# Noncooperative Power-Control Game and Throughput Game Over Wireless Networks

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***Abstract*—Resource allocation is an important means to increase system performance in wireless networks. In this letter, a game-theory approach for distributed resource allocation is proposed. Observing the bilinear matrix inequality nature of resource allocation, we construct two interrelated games: a power-control game at the user level, and a throughput game at the system level, respectively, to avoid local optima. An optimal complex centralized algorithm is developed as a performance bound. The simulations show that the proposed games have near-optimal system performance.**

***Index Terms*—Adaptive modulation, game theory, power control, rate adaptation.**

## I. Introduction

ONE OF THE major challenges in wireless networks is to efficiently use the limited radio resources, which are restrained by the cochannel interference (CCI) and time-varying nature of channels. Resource allocation [1], such as power control [2] and adaptive modulation, is an important means to combat these detrimental effects and increase the spectrum efficiency in the interference-limited wireless networks. Joint consideration of power control and rate adaptation can further improve the system performance [3]–[5]. Since individual mobile users do not have the knowledge of other users' conditions and cannot cooperate with each other, they act selfishly to maximize their own performances in a distributed fashion. Such a fact motivates us to adopt the game theory.

The resource allocation can be modeled as a noncooperative game that deals largely with how rational and intelligent individuals interact with each other in an effort to achieve their own goals. In the resource-allocation game, each mobile user is self-interested and trying to maximize his/her utility function, where the utility function represents the user's performance and controls the outcomes of the game. Some related work can be found in [6]–[10]. In most of the previous work, only one utility function is defined for both power and throughput, which can result in local optima because of nonlinearity. We can show that joint power control and adaptive rate-allocation problem can be formulated as a bilinear matrix inequality (BMI) [11] constraint if the bit-error rate (BER) is fixed, i.e., the throughput is linearly constrained if the power is fixed, and the power is linearly constrained if the throughput is fixed. So this gives us motivation to design two interconnected games for power and throughput, respectively, so that higher system performance can be more likely to be achieved. In [8], the idea of two-level optimization was first proposed, while only one utility was used and no adaptive modulation was applied.

In order to achieve better system performance, our primary concern is to design the utility functions and the rules of the games. One of the goals is to motive individual users to adopt a social behavior and enhance the system performance by sharing the resources. Consequently, we can make the distributed self-optimizing decisions compatible with the demand for a higher overall system performance. In doing so, we link both power control and adaptive modulation by designing games at both the user level and the system level. A noncooperative power-control game (NCPCG) is designed at the user level. At the system level, the optimization goal is to maximize the overall system throughput under the maximal transmitted power constraint. A noncooperative throughput game (NCTG) is designed. There may be multiple Nash equilibriums in this game. A distributed algorithm is constructed to achieve the better Nash equilibrium by employing a proposed game rule and an initialization method. An optimal but complex centralized algorithm that achieves the optimal system performance is developed as a performance upper bound. From simulations, the proposed games are optimal for the power at the user level, and can be optimal or near-optimal for network throughput at the system level.

## II. System Model and Bilinear Matrix Inequality

Consider $K$ cochannel uplinks that may exist in distinct cells of wireless networks. Each link consists of a mobile and its assigned base station (BS). We assume the average transmitted power for different modulation constellations is normalized. Define $N_i$ as the noise level. The $i$th user's signal-to-interference-plus-noise ratio (SINR) is $\Gamma_i = P_i h_{ii}/(\sum_{k\neq i} P_k h_{ki} + N_i)$, where $P_k$ is the $k$th user's transmitted power, and $h_{ki}$ is the channel gain from the $k$th user to the $i$th BS. Adaptive modulation provides the links with abilities to match the effective bit rates (throughput), according to interference and channel conditions. $M$-ary quadrature amplitude modulation (MQAM) is a modulation method that has high spectrum efficiency. We assume each user has a unit bandwidth. In [4], for a desired throughput $T_i$ of MQAM, the $i$th user's BER can be approximated as $\text{BER}_i \approx c_1 e^{-c_2(\Gamma_i/2^{T_i}-1)}$, where $c_1 \approx 0.2$ and $c_2 \approx 1.5$ when $\text{BER}_i$ is small. For a specific desired $\text{BER}_i$, the $i$th link's required SINR for the desired throughput $T_i$ can be expressed as $\gamma_i(T_i) = (2^{T_i} - 1)/c_3^i$, where $c_3^i = -c_2^i/\ln(\text{BER}_i/c_1^i)$.

If the users' throughput is too large, CCI is severe, and it is possible that there exists no feasible power allocation for the

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desired throughput and BER. In order to prevent the system from not being feasible, we need to analyze the feasibility condition. First, we use the targeted SINR $\gamma_i$ and require that the received SINR $\Gamma_i$ be larger than or equal to this targeted SINR, i.e., $\Gamma_i \geq \gamma_i \ \forall \ i$, in order to ensure the desired BER for the throughput $T_i$. Rewriting these inequalities in a matrix form, we have $(\mathbf{I} - \mathbf{DF})\mathbf{P} \geq \mathbf{Du}$, where $\mathbf{I}$ is an identity matrix, $\mathbf{u} = [u_1, \ldots, u_K]^T$ with $u_i = N_i/(|h_{ii}|^2)$, $\mathbf{D} = \text{diag}\{\gamma_1, \ldots, \gamma_K\}$, and $[\mathbf{F}_{ij}] = 0$, if $j = i$; $[\mathbf{F}_{ij}] = |h_{ji}|^2/|h_{ii}|^2$, if $j \neq i$. The above inequality is a BMI [10], i.e., the power vector is linearly constrained if the targeted SINR vector (throughput) is fixed, and vice versa. Since linearity can achieve global optimum, this motivates us using the two-game approach developed later. By the Perron–Frobenius theorem [1], there exists a feasible solution with positive power and rate allocation only if the maximum eigenvalue of $\mathbf{DF}$ [spectrum radius $\rho(\mathbf{DF})$] is inside the unit circle.

## III. Constructing Games for Distributive Resource Allocation

In wireless communication networks, because of the bandwidth limitation, it is impractical for the mobile users to communicate, thus cooperate with each other, so as to optimally use the wireless resources. Each individual mobile user tries to maximize his/her performance, based only on his/her perceived self-interest. However, this will cause the system to be balanced in some undesired nonoptimal equilibriums. We design the game rules for the users' competitions such that the system will be balanced in the desired optimal and efficient resource allocation. Because power and throughput are bilinearly constrained, it is natural to divide the optimization efforts into the system level and the user level. We define value function $v_i$ as the connection between two levels. The goals for both levels are given by the following.

1) **User Level:** The goal is to define a utility function $u_i$, and then each user can compete with other users in an NCPCG to maximize his/her utility function. There are some practical constraints, such as the maximum transmitted power $P_{\max}$. The proposed NCPCG is formulated as $\max_{P_i \leq P_{\max}} u_i(P_i, \mathbf{P}_{-i}, v_i)$, where $\mathbf{P}_{-i} = [P_1 \ldots P_{i-1} P_{i+1} \ldots P_K]^T$, and $v_i$ is the assigned value function that is related to throughput $T_i$. At the user level, the transmitted power $P_i$ is optimized by the proposed NCPCG, while $v_i$ is assigned by the BS. When the throughput $T_i$ is equal to zero, no transmitted power is needed and $v_i$ should be zero. Otherwise, we define the value function as a function of the desired throughput as $v_i = \ln((2^{T_i} - 1)/c_3^i) + 1$, if $T_i > 0$; $v_i = 0$, if $T_i = 0$, where $v_i$ is a function of only throughput $T_i$ and $c_3^i$. $c_3^i$ is related to the desired BER, and is usually predefined and fixed. When the CCI is high, from a system optimization point of view, the cost for the desired $v_i$ should be increased to reduce CCI. We represent this cost as $\ln \Gamma_i$, where $\Gamma_i$ reflects the severity of the CCI and can be fed back from the BS to the mobile. We define the utility function as $u_i = P_i(v_i - \ln \Gamma_i)$. NCPCG is played iteratively until convergence. It can be shown that $u_i$ satisfies the three requirements of the "standard function" in [2] for iterative function. Consequently, NCPCG converges to the unique optimum that achieves the minimal SNR for the desired BER.
2) **System Level:** The goal is to assign a user his/her value function $v_i$ by an NCTG, such that the overall system throughput $\sum_{i=1}^K T_i$ is maximized, under the constraint $P_i \leq P_{\max} \ \forall i$. When the system is balanced, $T_i$ and $P_i$ are functions of $\mathbf{v}$, where $\mathbf{v} = [v_1 \ldots v_K]^T$. The overall network throughput is optimized by NCTG, and the corresponding $v_i \ \forall i$ are assigned to the users for NCPCG. The problem can be formulated as $\max_{\mathbf{v}} \sum T_i(\mathbf{v})$, subject to $P_i(\mathbf{v}) < P_{\max} \ \forall i$.

### A. Noncooperative Throughput Game at the System Level

At the system level, NCTG is constructed for the users to compete distributively, while the system maintains feasibility. We define $\Lambda$ as an indication function for system feasibility. When the BS detects that all required transmitted power for the desired BER and throughput are less than or equal to $P_{\max}$, $\Lambda$ equals one, otherwise, it equals zero. Since the users compete with each other for the throughput, we define each user's utility function $\overline{u}_i$ for NCTG as a product of his/her throughput $T_i$ and $\Lambda$, i.e., **(NCTG)** $\max_{T_i} \ \overline{u}_i = T_i \Lambda$. The game starts from any feasible initial values and is balanced when no user can increase his/her throughput. The existence of the Nash equilibrium can be shown by a similar proof in [6]. However, there might be multiple Nash equilibriums, which will be shown in the simulations later. If the users with bad channels get high throughput, they will produce large CCI to other users. Consequently, the system's overall throughput will be reduced. So how to initialize the proposed game and how to design the game rule for each user to compete his/her throughput play a critical role on finding the better optimum. The idea for initialization comes from the following theorem.

*Theorem 1:* Define maximal achievable SINR as $\hat{\Gamma}_i$ when $P_i = P_{\max} \ \forall i$. Then the value $v_i = 1 + \ln((2^{\lfloor \log_2(1+c_3^i \hat{\Gamma}_i) \rfloor} - 1)/c_3^i)$ is feasible for both games, where $\lfloor \rfloor$ is floor function for the maximal integer smaller. For example, $\lfloor 3.2 \rfloor = 3$.

*Proof:* Define $\hat{\gamma}_i = e^{v_i - 1} = (2^{\lfloor \log_2(1+c_3^i \hat{\Gamma}_i) \rfloor} - 1)/c_3^i$. Since $\log_2$ is an increasing function, $\hat{\Gamma}_i \geq \hat{\gamma}_i \ \forall \ i$. Because $\mathbf{D}, \mathbf{F} \in \Re^{\mathbf{K} \times \mathbf{K}}$ and $|\rho(\mathbf{DF})| < 1$, we have $\mathbf{P}' = \sum_{j=0}^{\infty} (\mathbf{DF})^{\mathbf{j}} \mathbf{Du}$, since any component in $\mathbf{D}$, $\mathbf{F}$, and $\mathbf{u}$ is nonnegative. So all components in $\mathbf{P}'$ are nondecreasing functions of $\gamma_i$. When we select the targeted SINR $\hat{\gamma}_i \leq \hat{\Gamma}_i$, any component of the power vector must be smaller than or equal to $P_{\max}$, so the value functions satisfy the maximum power constraint and the system must be feasible. □

First, every user transmits the maximal power. The BS detects the received SINR. Using the above theorem, the BS decides what is the largest achievable throughput and sends back the corresponding $v_i$. The system is sure to be feasible, but not necessarily optimal. By doing this, the users with good channels will get higher throughput. Each user will then play NCPCG until convergence. The convergence is achieved when each user detects that the NCPCG utility is stable, because this means the interfering users' power and utilities are also stable. After

TABLE I
DISTRIBUTED SYSTEM ALGORITHM

| |
|---|
| Initial: $P_i = P_{max}$, $\forall$ $i$, calculate $v_i$ in Theorem 1 and send back to mobiles. |
| Iterations: 1. Wait until NCPCG converges |
| 2a. Power Increase Criteria at Users: |
| If $H_i$ condition is satisfied, send throughput increase request to the BS. |
| 2b. Feasibility Detection at the BS: |
| Increase throughput for requesting user with highest $H_i$, detect if feasible. |
| 2c. Feedback to Users: |
| If system not feasible: reduce throughput to original value; |
| else if no more request: Wait. |

NCPCG convergence, the users decide if they can increase their throughput, while the system is still feasible. We need to find the criteria for the users to decide when to send the requests to the BS for throughput increase. We define $\gamma_i(T_i)$ as the required SINR for the desired throughput $T_i$. When $T_i > 0$, if we assume the interferences, noise, and channel gains are fixed, the required power for throughput $T_i + 1$ will be $P_i(2^{(T_i+1)} - 1)/(2^{T_i} - 1)$, where $P_i$ is the current power. We compare this desired power with $\beta P_{\max}$, where $\beta$ is a constant and $0 \leq \beta \leq 1$. If the desired power is larger than $\beta P_{\max}$, the user can send a request to the BS to increase his/her throughput by one. When this user increases his/her power, he/she causes interference to the others. Others have to increase their power, which causes interference to this user, as well. So this user has to increase his/her power more. $\beta$ is such a factor that takes into consideration this "mutual interfering" effect. When $T_i = 0$, all received power is the interference plus noise power, defined as $I_i$. The channel gain $h_{ii}$ can be estimated during the initialization when this user transmits the maximal power. This user can calculate estimated received SINR $= \beta P_{\max} h_{ii}/I_i \gamma_i(1)$ by transmitting power $\beta P_{\max}$. If the received SINR is larger than $\gamma_i(1)$, the user will send the throughput increase request. Let us define $H_i$ as the throughput request factor, the criteria for the users to request their throughput to increase by one is when $H_i > 1$, where $H_i = \beta P_{\max}(2^{T_i} - 1)/P_i(2^{(T_i+1)} - 1)$, if $T_i > 0$ and $H_i = \beta P_{\max} h_{ii}/I_i \gamma_i(1)$, if $T_i = 0$. The above game rule for NCTG and the distributed adaptive algorithm are summarized in Table I.

After initialization, two games are played iteratively. First, users play NCPCG. After convergence, users play NCTG, calculate $H_i$, and send requests if necessary. If the request is granted, the selected user will increase the throughput, and then NCPCG is played again. Both games are distributive, because only local information is necessary to play. It is worth mentioning that in order to determine the largest $H_i$, adjacent BSs should exchange the values of $H_i$, which can be implemented with limited signaling.

## B. Centralized Scheme as a Performance Upper Bound

The distributed algorithm may not be optimal. First, there is a probability that the users do not send requests, while the system might be feasible if users sent requests. Second, there exist Nash equilibriums that are not global optimum. To understand the performance loss, we need to find the optimal solution as a performance bound. The most straightforward idea is to let the system centrally decide how to allocate throughput to users, with the assumption that all channel responses are known. This is not implementable, since the channel conditions from the users to the BS in other cells are hard to obtain. The problem becomes a constrained optimization: to maximize overall throughput under the maximum power and maximum eigenvalue constraints, which can be solved by standard nonlinear programming

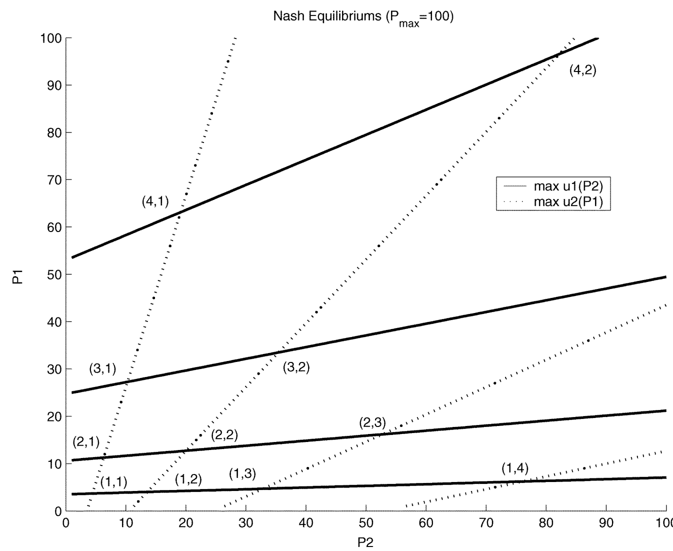


Fig. 1. Nash equilibriums of NCPCG.

$$\max_{v_i} \sum_{i=1}^{K} T_i(\mathbf{P}), \quad \text{s.t.} \; |\rho(\mathbf{DF})| < 1, \; P_i \leq P_{\max} \, \forall \, i.$$

## IV. SIMULATION RESULTS

We evaluate the performances of the proposed algorithms by two simulation setups. First, we consider a two-user case. Here we assume $h_{11} = h_{22} = 1$, $h_{21} = 0.01$, $h_{12} = 0.07$, $N_1 = N_2 = 1$, BER $= 10^{-3}$, and $P_{\max} = 100$. In Fig. 1, we show the Nash equilibriums of NCPCG when the different throughput allocations are given. On any solid line, $u_1$ gets the maxima. On any dotted line, $u_2$ has the optima. Starting from any feasible power allocation, each user tries to maximize his utility function by controlling his power, such that the power allocation is closer to the corresponding lines. When the system is balanced, any intersection is a Nash equilibrium, where we denote the throughput as (user1's throughput, user2's throughput). We can see that the maxima for $u_1$ obtained from $P_1$ will increase with increasing $P_2$. This is because the CCI increases. In Table II, we list the strategic form of NCTG at the system level for all the nonzero throughput allocations. Each row lists user1's

TABLE II
STRATEGIC FORM FOR TWO-USER NCTG EXAMPLE

| | | | | | |
|---|---|---|---|---|---|
| 5 | **0** | **0** | **0** | **0** | **0** |
| 4 | **5** | **0** | **0** | **0** | **0** |
| 3 | **4** | **5** | **0** | **0** | **0** |
| 2 | **3** | **4** | **5** | **6** | **0** |
| 1 | **2** | **3** | **4** | **5** | **0** |
| $(u_1, u_2)$ | 1 | 2 | 3 | 4 | 5 |

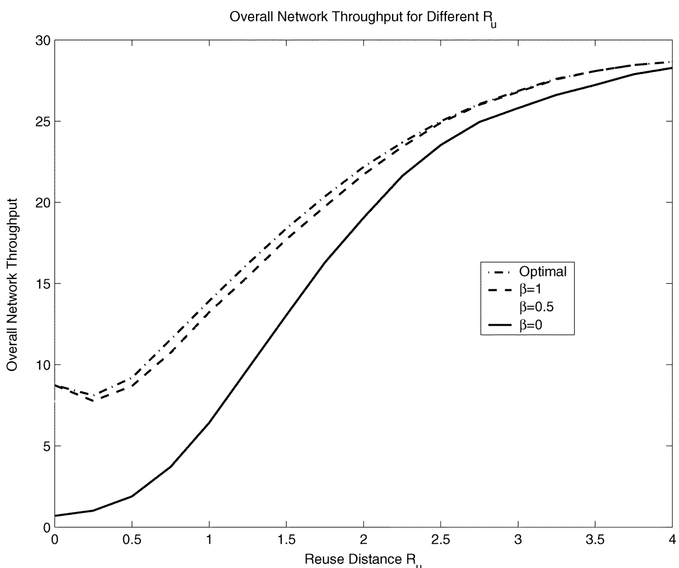


Fig. 2. System throughput versus $R_u$.

throughput, and each column lists user2's throughput. The bold numbers are the overall throughput. If the system is not feasible, the overall throughput is zero. We can see that (4,2), (2,3), and (1,4) are Nash equilibriums, because no user can improve his throughput alone. However, (2,3) and (1,4) are not desired Nash equilibriums for the optimal overall network throughput. The proposed distributed algorithm in Table I will be initialized at (3,2). If $\beta$ is properly selected (in this case, $\beta > 0.32$), because $H_1 > H_2$, the algorithm will increase user1's throughput first and converge to the optimal Nash equilibrium (4,2). So, from this example, we can see that we can achieve both power optimum and throughput optimum by playing the NCPCG at the user level and NCTG at the system level.

We set up another simulation to test the proposed algorithms. A network is constructed with one cell at the center and the other six at the degrees of [0, 60, 120, 180, 240, 300], respectively. One BS is located at the cell's center, and one user is randomly located within each cell. The cell radius is $r = 1000$ m, the minimal distance between the user and the BS is $r' = 50$ m, and the distance between the centers of two adjacent cells is $R = rR_u$ m, where $R_u$ is the reuse factor. $P_{\max} = 2$ W, $\eta = 3.5$, $\alpha = 10^{-3}$, BER $= 10^{-3}$, and $N_i = 10^{-11}$ W. The simulations run $10^5$ times.

Fig. 2 compares overall network throughput versus $R_u$ with different $\beta$. When $R_u$ is small, CCI is severe. After initializing by sending the maximal transmitted power, most users get throughput of zero. The overall network throughput is increased by the users' throughput-increasing requests. If $\beta$ is large enough, the proposed games can achieve the optimal

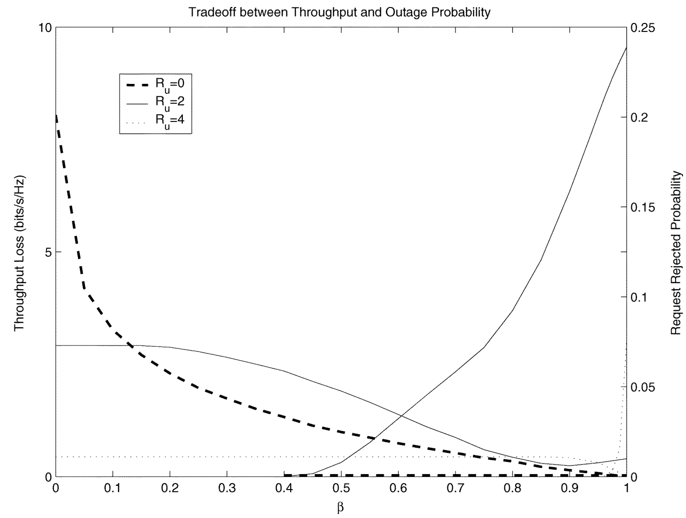


Fig. 3. Tradeoff between rejection and throughput loss for $\beta$.

system performance when $R_u$ is small enough. The overall network throughput is minimal when $R_u \approx 0.25$, because different BSs and users are mixed together, and CCI is most severe under this condition. When $R_u$ is large, CCI is minor. After the initialization, most users get the desired throughput. The overall network throughput is refined when $\beta$ is large. The optimal system performance can be achieved when both $R_u$ and $\beta$ are large enough. When $R_u$ is in the middle range, the proposed games may fall into some local minima and produce suboptimal solutions, even when $\beta = 1$. The overall throughput can be improved by increasing $\beta$.

However, the overall throughput improvement by increasing $\beta$ is at the expense of a possible high request-rejected probability defined as the ratio of the number of rejected requests over the total number of requests. Fig. 3 shows the throughput loss compared with the optimal solution and rejection probability versus $\beta$ for different $R_u$. When $R_u = 0$, the rejection probability is always zero, and the throughput loss is monotonically decreasing with $\beta$. This is because the optimal solution is that only the user with the best channel condition transmits,and there is no CCI from other users. So there is no penalty from other users if the transmitting user increases $\beta$ and aggressively sends the request. Therefore, it is optimal to select $\beta = 1$. When $R_u = 2$, the rejection probability monotonically increases with $\beta$. There is a tradeoff between the throughput loss and rejection probability. The higher the $\beta$, the lower the throughput loss, but the higher the rejection probability. If the system wants a very low rejection probability, we can select $\beta = 0.4$ with a performance loss of 2.35 b/s/Hz. When $R_u = 4$, the rejection probability is almost zero when $\beta < 0.97$, and the overall throughput loss is approximately 0.44 b/s/Hz when $\beta < 0.9$. The tradeoff only occurs when $\beta$ is large. The reason is that the users get almost optimal throughput after initialization. Consequently, the refinement only happens when the users are more aggressive for throughput requests.

We define the fairness factor as

$$\varrho = (1/\bar{T})\sqrt{(1/K-1)}\sum_{i=1}^{K}(T_i/\hat{T}_i - \bar{T})^2$$

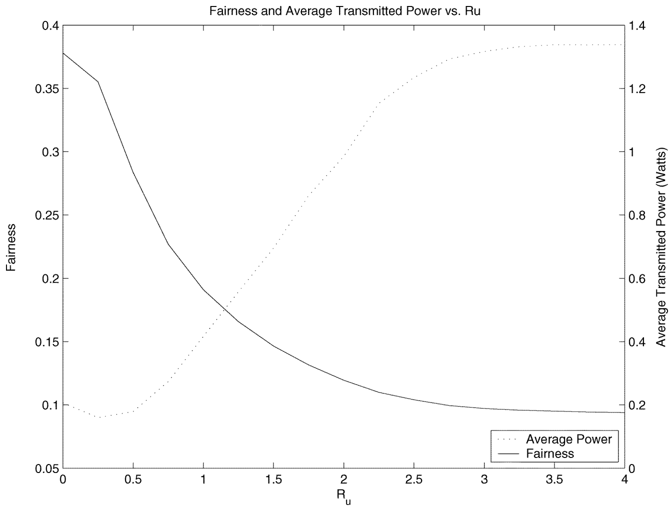


Fig. 4. Fairness and average transmitted power versus reuse distance.

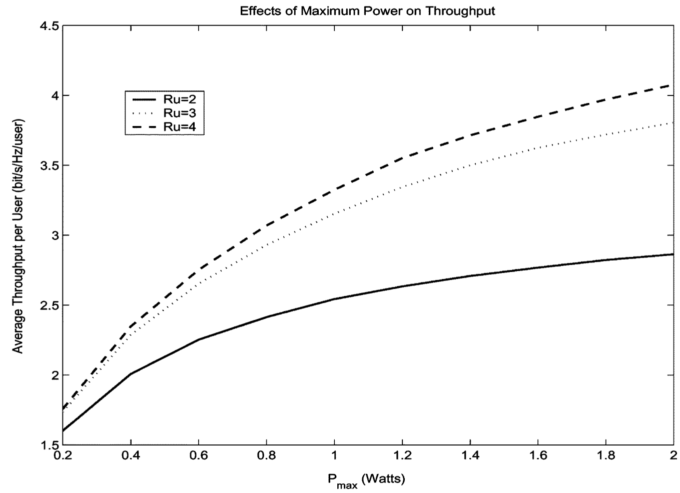


Fig. 5. Average throughput per user versus $P_{\max}$.

where $\hat{T}_i$ is the maximal throughput if user $i$ is the only transmitting user, and $\bar{T}_i$ = average $(T_i/\hat{T}_i)$. The physical meaning of $\varrho$ is the normalized variance of users' throughput compared with that of the single-user case. The higher the $\varrho$, the more unfair among users, i.e., the users throughput is more affected by CCI. $\varrho$ is one possible definition to measure the fairness. Fig. 4 shows the fairness and average transmitted power versus $R_u$ with $\beta = 1$. When $R_u$ is small and CCI is severe, $\varrho$ is large, and the users with the better channel condition occupy most of the resources. The average transmitted power is also low, because most users cannot transmit. When $R_u$ becomes large and CCI is reduced, the users with worse channel conditions can compete for their transmissions, while the users with better channel conditions are not so dominant. Consequently, $\varrho$ is reduced and users transmit more fairly, like the single-user case. The average transmitted power is increased and saturated with increase of $R_u$, because most users can transmit according to their own channel conditions, regardless of the low CCI from others.

Fig. 5 shows the average throughput per user versus $P_{\max}$ for different $R_u$ with $\beta = 1$. We can see that the average throughput increases more slowly when $P_{\max}$ is large. This is because the CCI is increasing, especially when $R_u$ is small. When $R_u$ is decreasing, the point where the average throughput per user saturates moves to the lower $P_{\max}$. There is no need for higher $P_{\max}$ if the performance curve is saturated already. So when $R_u$ is decreasing, we can reduce $P_{\max}$ accordingly.

## V. Conclusions

In order to achieve better overall network throughput, we construct NCPCG and NCTG at the user level and at the system level, respectively. From the simulations, the proposed games converge to the near-optimal solutions, compared with the optimal solutions obtained from the centralized scheme. The proposed two-game approach explores the BMI nature of the resource allocation to avoid local optima, and consequently, has high performance in a distributed implementation.

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