A Game-Theoretic Modeling of Popularity Dynamics

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***Abstract*—In the big data era, numerous items such as online memes and videos are generated everyday, some of which go viral, i.e., attract lots of attention, while most diminish quickly without any influence. The recorded people's interactions with these items constitute a rich amount of *popularity dynamics*, e.g., hashtags' mention count dynamics. It is crucial to understand the underlying mechanisms of popularity dynamics in order to utilize the valuable attention of people efficiently. In this work, we propose a game-theoretic model to analyze and understand popularity dynamics. The model takes into account both the instantaneous incentives and long-term incentives during people's decision making process. We analyze the equilibrium of the game and show several properties at the equilibrium, which confirm with the observations from real data. By using simulations as well as experiments based on real-world popularity dynamics data, we validate the effectiveness of the theory. We find that our theory can fit the real data well and can even predict the future dynamics.**

***Index Terms*—Game theory, popularity dynamics**

I. Introduction

In the big data era, people not only read lots of data but also create vast amount of data everyday through interactions. For instance, Twitter users may mention or retweet a hashtag; researchers may quote keywords in papers. All of these interactions lead to a notion of *popularity dynamics* such as: Twitter hashtags' mention count dynamics and research keywords' quotation dynamics. The popularity dynamics can track people's interactions with different types of items, some of which go viral, while most diminish quickly without any impact. To manage and utilize people's valuable interactions and attention better, it is crucial to understand the underlying mechanisms of the popularity dynamics. In the literature, tremendous efforts have been made to model and analyze network users' popularity dynamics (e.g., dynamics of websites and Twitter hashtag mention dynamics) [1]–[6]. One of the most important branch of popularity dynamics is information diffusion dynamics, which has been extensively investigated [7]–[15].

The process of the generation of popularity dynamics is complicated and involves the decision-making of many individuals. Individual's decision is influenced by many factors including the quality and timeliness of the item (e.g., a hashtag or a video), the personal preference of the individual and others' decisions. Since this involves the interactions between multiple decision makers, game theory [16] can be a very suitable mathematical modeling tool here. In the literature, game theory has been utilized to model various popularity dynamics [17]–[19]. All these existing game-theoretic models only consider the *instantaneous incentives* of players, i.e., the decision-makers are myopic in the sense that they only decide based on the current state of the system. However, in real-world, individuals are usually more farsighted: they may predict the subsequent behaviors of other individuals and then maximize their future benefits based on the predictions. In other words, individuals can have *long-term incentives* which depend on other individuals' future actions. For example, when a Twitter user is deciding whether to forward a tweet or not, he may take the future influence of the tweet and the future actions of other users into account. Different from the previous game-theoretic frameworks, our model incorporates both instantaneous incentives and long-term incentives of individuals. We show that the formulated game has a unique symmetric Nash equilibrium (SNE). We observe that the SNE is in pure strategy form and possesses a threshold structure. Furthermore, we design a backward induction algorithm to compute the SNE. From real data, we observe that: (i) most popularity dynamics first increase and then decrease and (ii) for some dynamics, when they are increasing, the increasing speed gradually slows down until they reach the peak and when they are decreasing, the decreasing speed also gradually slows down. We theoretically analyze these properties at the SNE of the proposed game-theoretic model. We find that the equilibrium behavior of the proposed game confirms with real data. The proposed theory is then validated by both simulations and experiments based on real data. It is shown that the proposed game-theoretic model can even predict future dynamics of real data.

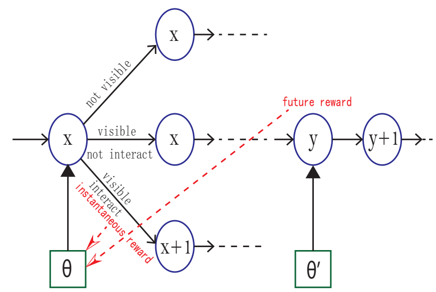


Fig. 1: Illustration of the state transition in the system model. The numbers inside the blue circle are the current states. The numbers inside the green square are the types of the arriving players.

II. Model

In this section, we will introduce a game-theoretic model of the popularity dynamics in detail. Suppose an item, item $\mathcal{A}$, is generated. The item can be an online meme, an online video or a keyword in scientific research. People decide whether to interact with item $\mathcal{A}$ or not sequentially. For instance, Twitter users decide whether to mention a hashtag or not; researchers decide whether to quote a keyword in their papers or not. We view the cumulative interaction dynamics x_t, i.e., the total number of interactions up to time t, as a stochastic dynamical system. We assume people, i.e., players of the game, arrive at the system at discrete time instants $t \in \mathbb{N}$ and decide whether to interact with item $\mathcal{A}$ or not. Players are heterogeneous and have different *types*, which indicate the relevances of the item to the different players. For example, for a Twitter hashtag related to football, football fans have higher types than normal users. We suppose that player's type θ is a random variable distributed in $[0, 1]$ with probability density function (PDF) $h(\theta)$.

As stated in Section I, the utility should encompass both the immediate effect and the future effect of the interactions. Besides, due to the preferential attachment property of networks [4], items which already get many interactions should be more visible, i.e., easier to be found by players arriving at the system. Combining all these factors, the proposed model can be illustrated in Fig. 1. When a player arrive at the system with state x, the item will be *visible* to him with some probability related to the current state of the system. If the item is visible to the player and he chooses to interact with the item, then he will get an *instantaneous reward* which depends on both the type of the player and the state of the system. Afterwards, whenever there is a new interaction with the item (occurs at say, state y), the aforementioned player will obtain a *future reward* because the current interacting player may pay attention to him. The overall utility of the player will be a discounted sum of the instantaneous reward and all the future rewards. In the following, we specify these components of the model in more detail.

A. Instantaneous Reward

Each player choosing to interact with item $\mathcal{A}$ gets an instantaneous reward $R(x, \theta)$, where x is the state of the system when the interaction occurs and θ is the type of the player. Note that $R(x, \theta)$ can also be negative since the interaction implicitly incurs a cost for the player, e.g., by mentioning a hashtag, a Twitter user needs to spend some time and efforts during the manipulation. Now, we impose five assumptions on the function $R(x, \theta)$ as follows.

1) $R(x, \theta)$ decreases with respect to x.
2) $R(x, \theta)$ strictly increases with respect to θ.
3) $R(x, \theta)$ is continuous with respect to θ, for each given x.
4) $R(0, 0) < 0$ and $R(0, 1) > 0$.
5) $\lim_{x\to\infty} R(x, 1) < 0$.

The five assumptions can be justified as follows respectively. (1) Taking timeliness of the item into account, players who interact with the item early (when x is small) should get higher utility than those who interact later (when x is large). For example, a Twitter user mentioning up to date hashtags should gain higher instantaneous reward than a user mentioning outdated hashtags. (2) Those players who find the item more relevant gain higher instantaneous reward by interacting with it. (3) A technical assumption. (4) Initially (i.e., $x = 0$), some players' instantaneous rewards are positive while some are negative. (5) Even for those who find the item very relevant ($\theta = 1$), if the item is very outdated ($x \to \infty$), then it is no longer attractive.

B. Future Reward

For a player B choosing to interact with item $\mathcal{A}$, whenever there is a subsequent interaction with item $\mathcal{A}$, this subsequent interacting player will pay attention to player B with probability $\frac{1}{x}$, where x is the system state when this subsequent interaction occurs. Thus player B will receive an expected reward of $\frac{1}{x}$ due to the possible attention he gets. This reward is called the *future reward* since it is obtained after the interaction occurs. We further assume players discount future reward with factor $0 < \lambda < 1$, which is a common assumption in dynamic games and sequential decision making. The instantaneous reward and the future reward together constitute the utility of an interacting player.

C. Visibility Probability

We assume one player arrives at the system at each time instant. Item $\mathcal{A}$ is *visible* to a player with probability $f(x) \in [0, 1]$, where x is the system state when the player arrives. In other words, after a player arrives, he/she will notice item $\mathcal{A}$ with probability $f(x)$. We also impose several assumptions on the visibility probability function $f(x)$ as follows.

- $f(x)$ increases with x. *Justification: Popular items are more visible. This is also refereed as the 'rich gets richer' or preferential attachment phenomenon in network science [4].*
- $f(0) > 0$. *Justification: Even the most unpopular item is visible with positive probability.*

D. Action Rule and Utility Function

When a player arrives at the system and sees item $\mathcal{A}$, he/she needs to decide whether to interact with the item or not based on the current system state x and his/her type θ. For sake of generality, we allow the players to use mixed action rule $\pi : \mathbb{N}\times[0, 1] :\to [0, 1]$, where $\pi(x, \theta)$ is the probability of choosing the action `interacting` when the state is x and the type of the player is θ. Denote the set of all possible mixed action rules as Π. We denote $g_\pi(x)$ the long-term utility of an interacting player starting from state x while the subsequent players use action rule π.

Denote $p^\pi(x) = \mathbb{E}_\theta[\pi(x, \theta)]$, i.e., the expected probability of a new interaction when the system state is x and users adopt action rule π. Thus, the long term utility $g_\pi(x)$ can be computed recursively as follows. $\forall x \geq 1$:

$$g_\pi(x) = \overbrace{\frac{f(x)p^\pi(x)}{x}}^{\text{instantaneous reward at the current time slot}} + \underbrace{\lambda \left\{ f(x)p^\pi(x)g_\pi(x+1) + [1 - f(x)p^\pi(x)]\, g_\pi(x) \right\}}_{\text{reward in future time slots}}. \quad (1)$$

Denote $u(x, \theta, a, \pi)$ the utility of a type-θ player who enters the system in state x and takes action a while other players adopt action rule π. Thus,

$$u(x, \theta, a, \pi) = \begin{cases} R(x, \theta) + \lambda g_\pi(x + 1), \\ \qquad\qquad \text{if } a = \texttt{interacting}, \\ 0 \qquad\qquad \text{if } a = \texttt{not interacting}. \end{cases} \quad (2)$$

If a player chooses mixed action, i.e., `interacting` with probability q, then his/her utility is:

$$\mathcal{U}(x, \theta, q, \pi) = q[R(x, \theta) + \lambda g_\pi(x + 1)]. \quad (3)$$

E. Solution Concept

In this work, the solution concept is chosen to be the *symmetric Nash equilibrium (SNE)*, which is defined in the following.

Definition 1. *An action rule π^* is said to be a symmetric Nash equilibrium (SNE) if:*

$$\pi^*(x, \theta) \in \arg\max_{q\in[0,1]} \mathcal{U}(x, \theta, q, \pi^*), \quad \forall x \in \mathbb{N}, \theta \in [0, 1]. \quad (4)$$

In an SNE, no player wants to deviate unilaterally, hence the action rule is self enforcing.

III. Equilibrium Analysis

In this section, we show that there is a unique SNE of the formulated game. A backward induction algorithm for computing this unique SNE is also presented.

The infinite-horizon sequential game is effectively of finite length, given the following lemma, which says that no one will interact with item $\mathcal{A}$ after a certain number of interactions is reached.

Lemma 1. *There exists an $\tilde{x} \in \mathbb{N}$ such that $\forall x \geq \tilde{x}, \theta \in [0, 1], \pi \in \Pi$:*

$$u(x, \theta, \mathit{interacting}, \pi) < u(x, \theta, \mathit{not\ interacting}, \pi), \quad (5)$$

i.e., the action `interacting` *is strictly dominated by the action* `not interacting` *regardless of the player type and other players' action rule.*

Denote $\hat{x} = \max\{x \in \mathbb{N} | R(x, 1) > 0\}$. We design a backward induction algorithm, Algorithm 1, to compute the SNE. We first show that the action rule obtained from Algorithm 1 is indeed an SNE.

Algorithm 1 Computation of the unique equilibrium

Inputs:

Instantaneous reward function $R(x, \theta)$.
Player type PDF $h(\theta)$.
Visibility probability function $f(x)$.
Discount factor λ.

Outputs:

Unique equilibrium action rule $\pi^*(x, \theta)$.

When $x \geq \hat{x} + 1$: $\pi^*(x, \theta) = 0, \theta \in [0, 1]$; $g_{\pi^*}(x) = 0$.
Let $x = \hat{x}$.
while $x \geq 0$ **do**
  if $R(x, 0) + \lambda g_{\pi^*}(x + 1) > 0$ **then**
    $\theta_x^* = 0$,
  else
    θ_x^* is the unique solution of $R(x, \theta) + \lambda g_{\pi^*}(x + 1) = 0$.
  end if
  Compute:

$$\pi^*(x, \theta) = \begin{cases} 1, & \text{if } \theta \geq \theta_x^*, \\ 0, & \text{if } \theta < \theta_x^*, \end{cases} \tag{6}$$

$$p^{\pi^*}(x) = \int_{\theta_x^*}^{1} h(\theta) d\theta, \tag{7}$$

$$g_{\pi^*}(x) = \frac{1}{1 - \lambda[1 - f(x) p^{\pi^*}(x)]} \tag{8}$$

$$\left[\frac{f(x) p^{\pi^*}(x)}{x} + \lambda f(x) p^{\pi^*}(x) g_{\pi^*}(x + 1) \right]. \tag{9}$$

  $x \leftarrow x - 1$.
end while

Theorem 1. *(Existence of the SNE) The action rule π^* computed by Algorithm 1 is an SNE.*

We further show that the π^* computed in Algorithm 1 is indeed the unique SNE.

Theorem 2. *(Uniqueness of the SNE) Suppose the distribution of player type θ is atomless, i.e., $h(\theta)$ is finite for every $\theta \in [0, 1]$. Then, any SNE $\tilde{\pi}$ differs with π^* for zero mass players, i.e., $\mathbb{P}\{\tilde{\pi}(x, \theta) \neq \pi^*(x, \theta)\} = 0$ for every $x \in \mathbb{N}$.*

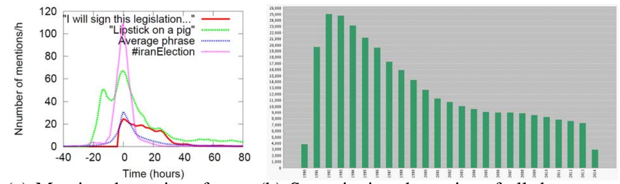
Remark 1. *The unique SNE of the game is in pure strategy form and possesses a threshold structure. For every state x, there exists a threshold θ_x^* such that a player of type θ will interact with item $\mathcal{A}$ if and only if $\theta \geq \theta_x^*$.*

IV. Popularity Dynamics at the Equilibrium

In this section, we first observe some properties of popularity dynamics from real data. Then, we analyze the corresponding properties at the equilibrium of the proposed game. We find that the equilibrium behavior of the proposed game-theoretic model confirms with the real data.

A. Observations from real data

In Fig. 2, we plot mention dynamics of popular memes and sum citation dynamics of all the papers published in *Nature* in 1990.



(a) Mention dynamics of popular memes. (b) Sum citation dynamics of all the papers published in Nature in 1990.

Fig. 2: Real-world popularity dynamics [7], [20].

Here, we use the dynamics of memes and the citation dynamics of papers as examples of popularity dynamics. We observe that, typically, the popularity dynamics of an item will first increase and then decrease, leading to a peak in the dynamics. This is a general *first order* property of popularity dynamics. Thus, a natural question is: does the equilibrium behavior of the proposed game-theoretic model possess this property? Furthermore, we observe that some popularity dynamics, especially the citation dynamics of papers as in Fig. 2-(b) and Fig. 5-(c)(d), have the following *second order* property: when it is increasing, its increasing speed gradually slows down and when it is decreasing, its decreasing speed also gradually slows down. This behavior is reasonable. Many items' interaction rates increase drastically when they first come out and keep increasing (but at a lower speed) until they reach the peak. Later, after the items are no longer that popular, their interaction rates decrease quickly and will keep decreasing for some time (but at a lower speed). In next subsection, we will formally state the aforementioned two properties under certain assumptions. The proofs are omitted due to space limitation.

B. Properties at the equilibrium

Generally, the unique SNE should be computed using the backward induction as specified in Algorithm 1, which is hard to analyze. To facilitate analysis, we further restrict attention to models satisfying the following three assumptions:

(1) (Linear reward) $R(x, \theta) = -x + a\theta - b$, where $a > b > 0$.
(2) (Uniform player type distribution) $h(\theta) = 1, \ \forall \theta \in [0, 1]$.
(3) (Saturated visibility probability) There exists a $\check{x} \in \mathbb{N}$ less than $\hat{x} = \lfloor a - b \rfloor$ such that: $\forall x \geq \check{x}: \ f(x) = 1$, and $\forall 1 \leq x \leq \check{x}$:

$$\frac{f(x)}{f(x-1)} \geq 1 + \frac{1 + \frac{\lambda}{x}}{a - b - \check{x}}. \tag{10}$$

These three assumptions can be justified as follows. (1) Linear reward is used to simplify the analysis and it indeed increases with θ and decreases with x, which coincides with the assumptions in Section III. (2) Uniform player type distribution is also to simplify calculations though our analysis is applicable to more complicated distributions in principle. (3) When the number of interactions is large enough, the item becomes 'famous' enough so that it is visible to everyone arriving at the system. Before this saturation occurs, however, it increases at a speed not too slow. Note that the R.H.S. of (10) is very close to 1 since the numerator of the ratio is close to 1 while the denominator is some integer much larger than 1 generally. So, the assumption is indeed very weak.

Denote $r(x) = f(x) p^{\pi^*}(x)$, i.e., the probability that there is a new interaction at state x in the SNE. We first show the first order property of the SNE.

Theorem 3. ***(First order characterization of the SNE)*** *Suppose the assumptions (1)(2)(3) hold, the SNE π^* satisfies the following:*

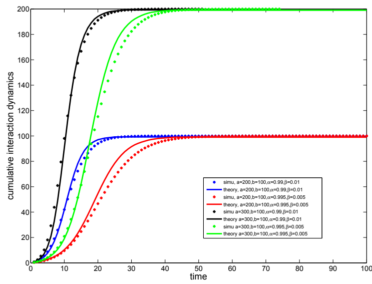
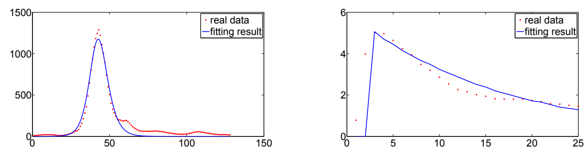


Fig. 3: Simulation results under different parameter setups.



(a) #googlewave

(b) Nature

Fig. 4: Fitting Twitter hashtag and citation dynamics.

- *For* $1 \leq x \leq \breve{x}$*:* $r(x) \geq r(x-1)$*;*
- *For* $x \geq \breve{x}$*:* $r(x) \geq r(x+1)$*.*

In other words, the interaction rate $r(x)$ *first increases and then decreases.*

Next we turn to the second order property of the SNE.

Theorem 4. ***(Second order characterization of the SNE)*** *Suppose that the assumptions (1)(2)(3) hold. Further assume that (i)* $\lambda \leq \frac{1}{a-b}$ *and (ii)* $\forall\ 2 \leq x \leq \breve{x}$*:* $f(x) + \left(1 + \frac{1+\frac{\lambda}{x}}{a-b-\breve{x}}\right) f(x-2) \leq 2f(x-1)$*. Then the SNE* π^* *satisfies the following:*

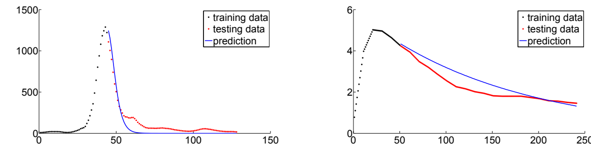
- *For* $2 \leq x \leq \breve{x}$*:* $0 \leq r(x) - r(x-1) \leq r(x-1) - r(x-2)$*;*
- *For* $x \geq \breve{x}+2$*:* $0 \leq r(x-1) - r(x) \leq r(x-2) - r(x-1)$*.*

In other words, we have: (a) when $r(x)$ *is increasing, its increasing speed gradually slows down; (b) when* $r(x)$ *is decreasing, its decreasing speed also gradually slows down.*

Remark 2. *Assumption (i) of Theorem 4 requires the discount factor* λ *to be sufficiently small, or in other words, players of the popularity dynamics game are myopic and don't care about future rewards very much. Assumption (ii) is basically equivalent to* $f(x) - f(x-1) \leq f(x-1) - f(x-2), \forall 2 \leq x \leq \breve{x}$ *since the ratio in the parenthesis of (ii) is usually very small. This requires* $f(x)$*'s increasing speed is slowing down as* x *approaches* $\breve{x}$*, which is a reasonable assumption. Moreover, we notice that Theorem 4 does not cover all the situations of popularity dynamics. There are real-world popularity dynamics, such as those in Fig. 2-(a), which have more complicated second order patterns. For example, during increasing phase of the dynamics, the increasing speed can first increase and then decrease. Due to the intricacy of these second order patterns, we don't give theoretical discussions about them here.*

V. Simulations and Real Data Experiments

In this section, we conduct simulations and real data experiments to validate the theoretical results obtained. We choose the form of instantaneous reward function to be linear, i.e., $R(x, \theta) = -x + a\theta - b$, where $a > b > 0$.



(a) #googlewave

(b) Nature

Fig. 5: Predicting future dynamics.

A. Simulations

In our simulation, we define the visibility probability function $f(x)$ in the following form:

$$f(x) = \alpha \left(1 - e^{-\beta x}\right) + 1 - \alpha, \tag{11}$$

where α, β are parameters controlling the initial visibility probability and the increasing speed of the visibility probability. We set the discount factor to be $\lambda = 0.5$. For different parameter setups of a, b, α, β, we stochastically simulate the equilibrium popularity dynamics calculated by Algorithm 1 many times and then take average of them. We also theoretically compute the expected equilibrium popularity dynamics by Algorithm 1, which serve as the theoretical dynamics. Both theoretical cumulative dynamics and simulated cumulative dynamics are shown in Fig. 3, from which we observe that (i) the theoretical dynamics match well with the simulated dynamics; (ii) the proposed game-theoretic model can flexibly generate popularity dynamics of different shapes by tuning the parameters.

B. Real data experiments

In this subsection, real data experiments are carried out to verify that the proposed theory matches well with the real-world popularity dynamics. The datasets we use here are Twitter hashtag dataset [7] and the citation data of papers from the Web of Science [20]. Due to space limitation, we only show results on individual examples to manifest typical behaviors. In Fig. 4, we use the equilibrium computed by Algorithm 1 to fit the mention dynamics of a popular Twitter hashtag #googlewave and the ciation dynamics of papers published in Nature 1990. We observe that the theoretical dynamics match well with the real-world dynamics, though the temporal shape of the citation dynamics are very different from that of the Twitter hashtag dynamics, confirming the universality of our theory for popularity dynamics.

Additionally, we can even exploit the equilibrium of the proposed game to predict future dynamics for real data. To this end, we use part of the dynamics data to train the proposed game-theoretic model and then predict future dynamics by using the trained model. The prediction results are reported in Fig. 5, from which we see that the prediction is quite accurate.

VI. Conclusion

In this paper, a sequential game is proposed to characterize the mechanisms of popularity dynamics. We prove that the proposed game has a unique SNE, which is a pure strategy action rule with a threshold structure and can be computed using a backward induction algorithm. Moreover, at the equilibrium of the proposed game, we analyze some properties observed from the real data, demonstrating that the equilibrium behavior of the proposed game confirms with real-world popularity dynamics. The theory is validated by both simulations and experiments based on real data. The proposed model can even predict future dynamics.

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