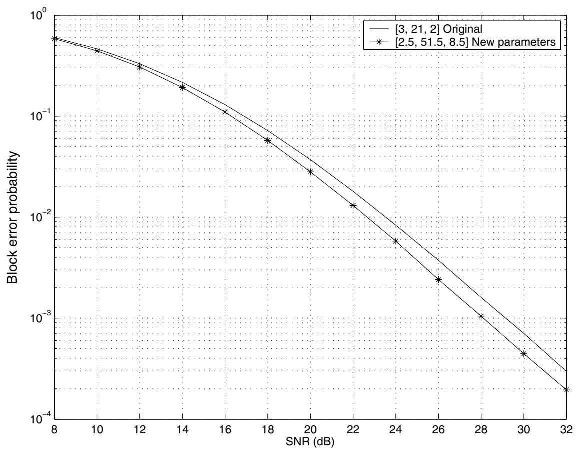

Fig. 5: Performance for L = 64, M_T = 2, and M_R = 1.

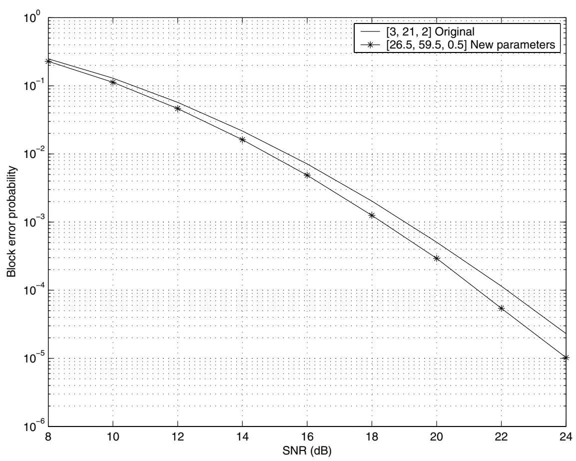

Fig. 6: Performance for L = 64, M_T = 2, and M_R = 2.

diversity sum and product in [10]; the so-called DS-DP code. Simulation results show that the performances of the MRB codes with new parameters are close to the DS-DP code performances. Although the DS-DP code provides slightly better performances, it is a hand-crafted signal constellation derived from sphere packings and it has no specific structure for other signal constellation sizes.

The BEP performances of constellation size L = 32 (R = 2.5 $b/s/Hz$) are illustrated in Figures 3 and 4 for M_R = 1 and 2, respectively. In comparison with the original parameters, our new parameters yield better performances in both cases. This confirms the merit of the design criterion in (13).

For L = 64 (R = 3 b/s/Hz) with M_T = 2, M_R = 1 and M_R = 2, the constellation performances are shown in Figures 5 and 6, respectively. We observe coding gains of 1 dB at a BEP of 10^{-3} and 10^{-4} for the single receive antenna system and the system with two receive antennas, respectively.

V. Conclusions

In this paper, we improved the MRB signal design for the DUST modulation system by using the design criterion of minimizing the union bound on pairwise block error probability ($PBEP$). Furthermore, we relaxed the set of search parameters from integers to non-integers to get better codes. By taking advantage of symmetric property of the full rotation matrix, we reduced search space for the best constellation remarkably. The approximated union bound was applied to further reduce computation time for large constellation sizes. Simulation results showed the performance improvement of the obtained signals, for example, about 0.75 - 1 dB for constellation size L = 4 and about 1 dB for L = 64 which support our numerical calculations.

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