

# Two-Way Network-Coded Relaying With Delay Constraint

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**Abstract**—Random channel fading may cause serious channel outage, and automatic repeat-request protocol has been widely used to mitigate packet loss by retransmitting the packet when the receiver fails to decode that packet. To restrict the transmission delay, a single packet might be retransmitted up to a limited number of times. For wireless two-way relay channels, the packet could be retransmitted by the original source node or the relay nodes, and the delay constraint might be imposed in a per-hop manner or end-to-end manner. For single-relay networks, we study the throughput of several relaying strategies including pure relaying and network-coded relaying subject to the delay constraint. We demonstrate that the binary XOR-based network coding can greatly improve the system throughput, but the throughput gain is upper bounded. We also develop a near-optimum power allocation scheme to maximize the system throughput. For multirelay networks, we show that using digital network coding alone may cause some throughput loss when the frame length is much smaller than the number of relays, and we develop a hybrid network coding scheme to recover such throughput loss.

**Index Terms**—Network coding, relay, throughput, delay.

## I. INTRODUCTION

TWO-WAY communication has found many applications in the cellular uplink/downlink. Traditionally, uplink and downlink communications operate on different time/frequency channels through direct link only, which has the disadvantage that the transmission could be in outage when the direct link experiences deep fading [2]. One solution to this issue is to place some relay nodes between the two source nodes. The relay nodes may help forward the source packets on both directions, thus providing another propagation path for each source packet to reach the other end. Due to cooperative diversity, the overall packet loss rate could be greatly reduced [3], [4].

Early work on wireless relaying focuses mainly on one-way communication, and various relaying schemes have been studied in [4]. Due to half-duplex constraint, wireless devices may not transmit and receive on the same channel simultaneously. Consequently, it takes two separate channels to relay a packet from the source to the destination, and a pre-log factor 1/2 is

Manuscript received January 9, 2014; revised May 3, 2014; accepted June 24, 2014. Date of publication July 2, 2014; date of current version January 7, 2015. This paper was presented in part at the IEEE GLOBECOM, December 2013. The associate editor coordinating the review of this paper and approving it for publication was H. Yousefi-zadeh.

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Digital Object Identifier 10.1109/TWC.2014.2335202

induced in the spectral efficiency [4], [5]. Those half-duplex relaying schemes might be applied for the two-way relay channel (TWRC) as well, where the relay nodes use separate channels to relay packets on each direction and as a result, a total of four channels are required. However, this natural extension may incur some loss in bandwidth efficiency, as the relay nodes serve as pure routers for each source packet. If some network coding [6] could be performed to mix different packets within the relay branch, certain channel uses might be put together and the spectral efficiency could be improved accordingly.

For TWRC, there are two generic ways to categorize different network coding schemes. Depending on *where* to mix the source packets, there are link-layer network coding (LLNC) and physical-layer network coding (PLNC) [7]. For LLNC, the relay nodes need two separate channels to obtain the copy of each source packet. The relay nodes then mix those packets locally and broadcast a single network-coded packet through the third channel [8], [9]. PLNC could further reduce the total channel use to two by letting both source packets reach the relay nodes through a common multiple-access channel. Because of the additive nature of wireless medium, those packets would be mixed automatically in the air [10]–[12]. Another way for categorizing different network coding schemes is according to *how* to mix the source packets. For digital network coding (DNC), the source packets are mixed in the finite field through bitwise XOR operations [8], [10], [12]. Therefore, the output of the relay nodes is a quantized message. By contrast, for analog network coding (ANC) the source messages are mixed in the complex field directly [11], [13], [14]. As a result, the output of the relay nodes is an analog signal containing additive noise. Note that both schemes have some shortcomings. For DNC, the quantized message may contain some errors, which may propagate to the intended receiver and cause diversity loss [15], [16]. For ANC, the co-channel interference and the noise amplification effect may reduce the diversity gain as well [17].

There is a large body of literature studying the achievable rate of TWRC with various relaying schemes [5], [8], [9], [12]–[14], [18]–[23]. Interestingly, it has been demonstrated that there does not exist a universally optimum scheme that strictly dominates all the other schemes, and a mixed relaying protocol may perform better under some channel conditions [20], [21]. The situation becomes even more complicated when there are multiple relays in the system, as different space-time coding schemes could be employed to pursue cooperation gain. In [5], an orthogonalize-and-forward scheme is proposed to realize multi-pair two-way communication, where joint processing within the relay array is performed to eliminate the co-channel interference. In [15], the authors study the diversity gain of

denoise-and-forward scheme and demonstrate that the detection error may reduce the diversity gain roughly by one-half. In [24], the authors study the diversity-multiplexing tradeoff of multiple-input multiple-output TWRC and demonstrate that the compress-and-forward strategy achieves the optimal tradeoff in the full-duplex case. In [25], the authors propose a family of multi-hop and multi-relay protocols and analyze the achievable rate regions.

Note that most of the aforementioned literature is from an information-theoretic view, which assumes ideal channel coding and thus ignores the packet transmission error. For practical systems, packet loss does occur due to random channel fading. To combat channel erasure and improve link reliability, automatic repeat-request (ARQ) protocol has been widely used. For simple ARQ, the same packet will be retransmitted if the receiver fails to decode that packet. An enhanced scheme, which is called hybrid ARQ, can be carried out by jointly using ARQ and forward error correction. For hybrid ARQ, all the previously erroneous packets are stored at the receiver, and those packets may be combined with the retransmitted packets before channel decoding. Compared to simple ARQ, hybrid ARQ scheme requires significantly more memory, as all previously packets are stored in terms of soft information [26]. This is a serious issue especially for block transmissions where multiple packets are transmitted altogether as a whole.

For wireless relay channels, the packets may be retransmitted by either the source node or relay nodes. The retransmitted packets intended for different receivers might be mixed through network coding to improve the bandwidth efficiency [27]. Such network-coded ARQ have found many applications in broadcast channel [28], wireless multicast [29] and multiple unicast flows [30]. In this work, we concentrate on the applications of network-coded ARQ for TWRC. Some related work can be found in [12], [13], and [31]–[34] and the references therein, where the authors propose different network-coded ARQ schemes and study the corresponding throughput. However, those studies on TWRC have various limitations.

1) In previous studies, it is usually assumed that the packet may be retransmitted infinitely many times until it is successfully delivered. Consequently, packet loss is ignored and the transmission delay could be indefinite. But for most real systems, the maximum number of retransmissions of the same packet is usually upper bounded for the purpose of reducing the latency. Consequently, packet loss does occur occasionally when, for example, the channel remains in outage for several retransmission sessions. To the best of our knowledge, the achievable throughput with such delay constraint has not been well studied before. In this work, we incorporate such delay constraint into our system model, and we derive the closed-form throughput after considering packet loss.

2) The previous studies [12], [13], [31]–[34] focus only on single-relay TWRC. But for a general multi-relay TWRC, how to design a sound network-coded ARQ scheme for combating channel erasure is still an open problem. The multi-relay scenario is naturally more involved in that the source packets could be decoded by very different relay nodes. Consequently, each relay node may have very limited freedom to mix the local packets only, and the gain of network coding could be quite

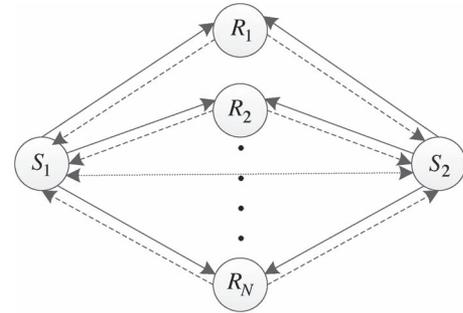


Fig. 1. Diagram of a two-way relay channel.

limited. To address this issue, we propose a hybrid network coding scheme in which the relay nodes could choose to mix the packets locally using DNC or mix the packets in the air using ANC according to the network dynamics, such that the packets intended for different source nodes could be mixed together whenever possible to fully leverage the power of network coding.

3) Power allocation in the presence of packet loss is an important system optimization problem, but it is totally ignored in the work [12], [13], [31]–[34]. In contrast, in this work we consider how to properly allocate the transmitted power within the network for maximizing the throughput. We derive a near-optimum power allocation scheme, which demonstrates significant throughput gain for asymmetric networks.

The rest of this paper is organized as follows: We describe the system model in Section II, and study several ARQ schemes for single-relay networks in Section III. Then in Section IV we turn to the multi-relay scenario. Some simulation results can be found in Section V, and some conclusions are drawn in Section VI.

*Notations:* The abbreviation i.i.d. stands for independent and identically distributed. The probability of an event  $\mathcal{A}$  is denoted by  $\Pr(\mathcal{A})$ . The cumulative distribution function (CDF) and the probability mass function (PMF) of a random variable  $Z$  are denoted by  $F_Z(z)$  and  $f_Z(z)$ , respectively. We shall use  $Z \sim \mathcal{CN}(\mathbf{u}, \Sigma)$  as a circularly symmetric complex Gaussian random variable vector with mean  $\mathbf{u}$  and covariance matrix  $\Sigma$ .  $Z \sim \text{Bin}(p, n)$  stands for binomial distribution with parameter  $p$  and  $n$ .  $Z \sim \text{Geom}(p)$  stands for geometric distribution with parameter  $p$ , i.e.,  $f_Z(k) = (1-p)p^{k-1}$  for  $k = 1, 2, \dots$

## II. SYSTEM MODEL

Consider a general multi-relay TWRC shown in Fig. 1, where the two source nodes  $S_1$  and  $S_2$  want to exchange data with the help of  $N$  relay nodes  $R_k$  for  $k = 1, 2, \dots, N$ . Suppose the source packets are sent in a frame-by-frame manner, and each frame consists of  $K$  packets. The  $k$ th packet of  $S_i$  is denoted by  $X_i(k)$  for  $i = 1, 2$ . Each packet consists of packet header and packet body. The header usually encapsulates some control information, and the body basically carries the data. For simplicity, we assume all the packets are of the same length, and the data rate is fixed and is denoted by  $r$ . The time for transmitting a packet through any point-to-point channel is referred to as one time unit.

The three-phase relaying protocols are studied in this work. Specifically, in the first two source transmission phases, one source node is the transmitter and the relay nodes and the other source node are the receivers. In the third relaying phase, the relay nodes are the transmitters and the two sources are the receivers. Under all conditions, the source node would not transmit concurrently with any relay node. The signal model for data transmission from node  $i$  to node  $j$  is given by

$$y_{ij} = h_{ij} \sqrt{P_i} x_i + n_{ij}. \quad (1)$$

Here  $P_i$  is the transmitted power of node  $i$ ,  $x_i$ , and  $y_{ij}$  are the transmitted signal and the received signal, respectively,  $n_{ij} \sim \mathcal{CN}(0, 1)$  is the additive white Gaussian noise,  $h_{ij} \sim \mathcal{CN}(0, \lambda_{ij})$  is the fading channel coefficient, where  $\lambda_{ij}$  is the channel gain determined by the distance between the transmitter and receiver. All channel coefficients and additive noises are independent.

Due to random channel fading, the receiver may be unable to decode the transmitted packet. Usually, some parity bits are appended to the raw data, based on which the receiver can perform cyclic redundant check to tell whether the decoding is successful. In this work, we assume such error detection is perfect. The packet error rate is denoted by  $q_{ij}$ . Suppose some capacity-achieving channel coding scheme is applied in the physical layer, then the packet loss rate is approximately equal to the channel outage rate given by

$$q_{ij} \approx \Pr(\log_2(1 + |h_{ij}|^2 P_i) \leq r) = 1 - \exp\left(-\frac{2^r - 1}{P_i \lambda_{ij}}\right). \quad (2)$$

Depending on the decoding status of the packet, the receiver needs to feed back the acknowledge (ACK) or negative acknowledge (NACK) signal to inform the transmitter that it has successfully decoded a packet or not. In this work, we assume that the feedback channel is perfect. We also ignore the feedback overhead, which is justifiable when the size of the data packet is much larger than the size of control signals.

We consider using ARQ protocol to combat channel erasure.<sup>1</sup> That is, once the receiver fails to decode the packet, it feeds back the NACK signal and discards the wrong packet immediately. Afterwards, the transmitter will retransmit the same packet until the receiver successfully decodes that packet or the number of retransmissions of the current packet has reached an upper bound. The purpose of imposing such constraint is to restrict the transmission delay of the entire frame. In this work, we consider two types of delay constraints, i.e., per-hop constraint and end-to-end (E2E) constraint, both of which could be justified from the engineering perspective. For per-hop constraint, packet transmissions on different point-to-point channels are regarded as independent sessions, and the number

<sup>1</sup>Hybrid ARQ is not considered because of its significant memory consumption [26]. This issue is especially serious for the block transmission scenario studied in this work, as a single block may involve multiple packets. Indeed, the retransmission protocol (i.e., ARQ or hybrid ARQ) would impact only the decoding performance and throughput, and it is independent of the relaying protocols which specify the actions under each possible decoding state of packets. Therefore, the relaying protocols we shall study later work equally well for hybrid ARQ.

of transmissions of each session cannot exceed a threshold denoted by  $L$ . In contrast, for E2E constraint the transmission of the same packet from the original source to the final sink is regarded as an entire session, and the total number of transmissions of this E2E session cannot exceed the threshold  $L$ , regardless of the transmission routes. Here  $L$  is a system parameter and could be determined by the maximum tolerable delay. To give an example, suppose there is packet sent from  $S_1$  to  $S_2$  and the transmission route is  $S_1 \rightarrow R_1 \rightarrow S_2$ . Then for per-hop constraint,  $S_1$  can transmit the packet up to  $L$  times and if  $R_1$  can successfully decode this packet within  $L$  attempts, it can retransmit this packet to  $S_2$  for another  $L$  times. In contrast, for E2E constraint the total number of transmissions of this packet cannot exceed  $L$ , regardless if it is sent by  $S_1$  or  $R_1$ .

As the performance measure, throughput is defined as the average number of successfully delivered packets per time unit for each frame, which is given by

$$\eta = \frac{E[M]}{E[T]}, \quad (3)$$

where  $M$  and  $T$  are the total number of successfully delivered packets and the total number of transmissions to exchange a frame of  $K$  packets between  $S_1$  and  $S_2$ , respectively. In this work, we assume all nodes are subject to half-duplex constraint such that they cannot transmit and receive at the same time. As a result, the throughput is bounded by  $0 \leq \eta \leq 1$ . The exact throughput is dependent on the transmission strategies, which will be described in the next section.

### A. Some Preliminaries

Before leaving this section, we first present some mathematical results that will be used repeatedly in later sections for analyzing the throughput performance. For notational convenience, we define the bounded geometric distribution (BGeom) as  $Z = \min(X, L) \sim \text{BGeom}(p, L)$ , where  $X \sim \text{Geom}(p)$  for  $0 \leq p \leq 1$  and  $L > 0$  is some integer. After some simple algebra, it is easy to show that the PMF and CDF of  $Z$  are respectively given by

$$f_Z(k) = \begin{cases} (1-p)p^{k-1}, & k = 1, 2, \dots, L-1 \\ p^{L-1}, & k = L, \end{cases} \quad (4)$$

and

$$F_Z(k) = \begin{cases} 0, & k < 0 \\ 1 - p^k, & k = 0, 1, \dots, L-1 \\ 1, & k \geq L. \end{cases} \quad (5)$$

Besides, we can also show that

$$E[Z] = \sum_{k=1}^L \Pr(Z \geq k) = \sum_{k=1}^L p^{k-1} = \begin{cases} \frac{1-p^L}{1-p}, & p \neq 1 \triangleq g_L(p). \\ L, & p = 1 \end{cases} \quad (6)$$

With these results, we can prove the following lemma.

*Lemma 1:* Let  $X \sim \text{BGeom}(p, L)$  for  $0 \leq p \leq 1$  and  $Y \sim \text{BGeom}(q, L)$  for  $0 \leq q \leq 1$  be independent, then

- 1)  $Z = \min(X, Y) \sim \text{BGeom}(pq, L)$  and  $E[Z] = g_L(pq)$ .
- 2) Let  $Z = \max(X, Y)$ , then

$$E[Z] = g_L(p) + g_L(q) - g_L(pq). \quad (7)$$

- 3) Let  $Z \sim \text{BGeom}(a, L - X)$  for  $0 \leq a \leq 1$ . Then for  $0 \leq p < 1$ , we have

$$\begin{aligned} E[Z|X \leq L - 1] &= \frac{1}{1 - a} \left( 1 - \frac{1 - p}{1 - p^{L-1}} a^{L-1} g_{L-1} \left( \frac{p}{a} \right) \right) \\ &\triangleq h_0(a, p; L). \end{aligned} \quad (8)$$

- 4) Let  $W \sim \text{BGeom}(a, L - X)$  for  $0 \leq a \leq 1$  and  $T \sim \text{BGeom}(b, L - Y)$  for  $0 \leq b \leq 1$ , and define  $Z = \max(W, T)$ . Then for  $0 \leq p, q < 1$ , we have

$$\begin{aligned} E[Z|\max(X, Y) \leq L - 1] &= \frac{1}{1 - p^{L-1}} \left( g_{L-1}(a) - p^{L-1} g_{L-1} \left( \frac{a}{p} \right) \right) \\ &+ \frac{1}{1 - q^{L-1}} \left( g_{L-1}(b) - q^{L-1} g_{L-1} \left( \frac{b}{q} \right) \right) \\ &- \frac{g_{L-1}(ab) + (pq)^{L-1} g_{L-1} \left( \frac{ab}{pq} \right)}{(1 - p^{L-1})(1 - q^{L-1})} \\ &+ \frac{q^{L-1} g_{L-1} \left( \frac{ab}{q} \right) + p^{L-1} g_{L-1} \left( \frac{ab}{p} \right)}{(1 - p^{L-1})(1 - q^{L-1})} \\ &\triangleq h_1(a, b, p, q; L). \end{aligned} \quad (9)$$

*Proof:* See Appendix. ■

### III. SINGLE-RELAY NETWORK

In this section, we first study several transmission strategies for the single-relay network, i.e.,  $N = 1$ . The multi-relay network will be considered in the next section. For notational convenience, we use  $P_1$ ,  $P_2$ , and  $P_R$  to represent the transmitted power of  $S_1$ ,  $S_2$ , and  $R_1$ , respectively. The packet loss rate for the channel  $i \rightarrow j$  is denoted by  $q_{i,j}$  for  $i, j \in \{1, 2, R\}$ .

#### A. Direct Transmission

A simple strategy is to let the two-way communication go through the direct link alone without using the relay. Due to half-duplex constraint, the two source nodes need take turns to send a frame of  $K$  packets to the other end in a time-division multiplex manner. For each packet of  $S_1$ , it would be either successfully delivered to  $S_2$  (denoted by the event  $I_1 = 1$ ) or discarded after reaching the maximum number of transmissions (denoted by the event  $I_1 = 0$ ). The total number of transmissions of each packet can be represented by  $T_1 = \min(X_1, L) \sim \text{BGeom}(q_{1,2}, L)$ , where  $X_1 \sim \text{Geom}(q_{1,2})$ . The probability of successful delivery is given by

$$\Pr(I_1 = 1) = \Pr(X_1 \leq L) = 1 - q_{1,2}^L, \quad (10)$$

and the average number of transmissions is  $E[T_1] = g_L(q_{1,2})$ . Due to symmetry, the above statistics should be similar for any packet delivered along the reverse direction after properly modifying the subscripts. Therefore, the throughput of direct transmission is

$$\begin{aligned} \eta_{DT} &= \frac{K \times E[\mathbb{I}\{I_1 = 1\} + \mathbb{I}\{I_2 = 1\}]}{K \times E[T_1 + T_2]} \\ &= \frac{2 - q_{1,2}^L - q_{2,1}^L}{\frac{1 - q_{1,2}^L}{1 - q_{1,2}} + \frac{1 - q_{2,1}^L}{1 - q_{2,1}}} \stackrel{L \gg 1}{\approx} \frac{2(1 - q_{1,2})(1 - q_{2,1})}{2 - q_{1,2} - q_{2,1}}, \end{aligned} \quad (11)$$

where  $\mathbb{I}\{\cdot\}$  is the indicator function.

#### B. Pure Relaying

If the relay node is located somewhere between the two source nodes, the relay link could be much better than the direct link. As a result, if the relay node happens to successfully decode the source packets, it can retransmit the packet for the original source to improve the transmission quality. One intuitive relaying strategy is to let the relay node purely forward the decoded packets in different time slots without performing any coding. Although a similar scenario has been studied in the context of one-way relaying [35], the authors mainly study the throughput through simulations and no analytical results are derived. In contrast, we are seeking to derive the closed-form throughput subject to delay constraint.

The entire two-way communication completes in three phases. In the first two phases, the two sources take turns to send out a frame of  $K$  packets. Then in the third relaying phase, the relay node would forward all the packets stored in the local buffer to the intended receivers. For each packet sent by  $S_1$ , it will be retransmitted for up to  $L$  times until it is successfully decoded by either the relay node  $R_1$  or the receiver  $S_2$ , otherwise the packet will be discarded after the  $L$ th attempt. The relay node  $R_1$  then stores all the decoded packets in the local buffer, provided that the same packet has not been decoded by  $S_2$ . For E2E constraint, the extra requirement is that when the relay is able to decode that packet, the number of transmissions has not reached the upper limit  $L$ .

Eventually, each packet sent by  $S_1$  will be either successfully delivered to  $S_2$  (denoted by the event  $I_1 = 1$ ), or decoded and stored by  $R_1$  but yet not decoded by  $S_2$  (denoted by the event  $I_1 = -1$ ), or discarded after reaching the maximum number of transmissions (denoted by the event  $I_1 = 0$ ). The total number of transmissions is given by  $T_1 = \min(T_{1,R}, T_{1,2})$ , where  $T_{1,2} = \min(X_{1,2}, L)$  and  $T_{1,R} = \min(X_{1,R}, L)$ , and  $X_{1,2} \sim \text{Geom}(q_{1,2})$  and  $X_{1,R} \sim \text{Geom}(q_{1,R})$  are independent. The probability that  $S_2$  decodes the packet is given by

$$\begin{aligned} \Pr(I_1 = 1) &= \sum_{n=1}^L \Pr(X_{1,2} = n, X_{1,R} \geq n) \\ &= \frac{1 - q_{1,2}}{1 - q_{1,R}q_{1,2}} (1 - q_{1,R}^L q_{1,2}^L). \end{aligned} \quad (12)$$

The probability that the packet is decoded and stored by  $R_1$  but yet not decoded by  $S_2$  is given by

$$\begin{aligned} \Pr(I_1 = -1) &= \sum_{n=1}^L \Pr(X_{1,R} = n, X_{1,2} > n) \\ &= \frac{(1 - q_{1,R})q_{1,2}}{1 - q_{1,R}q_{1,2}} (1 - q_{1,R}^L q_{1,2}^L) \end{aligned} \quad (13)$$

for per-hop constraint, and

$$\begin{aligned} \Pr(I_1 = -1) &= \sum_{n=1}^{L-1} \Pr(X_{1,R} = n, X_{1,2} > n) \\ &= \frac{(1 - q_{1,R})q_{1,2}}{1 - q_{1,R}q_{1,2}} (1 - q_{1,R}^{L-1} q_{1,2}^{L-1}) \end{aligned} \quad (14)$$

for E2E constraint. By using property 1 of *Lemma 1*, we can also calculate the average number of transmissions of each packet as  $E[T_1] = g_L(q_{1,R}q_{1,2})$ .

After the two source transmission phases, the relay  $R_1$  forwards the packets stored in the local buffer to the intended receivers sequentially. This is essentially a direct transmission process from  $R_1$  to  $S_1$  or  $S_2$ . Let  $\mathbb{D}_i$  be the set of packets of  $S_i$  that are stored in the buffer, and the set size is denoted by  $D_i = |\mathbb{D}_i|$ . The relay needs to manage a set  $\mathbb{R}_i$  that records the maximum number of transmissions of each packet in  $\mathbb{D}_i$ . At the beginning of the relaying phase, the maximum number of transmissions of a packet is equal to  $L$  for per-hop constraint and  $L - T_i$  for E2E constraint,<sup>2</sup> respectively. Note that the two sets  $\mathbb{D}_i$  and  $\mathbb{R}_i$  have the same size  $D_i$  that satisfies binomial distribution, i.e.,  $D_i \sim \text{Bin}(Q_i, K)$  where  $Q_i = \Pr(I_i = -1)$  is given by (13) and (14). For each packet transmitted from  $R_1$  to  $S_2$ , the total number of transmissions are respectively given by  $T_{R,2} = \min(X_{R,2}, L)$  for per-hop constraint, and  $T_{R,2} = \min(X_{R,2}, L - T_1)$  for E2E constraint under the condition that  $T_1 \leq L - 1$ , where  $X_{R,2} \sim \text{Geom}(q_{R,2})$ . Let  $\{I_{R,2} = 1\}$  and  $\{I_{R,2} = 0\}$  represent the event of successful delivery and the event of packet drop after reaching the maximum number of transmissions, respectively. Then for per-hop constraint, the probability of successful delivery is

$$\Pr(I_{R,2} = 1) = \Pr(X_{R,2} \leq L) = 1 - q_{R,2}^L \quad (15)$$

and we have  $E[T_{R,2}] = g_L(q_{R,2})$ . For E2E constraint, we have

$$\begin{aligned} \Pr(I_{R,2} = 1) &= \Pr(X_{R,2} \leq L - T_1 | T_1 \leq L - 1) \\ &= 1 - \frac{1 - q_{1,R}q_{1,2}}{1 - q_{1,R}^{L-1} q_{1,2}^{L-1}} q_{R,2}^{L-1} g_{L-1} \left( \frac{q_{1,R}q_{1,2}}{q_{R,2}} \right). \end{aligned} \quad (16)$$

By using property 1 and property 3 of *Lemma 1*, we have  $E[T_{R,2}] = h_0(q_{R,2}, q_{1,R}q_{1,2}; L)$ .

<sup>2</sup>For E2E constraint, the maximum number of transmissions could be different for packets stored at the relay. Here we omit the time index of different packets for notational convenience.

With the above results, we can write the total number of successfully delivered packets in all three phases as

$$\begin{aligned} E[M] &= \sum_{j=\{1,2\} \setminus \{i\}}^2 (KE \mathbb{I}\{I_i = 1\} + E[D_i]E \mathbb{I}\{I_{R,j} = 1\}) \\ &= K \sum_{j=\{1,2\} \setminus \{i\}}^2 (\Pr(I_i = 1) + \Pr(I_i = -1) \Pr(I_{R,j} = 1)), \end{aligned} \quad (17)$$

and the total number of transmissions for exchanging the entire frame between  $S_1$  and  $S_2$  is given by

$$\begin{aligned} E[T] &= \sum_{i=1, j=\{1,2\} \setminus \{i\}}^2 (KE[T_i] + E[D_i]E[T_{R,j}]) \\ &= K \sum_{i=1, j=\{1,2\} \setminus \{i\}}^2 (E[T_i] + \Pr(I_i = -1)E[T_{R,j}]), \end{aligned} \quad (18)$$

After inserting (12)–(16) into the above two expressions, we could obtain the closed-form throughput using (3). When the maximum number of transmissions is sufficiently large (i.e.,  $L \gg 1$ ), the two types of constraints lead to the same asymptotic throughput given by

$$\eta_{\text{Relay}} \stackrel{L \gg 1}{\approx} 2 \left[ \sum_{i=1, j=\{1,2\} \setminus \{i\}}^2 \frac{1 - q_{R,j} + (1 - q_{i,R})q_{i,j}}{(1 - q_{i,R}q_{i,j})(1 - q_{R,j})} \right]^{-1}. \quad (19)$$

### C. Static Network Coding

Pure relaying is not bandwidth efficient in that the relay node retransmits packets using orthogonal channels. If the relay node can somehow mix the two packets intended for different sources, some channel use can be saved accordingly. For example, suppose the relay node needs to deliver  $X_1$  to  $S_2$  and  $X_2$  to  $S_1$ , respectively. Then instead of transmitting these two packets separately, the relay can perform DNC to combine these two packets as  $X_R = X_1 \oplus X_2$ , and then send out this single network-coded packet  $X_R$ . It should be pointed out that only the packet body is combined through DNC. The packet header remains uncoded and needs to indicate which packets are combined. If  $S_1$  is able to successfully decode this packet, it first extracts control information in the packet header of  $X_R$  and then decodes the message sent from  $S_2$  by  $X_2 = X_1 \oplus X_R$ . Likewise,  $S_2$  can also decode  $X_1$  based on  $X_R$  through similar operation. In this way, both receivers can extract the desired message while the relay node only needs to broadcast a single packet. This strategy has been studied in [8] and [18]–[21] by ignoring the channel erasure, and in [12], [13], and [31]–[33] by ignoring the delay constraint. In the sequel, we

incorporate those two factors into the protocol and analyze the corresponding throughput.

The overall data exchange still completes in three phases, i.e., two source transmission phases followed by a data relaying phase. The two source transmission phases are exactly the same as what we studied in the previous subsection for pure relaying. The only difference is how the relay node shall forward the packets stored in the local buffer during the data relaying phase. Suppose the  $D_i$  packets in the set  $\mathbb{D}_i$  are denoted by  $\{X_i(1), X_i(2), \dots, X_i(D_i)\}$  for  $i = 1, 2$ . Let  $D_{NC} = \min(D_1, D_2)$  and  $D_{REG} = |D_1 - D_2|$  be the number of network-coded packets and the number of regular packets, respectively. Without loss of generality, we assume  $D_1 \geq D_2 \geq 0$ . Then the relay node could mix the first  $D_{NC}$  packets in  $\mathbb{D}_1$  and  $\mathbb{D}_2$  by  $X_R(k) = X_1(k) \oplus X_2(k)$  for  $k = 1, 2, \dots, D_{NC}$ . Afterwards, the relay node broadcasts  $D_{NC}$  network-coded packets  $\{X_R(k)\}_{k=1}^{D_{NC}}$  intended for both receivers, and forwards  $D_{REG}$  regular packets  $\{X_1(k)\}_{k=D_{NC}+1}^{D_2}$  intended for  $S_2$  alone.

For regular packets, it is essentially direct transmission from  $R_1$  to  $S_2$ , which we have studied in the previous subsection. For network-coded packets, each packet involves two primitive packets that are subject to different retransmission constraints. Each time any receiver is able to decode the desired primitive packet, or any primitive packet has reached the maximum number of transmission, that primitive packet would be discarded by the relay. Afterwards, only the other primitive packet will be retransmitted. Consequently, the transmission of any network-coded packet will terminate until both primitive packets are successfully delivered or discarded after reaching the maximum number of transmissions. For example, suppose  $X_R = X_1 \oplus X_2$  where  $X_1$  and  $X_2$  can be respectively transmitted up to 2 and 4 times. If neither source node is able to decode  $X_R$  in the first 2 attempts, then  $X_1$  would be discarded and in the following time slots, the relay node only forwards  $X_R = X_2$  as a regular packet to  $S_1$  alone for up to 2 more times.

Next we analyze the throughput. The first thing to note is that the packet loss rate remains the same for network-coded packet and regular packet. Consequently, the average number of successfully delivered packets is still given by (17). The total number of transmissions for all three phases is given by

$$\begin{aligned} E[T] &= KE[T_1] + KE[T_2] + E[T_R] \\ &= Kg_L(q_{1,2}q_{1,R}) + Kg_L(q_{2,1}q_{2,R}) + E[T_R], \end{aligned} \quad (20)$$

where  $T_i \sim \text{BGeom}(q_{i,j}q_{i,R}, L)$  for  $i = 1, 2$  and  $j = \{1, 2\} \setminus \{i\}$  is the total number of transmissions of a single packet sent from  $S_i$ , and

$$\begin{aligned} E[T_R] &= E[(D_1 - D_2)\mathbb{I}\{D_1 > D_2\}] E[T_{R,2}] \\ &\quad + E[(D_2 - D_1)\mathbb{I}\{D_2 > D_1\}] E[T_{R,1}] \\ &\quad + E[\min(D_1, D_2)] E[T_{R,NC}], \end{aligned} \quad (21)$$

is the total number of transmissions of all packets stored at relay  $R_1$ . Here  $D_i \sim \text{Bin}(Q_i, K)$ , where  $Q_i = \Pr(I_i = -1)$  is given by (13) and (14), and  $E[T_{R,i}]$  is the average number of transmissions of a regular packet sent from  $R_1$  to  $S_i$  and

this quantity has been derived in the previous subsection. As a result, we only need to compute the average number of transmissions of a network-coded packet, i.e.,  $E[T_{R,NC}]$  where  $T_{R,NC} = \max(T_{R,1}, T_{R,2})$ . For per-hop constraint,  $T_{R,i} \sim \text{BGeom}(q_{R,i}, L)$  for  $i = 1, 2$  are independent. By using property 2 of Lemma 1, we have

$$E[T_{R,NC}] = g_L(q_{R,1}) + g_L(q_{R,2}) - g_L(q_{R,1}q_{R,2}). \quad (22)$$

For E2E constraint, we have  $T_{R,i} \sim \text{BGeom}(q_{R,i}, L - T_j)$  under the condition that  $T_j \leq L - 1$  for  $i = 1, 2$  and  $j = \{1, 2\} \setminus \{i\}$ . By using property 4 of Lemma 1, we have

$$E[T_{R,NC}] = h_1(q_{R,1}, q_{R,2}, q_{1,R}q_{1,2}, q_{2,R}q_{2,1}; L). \quad (23)$$

#### D. Dynamic Network Coding

The network coding scheme studied in the previous subsection is static in that the pairing pattern is fixed after scheduling. Specifically, once two packets are mixed they cannot be paired with other packets later. As a result, that scheme does not leverage the power of network coding to the full. For example, if any primitive packet is discarded or delivered earlier than the other one, the corresponding receiver has to stay idle until the other primitive packet transmission terminates. If the relay can somehow combine that primitive packet with another primitive packet in the buffer and broadcast a new network-coded packet, such bandwidth efficiency loss could be avoided. We remark that a similar idea has been studied in [31], where the relay node stores only the network-coded packets and different network-coded packets could be dynamically re-mixed. However, that scheme will not work properly after incorporating the delay constraint, since some primitive packet may be discarded earlier. As a result, the relay has to store all the primitive packets for re-mixing purpose.

To carry out the above idea, the pairing pattern must be determined dynamically. Specifically, once the two sets  $\mathbb{D}_1$  and  $\mathbb{D}_2$  are not empty, the first packet in those two sets are always mixed to form a network-coded packet. This process continues until one set becomes empty. Since then the relay would forward the packets in the remaining set as regular packets sequentially. For example, suppose  $\mathbb{D}_1 = \{X_1(1), X_1(2), X_1(3)\}$  and  $\mathbb{D}_2 = \{X_2(1)\}$ . Then the first network-coded packet is  $X_R(1) = X_1(1) \oplus X_2(1)$ . Suppose after several attempts, the transmission of  $X_1(1)$  terminates and  $X_1(1)$  is removed from  $\mathbb{D}_1$ , but the transmission of  $X_2(1)$  is still unfinished. Then the relay would form a new network-coded packet given by  $X_R(2) = X_1(2) \oplus X_2(1)$ . In contrast, for static network coding scheme, the relay node would keep sending  $X_R(2) = X_2(1)$  after dropping  $X_1(1)$ , during which period  $S_2$  always remains idle and the channel  $R_1 \rightarrow S_2$  is not in use at all. For dynamic network coding, the remaining packets are always mixed whenever possible. In the above example, if  $X_2(1)$  terminates earlier than  $X_1(2)$ ,  $\mathbb{D}_2$  becomes empty and  $\mathbb{D}_1 = \{X_1(2), X_1(3)\}$ . Then the relay simply sends  $X_1(2)$  and  $X_1(3)$  as regular packets to  $S_2$  sequentially.

As dynamic network coding does not affect the probability of successful delivery, the total number of transmissions for all

three phases is still given by (20). The only difference lies in the total number of transmissions of all packets stored at the relay, i.e.,  $E[T_R]$ . Note that given the set size  $D_i$ , the relay node only needs to deliver  $D_i$  packets  $X_i(1), X_i(2), \dots, X_i(D_i)$  to  $S_j$  for  $i = 1, 2$  and  $j = \{1, 2\} \setminus \{i\}$ . Those packets may be in the form of network-coded packets or regular packets depending on the network dynamics; nevertheless, the packet loss rate remains the same. Therefore, if we denote  $T_{R,j}(k)$  as the total number of transmissions of all packets containing  $X_i(k)$ , we have  $T_{R,j}(k) \sim \text{BGeom}(q_{R,j}, L)$  for per-hop constraint and  $T_{R,j} \sim \text{BGeom}(q_{R,j}, L - T_i)$  for E2E constraint under the condition that  $T_i \sim \text{BGeom}(q_{i,j}q_{i,R}, L) \leq L - 1$ , respectively. As the transmission from  $R_1$  to  $S_j$  terminates when  $\mathbb{D}_i$  becomes empty, its duration is equal to the summation of the total number of transmissions of individual packets, i.e.,  $\sum_{k=1}^{D_i} T_{R,j}(k)$ . Finally, as the relaying phase terminates when both  $\mathbb{D}_1$  and  $\mathbb{D}_2$  become empty, we have

$$T_R|_{D_1, D_2} = \max \left( \sum_{k=1}^{D_2} T_{R,1}(k), \sum_{k=1}^{D_1} T_{R,2}(k) \right). \quad (24)$$

Analytically, it is hard to compute  $E[T_R]$  due to the maximum operation. In practice, we can use the following lower bound to get an estimate of  $E[T_R]$ , i.e.,

$$E[T_R|D_1, D_2] \geq \max(D_2 E[T_{R,1}], D_1 E[T_{R,2}]). \quad (25)$$

Here  $D_i$  and  $T_{R,i}$  for  $i = 1, 2$  have exactly the same distribution as in the static network coding case and have been derived in the previous subsection. Averaging the above expression over the distribution of  $D_i$  leads to  $E[T_R]$ , and the theoretical throughput thus obtained is a tight upper bound. In the special case when there is no maximum number of transmissions constraint (i.e.,  $L \rightarrow \infty$ ), all the packets could be delivered successfully, and the total number of transmissions for the relaying phase could be obtained recursively as

$$\begin{aligned} & (1 - q_{R,1}q_{R,2})E[T_R|D_1, D_2] \\ &= 1 + (1 - q_{R,1})(1 - q_{R,2})E[T_R|D_1 - 1, D_2 - 1] \\ & \quad + (1 - q_{R,1})q_{R,2}E[T_R|D_1, D_2 - 1] \\ & \quad + q_{R,1}(1 - q_{R,2})E[T_R|D_1 - 1, D_2] \end{aligned} \quad (26)$$

for  $D_1, D_2 > 0$ , and the boundary conditions are

$$E[T_R|D_i, D_j = 0] = \frac{D_i}{1 - q_{R,j}} \quad (27)$$

for  $i = 1, 2$  and  $j = \{1, 2\} \setminus \{i\}$ .

### E. Throughput Comparison

In this subsection, we compare the throughput of the aforementioned schemes. To make the analysis tractable, we focus on the symmetric networks where the relay node is located at the middle between  $S_1$  and  $S_2$ . Besides, we assume all nodes use the same power and transmit at the same rate. The channel gain is supposed to be dependent on the distance only. Under those assumptions, we have  $q_{1,R} = q_{2,R} = q_{R,1} = q_{R,2} \triangleq q_R$

and  $q_{1,2} = q_{2,1} \triangleq q_S$ . In addition, we assume that the maximum number of transmissions is sufficiently large, i.e.,  $L \gg 1$ . From (11) and (19), we can derive

$$\eta_{\text{DT}} = 1 - q_S, \quad (28)$$

$$\eta_{\text{Relay}} = \frac{1 - q_R q_S}{1 + q_S} \stackrel{q_S=1}{=} \frac{1 - q_R}{2}. \quad (29)$$

For network coding schemes, the throughput also depends on the frame length  $K$ . When  $K \gg 1$ , we have

$$D_i \rightarrow K \frac{(1 - q_R)q_S}{1 - q_R q_S}. \quad (30)$$

After some simple algebra, we can show that the throughputs of static and dynamic network coding are respectively given by

$$\eta_{\text{S-NC}} = \frac{2(1 - q_R q_S)(1 + q_R)}{2(1 + q_R) + (1 + 2q_R)q_S} \stackrel{q_S=1}{=} \frac{2(1 - q_R^2)}{3 + 4q_R}, \quad (31)$$

$$\eta_{\text{D-NC}} = \frac{2(1 - q_R q_S)}{2 + q_S} \stackrel{q_S=1}{=} \frac{2(1 - q_R)}{3}. \quad (32)$$

We first consider the special case when there is no direct link, i.e.,  $q_S = 1$ . In this case,  $\eta_{\text{DT}} \equiv 0$  because no information could be delivered through the direct link alone. Besides, we observe that  $0 \leq \eta_{\text{Relay}} \leq 1/2$  and  $0 \leq \eta_{\text{S-NC}}, \eta_{\text{D-NC}} \leq 2/3$ , and the maximum throughput is achieved when  $q_R = 0$ . Next we consider the general case with direct link, i.e.,  $0 \leq q_S < 1$ . It is easy to see that  $0 \leq \eta_{\text{DT}}, \eta_{\text{Relay}}, \eta_{\text{S-NC}}, \eta_{\text{D-NC}} \leq 1$ , and the upper bound is achieved when  $q_S = 0$ . Note that when the direct link is in good quality (i.e.,  $q_S \ll 1$ ), using direct transmission alone is able to achieve the throughput bound. In that case, the relay nodes could stay idle to save the transmitted power and reduce the channel use. Incorporating the direct link is also important in that the throughput upper bound increases from 2/3 to 1.

Next we compare the relative throughput gain. By comparing direct transmission and pure relaying, we have

$$\frac{\eta_{\text{Relay}}}{\eta_{\text{DT}}} = \frac{1 - q_R q_S}{1 - q_S^2} > 1 \Leftrightarrow q_S > q_R. \quad (33)$$

Therefore, wireless relaying can improve the throughput if and only if the relay link has better quality than direct link. Next we study the gain of static network coding, which is given by

$$1 \leq \frac{\eta_{\text{S-NC}}}{\eta_{\text{Relay}}} = 1 + \frac{q_S}{2 + 2q_R + q_S + 2q_R q_S} \leq \frac{4}{3}. \quad (34)$$

Clearly, static network coding is strictly better than pure relaying in terms of the achievable throughput. However, the relative throughput gain is bounded by 33.3%, which occurs when the direct link is always in outage and the relay link is error-free. This result is consistent with that obtained in [8], [12]. Finally, we study the gain of dynamic network coding, which is given by

$$1 \leq \frac{\eta_{\text{D-NC}}}{\eta_{\text{S-NC}}} = 1 + \frac{q_R q_S}{2 + 2q_R + q_S + q_R q_S} \leq \frac{7}{6}. \quad (35)$$

It is observed that dynamic network coding can further improve the throughput. However, the relative gain is at most 16.7%, and this upper bound is achieved when both direct link and relay link are nearly in outage. For most practical system settings, the gain of dynamic network coding would not be very significant, as will be justified by simulations in later sections.

#### F. Power Allocation

In the last subsection, we compared the throughput by assuming equal power allocation. For practical wireless networks, the node distribution could be quite random, and the transmitted power could be properly allocated to further improve the throughput. However, the optimum transmitted power can be found only through exhaustive search, as the closed-form throughput expression is hard to manipulate. In the sequel, we seek to develop a near-optimum power allocation strategy with closed-form solution.

For the packet loss rate, we use the channel outage model given by (2). To make a step further, we intentionally ignore the maximum number of transmissions constraint and the impact of direct link, i.e., let  $L \rightarrow \infty$  and  $q_{1,2} = q_{2,1} = 1$ . From (19), the throughput of pure relaying is then given by (36), shown at the bottom of the page, where  $\alpha = 2^r - 1$  and we use the arithmetic-geometric mean inequality in the last step. To maximize the throughput, we use the upper bound instead. The power allocation problem is formulated as

$$\begin{aligned} \min & \left( \frac{1}{\lambda_{1,R}P_1} + \frac{1}{\lambda_{2,R}P_2} + \frac{\lambda_{R,1} + \lambda_{R,2}}{\lambda_{R,1}\lambda_{R,2}P_R} \right) \\ \text{s.t. } & P_1 + P_2 + P_R \leq 3P. \end{aligned} \quad (37)$$

By using the method of Lagrange multipliers, we could derive the optimum solution as

$$\begin{cases} P_1 = \frac{3P\lambda_{1,R}^{-1/2}}{\lambda_{1,R}^{-1/2} + \lambda_{2,R}^{-1/2} + (\lambda_{R,1}^{-1} + \lambda_{R,2}^{-1})^{\frac{1}{2}}} \\ P_2 = \frac{3P\lambda_{2,R}^{-1/2}}{\lambda_{1,R}^{-1/2} + \lambda_{2,R}^{-1/2} + (\lambda_{R,1}^{-1} + \lambda_{R,2}^{-1})^{\frac{1}{2}}} \\ P_R = \frac{3P(\lambda_{R,1}^{-1} + \lambda_{R,2}^{-1})^{\frac{1}{2}}}{\lambda_{1,R}^{-1/2} + \lambda_{2,R}^{-1/2} + (\lambda_{R,1}^{-1} + \lambda_{R,2}^{-1})^{\frac{1}{2}}}. \end{cases} \quad (38)$$

For dynamic network coding,  $L \rightarrow \infty$  and  $q_{1,2} = q_{2,1} = 1$  imply that  $D_1 = D_2 \equiv K$  in (24), which leads to (39), shown at the bottom of the page. Likewise, the power allocation problem could be formulated as

$$\begin{aligned} \min & \left( \frac{1}{\lambda_{1,R}P_1} + \frac{1}{\lambda_{2,R}P_2} + \frac{1}{\min(\lambda_{R,1}, \lambda_{R,2})P_R} \right) \\ \text{s.t. } & P_1 + P_2 + P_R \leq 3P. \end{aligned} \quad (40)$$

The solution to the above optimization problem is given by

$$\begin{cases} P_1 = \frac{3P\lambda_{1,R}^{-1/2}}{\lambda_{1,R}^{-1/2} + \lambda_{2,R}^{-1/2} + \max(\lambda_{R,1}^{-1/2}, \lambda_{R,2}^{-1/2})} \\ P_2 = \frac{3P\lambda_{2,R}^{-1/2}}{\lambda_{1,R}^{-1/2} + \lambda_{2,R}^{-1/2} + \max(\lambda_{R,1}^{-1/2}, \lambda_{R,2}^{-1/2})} \\ P_R = \frac{3P \max(\lambda_{R,1}^{-1/2}, \lambda_{R,2}^{-1/2})}{\lambda_{1,R}^{-1/2} + \lambda_{2,R}^{-1/2} + \max(\lambda_{R,1}^{-1/2}, \lambda_{R,2}^{-1/2})}. \end{cases} \quad (41)$$

The throughput of static network coding is very close to that of dynamic network coding. Consequently, (41) is also a sound power allocation scheme for static network coding.

#### IV. MULTI-RELAY NETWORK

In this section, we turn to the multi-relay network. The packet loss rates of the channel  $S_i \rightarrow R_k$  and the channel  $R_k \rightarrow S_i$  are respectively denoted by  $q_{i,R_k}$  and  $q_{R_k,i}$  for  $i = 1, 2$  and  $k = 1, 2, \dots, N$ . The two sets of packets stored at  $R_k$  that are intended for different source nodes are denoted by  $\mathbb{D}_{1,R_k}$  and  $\mathbb{D}_{2,R_k}$ . The power of  $S_i$  is denoted by  $P_i$  and the power of  $R_k$  is denoted by  $P_{R_k}$ . From (2), it is observed that the packet loss rate of the channel  $R_k \rightarrow S_j$  depends only on the average channel signal-to-noise ratio (SNR), i.e.,  $P_{R_k}\lambda_{R_k,S_j}$ . We assume that the transmitted power  $P_{R_k}$  is fixed and the channel gain  $\lambda_{R_k,S_j}$  is determined by the distance only, which could be obtained right before the data communication starts. Consequently, those average channel SNRs are supposed to be a-priori known to the whole network.

When there are multiple relays, the entire two-way communication still completes in three phases. During the source transmission phase, each source packet is either discarded after reaching the maximum number of transmissions, or decoded

$$\begin{aligned} \eta_{\text{Relay}} & \approx 2 \left( \exp\left(\frac{\alpha}{\lambda_{1,R}P_1}\right) + \exp\left(\frac{\alpha}{\lambda_{2,R}P_2}\right) + \exp\left(\frac{\alpha}{\lambda_{R,1}P_R}\right) + \exp\left(\frac{\alpha}{\lambda_{R,2}P_R}\right) \right)^{-1} \\ & \leq \frac{1}{2} \exp\left(-\frac{\alpha}{4} \left( \frac{1}{\lambda_{1,R}P_1} + \frac{1}{\lambda_{2,R}P_2} + \frac{\lambda_{R,1} + \lambda_{R,2}}{\lambda_{R,1}\lambda_{R,2}P_R} \right)\right) \end{aligned} \quad (36)$$

$$\begin{aligned} \eta_{\text{D-NC}} & \approx 2 \left( \exp\left(\frac{\alpha}{\lambda_{1,R}P_1}\right) + \exp\left(\frac{\alpha}{\lambda_{2,R}P_2}\right) + \exp\left(\frac{\alpha}{\min(\lambda_{R,1}, \lambda_{R,2})P_R}\right) \right)^{-1} \\ & \leq \frac{2}{3} \exp\left(-\frac{\alpha}{3} \left( \frac{1}{\lambda_{1,R}P_1} + \frac{1}{\lambda_{2,R}P_2} + \frac{1}{\min(\lambda_{R,1}, \lambda_{R,2})P_R} \right)\right) \end{aligned} \quad (39)$$

by the intended receiver, or decoded and stored by some relay nodes. It is likely that multiple relay nodes happen to decode the same source packet simultaneously. In this work, we consider the best-path routing strategy that has been widely used for the one-way relay channel [35]. That is, among the relay nodes that have successfully decoded the same packet of  $S_i$  for  $i = 1, 2$ , we assume that only the relay node that has the largest average channel SNR to the intended receiver  $S_j$  for  $j = \{1, 2\} \setminus \{i\}$  would store and retransmit that packet later. If there are several such relay nodes that happen to have the same average channel SNR, only one relay node is selected randomly. Consequently, after the two source transmission phases all relays would store some mutually exclusive packets in the local buffers.

In the sequel, we focus only on the relaying phase and study several relaying strategies. It is presumably assumed that all relay nodes are perfectly synchronized in timing. In the cellular systems, such timing information may come from the base station. As the exact throughput is hard to analyze in general, we also assume<sup>3</sup>  $L \gg 1$  and investigate the limiting performance instead. Besides, the synchronization overhead is ignored in the throughput analysis, because such overhead is usually negligible compared to the size of data packets.

#### A. Parallel Pure Relaying

One simple strategy is to let all relay nodes take turns to transmit the local packets in different time slots. The transmission order could be predetermined (e.g., from  $R_1$  to  $R_N$ ) to reduce the scheduling overhead. In this way, there would be no interaction between different relay nodes, and there are basically a set of parallel relay channels between the two source nodes. For throughput analysis, we consider only the symmetric settings such that  $q_{i,R_k} = q_{R_k,i} \triangleq q_R$  and  $q_{1,2} = q_{2,1} \triangleq q_S$  for  $i = 1, 2$  and  $k = 1, 2, \dots, N$ . Under those assumptions, the throughput could be obtained from the single-relay case by regarding the entire relay array as a single super relay node. During the source transmission phases, each packet would be captured by the super relay node if any relay node within the array is able to decode that packet correctly. Consequently, the packet loss rate of the channel from any source node to this super relay node is equal to  $q_R^N$ . Then in the relaying phase, the packet loss rate of the channel from any relay node to each source node is identical and is equal to  $q_R$ . So no matter which relay node decodes and stores a specific source packet, the average number of transmissions of that packet during the relaying phase is identical. Consequently, the throughput could be easily obtained from (17) and (18) by inserting  $q_{i,R} = q_R^N$ ,  $q_{R,j} = q_R$ , and  $q_{i,j} = q_S$  for  $i = 1, 2$  and  $j = \{1, 2\} \setminus \{i\}$ . When  $N \gg 1$ , the channel between any source node and the relay array is nearly error-free since  $q_R^N \xrightarrow{N \gg 1} 0$ . The asymptotic throughput in the case of  $N \gg 1$  could be obtained from (19) as

$$\eta_{\text{Parallel}} = \frac{1 - q_R}{1 - q_R + q_S}. \quad (42)$$

<sup>3</sup>This assumption is just for analysis purpose. The protocols can still work in the case of finite  $L$ .

#### B. Digital Network Coding Only

Alternatively, each relay node may choose to mix the local packets using network coding. However, as there is no interaction between the relay nodes, each relay node may mix the local packets by using DNC alone. Afterwards, all relay nodes still take turns to forward the local network-coded packets and/or regular packets in a predetermined order. To analyze the throughput of this strategy, we focus again on the symmetric settings. Without loss of generality, we assume dynamic network coding is applied at all relay nodes. The throughput of static network coding could be obtained in a similar way.

We start with the scenario  $N \gg K = 1$ , in which case the frame length is equal to 1 and only one packet is exchanged between source nodes. Depending on the network dynamics, either both packets are successfully delivered through direct link, or only one packet is delivered through direct link and the other one is decoded by the relay but not the intended receiver, or both source packets are decoded by some relay nodes. The probabilities of those three events are  $(1 - q_S)^2$ ,  $2q_S(1 - q_S)$  and  $q_S^2$ , respectively. As we assume  $N \gg 1$ , the channel between any source node and the relay array is nearly error-free. Consequently, during the source transmission phases each source packet could be successfully decoded by some relay node after one attempt almost surely, and the total number of transmissions of the two source transmission phases is exactly 2. For each regular packet transmitted by the relay node, the average number of transmissions is  $1/(1 - q_R)$ . For each network-coded packet, the average number of transmissions is  $(1 + 2q_R)/(1 - q_R^2)$  from (22). Note that when  $K = 1$ , static network coding becomes equivalent to dynamic network coding, and network coding could be performed only when a single relay node happens to decode and store both source packets. After some simple algebra, we can obtain the throughput as

$$\eta_{\text{DNC}} = \left( \frac{1 - q_R + q_S}{1 - q_R} - \frac{q_S^2}{2N(1 - q_R^2)} \right)^{-1} \stackrel{N \gg 1}{\approx} \eta_{\text{Parallel}}. \quad (43)$$

It is observed that using DNC alone can hardly boost throughput when the number of relays is much larger than the frame length. This is because the source packets would be decoded and stored by different relay nodes with high probability, in which case the gain of network coding is very limited. Next we study the throughput when the frame length is sufficiently large, i.e.,  $K \gg N \gg 1$ . On average, each relay node can decode  $(K/N)q_S$  packets from both sources. So from (25), the total number of transmissions of each relay node is approximately equal to  $(K/N)(q_S/(1 - q_R))$  for dynamic network coding. Consequently, we have

$$\eta_{\text{DNC}} = \frac{2(1 - q_R)}{2 - 2q_R + q_S}. \quad (44)$$

The throughput of parallel pure relaying is independent of  $K$  and is still given by (42). The relative throughput gain is

$$1 \leq \frac{\eta_{\text{DNC}}}{\eta_{\text{Parallel}}} = 1 + \frac{q_S}{2 - 2q_R + q_S} \leq 2 \quad (45)$$

for large  $K$ . It is observed that network coding can always improve the throughput and the throughput gain is up to 100%.

### C. Hybrid Network Coding

The problem of the previous strategy is that there is no interaction between different relay nodes. Consequently, each relay node is only able to mix the local packets using DNC alone. However, the gain of network coding is quite limited in the case of small frame length. In this subsection, we develop a hybrid network coding scheme to address this issue. Let us start with an example, supposing that there are  $N = 2$  relay nodes and there is  $K = 1$  packet per frame. Suppose after the two source transmission phases, we have  $\mathbb{D}_{1,R_1} = \{X_1\}$ ,  $\mathbb{D}_{2,R_2} = \{X_2\}$ , and  $\mathbb{D}_{2,R_1} = \mathbb{D}_{1,R_2} = \phi$ . That is, each relay node is able to decode only one source packet, and it is impossible to perform DNC to mix packets at any relay node. However, if the two relay nodes could somehow send  $X_1$  and  $X_2$  simultaneously, the two transmitted packets would be mixed automatically in the air, which is a form of ANC [11]. The received signal at  $S_i$  is given by

$$y_i = h_{R_1,i} \sqrt{P_{R_1}} X_1 + h_{R_2,i} \sqrt{P_{R_2}} X_2 + n_i \quad (46)$$

for  $i = 1, 2$ . At  $S_1$ , it can first subtract its own packet  $X_1$  from the mixed signal and then decodes  $X_2$ . The packet loss rate is still given by  $q_{R_2,1}$  after the self-interference is perfectly eliminated. Similar operations could be performed at  $S_2$  to decode  $X_1$ . In this way, both packets may be delivered in a single channel use even though they are stored at different relays.

For the general case, all relay nodes still take turns to forward packets in a predetermined order. Each transmitting relay node can first perform DNC to mix as many local packets as possible. Whenever one of the two sets becomes empty, say  $\mathbb{D}_1$ , it can invite another relay node with non-empty buffer  $\mathbb{D}_1$  to jointly perform ANC. This process continues until it is impossible to mix any packets further within the relay array. Afterwards, all relay nodes just take turns to transmit the remaining packets as regular packets. In some sense, the relay array is like a super relay node that has distributed buffers. This super relay node can smartly choose to perform DNC or ANC according to the network dynamics. However, each relay node within the relay array need not know the packets stored in other locations.

For analyzing the throughput, we still consider symmetric settings and focus only on dynamic network coding. We ignore the overhead incurred by letting two relay nodes jointly perform ANC, because such overhead is usually negligible compared to the size of data packets. The exact throughput could be obtained from the results in Section III-D by regarding the relay array as a single super relay node. During source transmission phases, the packet loss rate of the channel from any source to this super relay node is still equal to  $q_R^N$ . The next trick to note is that for each packet transmitted by some relay node within the relay array, it could be in the form of a regular packet, a network-coded packet that is mixed with another local packet by DNC, or a network-coded packet that is mixed with a packet stored at another relay by ANC. No matter which scenario applies, the packet loss rate of the channel from any relay to the intended source during the relaying phase is unaffected and is equal to  $q_R$ . As a result, the throughput could be obtained from (25) by

inserting  $q_{i,R} = q_R^N$ ,  $q_{R,j} = q_R$ , and  $q_{i,j} = q_S$  for  $i = 1, 2$  and  $j = \{1, 2\} \setminus \{i\}$ . When  $N \gg 1$ , we have

$$\eta_{\text{Hybrid}} = \begin{cases} \frac{2(1-q_R)}{2-2q_R+q_S}, & K \gg 1 \\ \frac{2(1-q_R^2)}{2(1-q_R^2)+2q_S(1+q_R)-q_S^2}, & K = 1 \end{cases} \quad (47a) \quad (47b)$$

By comparing (44) and (47a), we observe that the two strategies may achieve the same asymptotic throughput for large  $K$ . For small  $K$  (i.e.,  $K = 1 \ll N$ ), the gain of hybrid network coding is given by

$$1 \leq \frac{\eta_{\text{Hybrid}}}{\eta_{\text{DNC}}} = 1 + \frac{q_S^2}{2(1-q_R^2) + 2q_S(1+q_R) - q_S^2} \leq \frac{4}{3}. \quad (48)$$

In this case, hybrid network coding can greatly improve the throughput, but the throughput gain is up to 33.3%.

## V. SIMULATIONS

In this section, we present some simulation results to justify our analytical results. Throughout simulations, we use the model  $\lambda = d^{-3}$ , where  $\lambda$  is the channel gain and  $d$  is the distance. The noise power is always normalized, and the average transmitted power of all nodes is referred to as system SNR. The two source nodes are always located at  $(0, 0)$  and  $(1, 0)$ , respectively.

In Fig. 2 we first compare the throughput of various transmission schemes for single-relay networks. It is observed that the simulation results match perfectly with our theoretical results. Compared with direct transmission, wireless relaying can greatly boost the throughput when the relay link is much better than the direct link. Network coding can further improve the throughput at moderate SNRs. For example, at 0 dB the throughput goes from 0.57 to 0.68, and the relative gain is around 20%. Comparatively, dynamic network coding has the best performance in all situations, but the throughput gain against static network coding is very limited over the entire SNR range, which is consistent with the analytical results in Section III-E. It is also observed that the four schemes have the same performance at high SNRs, in which case most packets are delivered through the direct link and the relay link is active only occasionally.

By comparing Fig. 2(a) and (b), we also observe that per-hop constraint performs better than E2E constraint at low SNRs, while the throughput is almost identical at medium-to-high SNRs. This is because for per-hop constraint, the same packet can be transmitted for up to  $L$  times each hop, whereas for E2E constraint the total number of transmissions is bounded by  $L$  regardless of the sender. Therefore, the probability of successful delivery is higher for per-hop constraint. To fairly compare these two types of constraints, we normalize the aggregate E2E maximum number of transmissions and plot the throughput in Fig. 3. For example, if  $L = 4$  for E2E constraint, then the same packet can be transmitted for up to 2 times each hop for the per-hop constraint. We observe that at low SNRs, the throughput increases quickly as a function of the normalized E2E maximum number of transmissions, as allocating more retransmission channels could significantly improve the probability of successful delivery. In contrast, at high SNRs the throughput

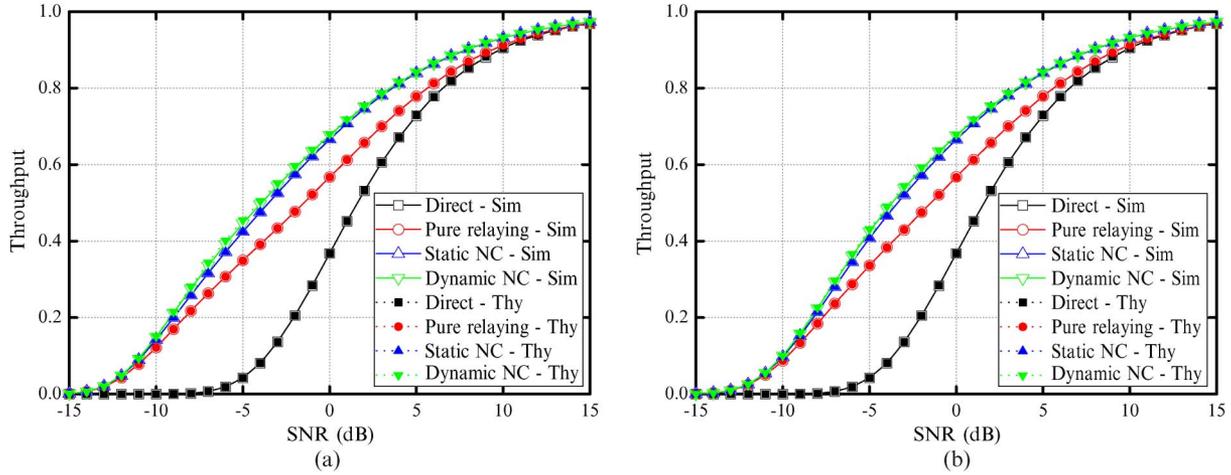


Fig. 2. Throughput versus SNR for  $L = 4$  and  $K = 10$ . The relay node is located at  $(0.5, 0)$ . (a) Per-hop constraint. (b) E2E constraint.

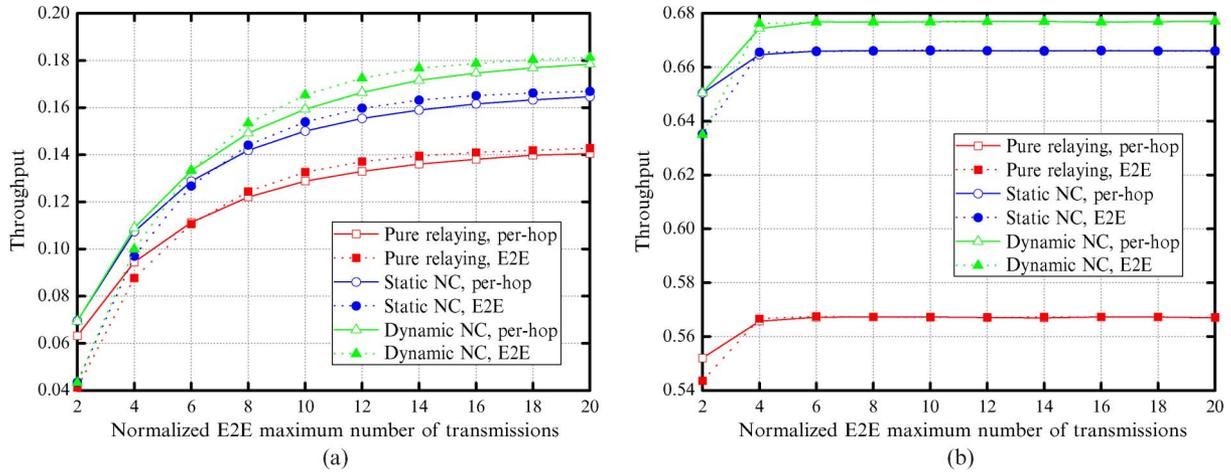


Fig. 3. Throughput versus normalized E2E maximum number of transmissions for  $K = 10$ . The relay node is located at  $(0.5, 0)$ . (a) SNR =  $-10$  dB. (b) SNR =  $0$  dB.

almost remains the same when  $L \geq 4$ , as most packets can be successfully delivered within 4 attempts. It is also observed that E2E constraint performs better when  $L$  is relatively large. This is because for E2E constraint, the retransmission channels are dynamically allocated between the two hops, whereas for per-hop constraint such split is fixed. So there is some throughput loss due to early give-up under per-hop constraint. In contrast, per-hop constraint leads to higher throughput when  $L$  is relatively small. This is because when the first few attempts have failed, that packet should be discarded early to save the channel for transmitting a new packet. Otherwise even the relay node is able to decode the packet later, there would be very limited number of retransmission channels available and the probability of successful delivery is still very low. As a result, early termination appears to be a better choice for small  $L$ .

Next we study the impact of power allocation in Fig. 4. The relay node is located at  $(D_{sr}, 0)$ , and we plot the throughput with different relay locations. It is observed that our power allocation schemes (38) and (41) perform very close to the optimum schemes obtained through exhaustive search. When the network topology is highly asymmetric, i.e., when the relay node is very close to one source, optimum power allocation can almost double the throughput against equal power allocation.

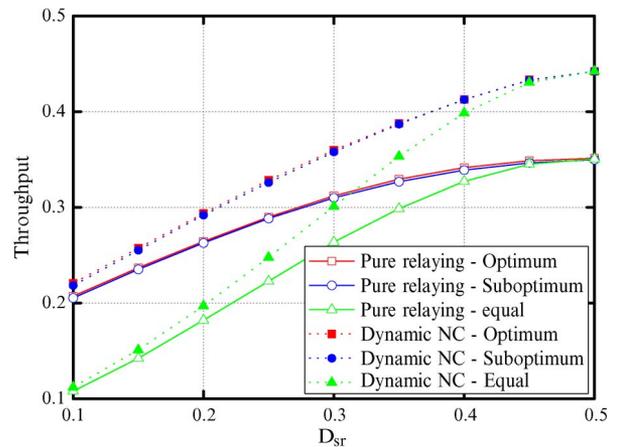


Fig. 4. Throughput versus relay position with power allocation for SNR =  $-5$  dB,  $K = 5$  and  $L = \infty$ .

This is because some source-relay link becomes the system bottleneck, and that bottleneck link limits the throughput of the whole system. Our power allocation schemes attempt to address this issue by allocating more power to the end terminal associated with the bottleneck link, such that the packet loss rate of that link could be reduced to some extent.

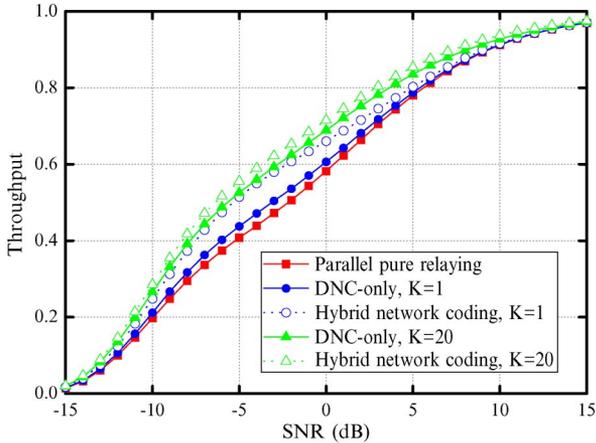


Fig. 5. Throughput versus SNR with  $N = 3$  relays for  $L = \infty$ . All relay nodes are located at  $(0.5, 0)$ .

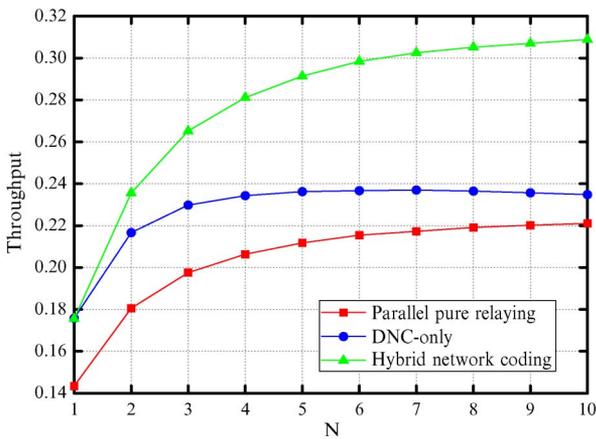


Fig. 6. Throughput versus the number of relays for SNR =  $-10$  dB,  $K = 3$  and  $L = \infty$ . All relay nodes are located at  $(0.5, 0)$ .

Finally we study the throughput of multi-relay networks in Figs. 5 and 6, where all the relays are assumed to locate at  $(0.5, 0)$ . In Fig. 5, we fix the number of relays and change the frame length. It is observed that DNC-only performs very close to parallel pure relaying when the frame length is only 1, as the chance of mixing local packets is very low. As the frame length becomes much larger than the number of relays, most packets could be mixed together and the power of network coding comes into play. That is why the throughput is increasing as a function of the frame length. When the frame length  $K$  is relatively large, DNC-only performs almost as well as hybrid network coding, which is consistent with our analysis in Section IV-B. Then in Fig. 6, we investigate the throughput with fixed frame length but different number of relays. It is observed that the gap between hybrid network coding and DNC-only becomes wider as  $N$  increases. Besides, the throughput of DNC-only converges to that of parallel pure relaying when the number of relays exceeds the frame length, as expected in Section IV-A. In sum, the gain of DNC-only is affected by the frame length and the number of relays, whereas hybrid network coding can overcome such shortcoming by smartly switching between DNC and ANC.

## VI. CONCLUSION

In this work, we studied several relaying schemes for TWRC. We obtained the closed-form throughput after taking into account the delay constraint. We showed that the binary XOR-based network coding may greatly improve the throughput, but the gain is always upper bounded. We also developed a near-optimum power allocation strategy and demonstrated that the source node of the bottleneck link should use more power to improve the throughput. For multi-relay networks, we showed that using DNC alone may incur significant throughput loss when the frame length is smaller than the number of relays, whereas our hybrid network coding scheme may fully leverage the power of network coding by using both DNC and ANC.

## APPENDIX PROOF OF LEMMA 1

For property 1, we use (4) and (5) and can obtain

$$\begin{aligned} f_Z(k) &= \Pr(X = k, Y > k) + \Pr(X > k, Y = k) \\ &\quad + \Pr(X = Y = k) \\ &= (1 - pq)(pq)^{k-1} \end{aligned} \quad (49)$$

for  $k = 1, 2, \dots, L - 1$  and  $f_Z(L) = \Pr(X = Y = L) = p^{L-1}q^{L-1}$ . Comparing with (4), we have  $Z \sim \text{BGeom}(pq, L)$ .

For property 2, we have

$$\begin{aligned} f_Z(k) &= \Pr(X = k, Y < k) + \Pr(X < k, Y = k) \\ &\quad + \Pr(X = Y = k) \\ &= (1 - p)p^{k-1} + (1 - q)q^{k-1} - (1 - pq)(pq)^{k-1} \end{aligned} \quad (50)$$

for  $k = 1, 2, \dots, L - 1$  and  $f_Z(L) = p^{L-1} + q^{L-1} - p^{L-1}q^{L-1}$ . By using (6), we can obtain (7) immediately.

For property 3, we need to first derive the conditional PMF of  $X$  given by

$$f_X(k | X \leq L - 1) = \frac{1 - p}{1 - p^{L-1}} p^{k-1} \quad (51)$$

for  $k = 1, 2, \dots, L - 1$ . Now we can calculate the conditional expectation of  $Z$  given  $X$  from (6) and obtain

$$\begin{aligned} E[Z | X \leq L - 1] &= E\left[\frac{1 - a^{L-X}}{1 - a} \mid X \leq L - 1\right] \\ &= \frac{1}{1 - a} \left(1 - \frac{1 - p}{1 - p^{L-1}} \sum_{k=1}^{L-1} p^{k-1} a^{L-k}\right) \\ &= \frac{1}{1 - a} \left(1 - \frac{1 - p}{1 - p^{L-1}} a^{L-1} g_{L-1}\left(\frac{p}{a}\right)\right). \end{aligned} \quad (52)$$

Finally for property 4, we have

$$\begin{aligned}
 f_W(k|X \leq L-1) &= \Pr(W = k, X < L - k|X \leq L - 1) \\
 &\quad + \Pr(W = k, X = L - k|X \leq L - 1) \\
 &= (1 - a)a^{k-1} \frac{1 - p^{L-k-1}}{1 - p^{L-1}} + a^{k-1} \frac{(1 - p)p^{L-k-1}}{1 - p^{L-1}} \\
 &= \frac{a^{k-1}}{1 - p^{L-1}} (1 - a + (a - p)p^{L-k-1}) \tag{53}
 \end{aligned}$$

for  $k = 1, 2, \dots, L - 1$ . Now we can compute the conditional CDF of  $W$  given by

$$\begin{aligned}
 F_W(k|X \leq L-1) &= \frac{1 - a}{1 - p^{L-1}} \sum_{i=1}^k a^{i-1} + \frac{a - p}{1 - p^{L-1}} p^{L-2} \sum_{i=1}^k a^{i-1} p^{-(i-1)} \\
 &= \frac{1 - a^k - p^{L-1} + a^k p^{L-k-1}}{1 - p^{L-1}} \tag{54}
 \end{aligned}$$

for  $k = 0, 1, \dots, L - 1$ . As  $W$  and  $T$  are independent, we have

$$\begin{aligned}
 F_Z(k|\max(X, Y) \leq L-1) &= F_W(k|X \leq L-1)F_T(k|X \leq L-1) \\
 &= 1 - \frac{1 - p^{L-k-1}}{1 - p^{L-1}} a^k - \frac{1 - q^{L-k-1}}{1 - q^{L-1}} b^k \\
 &\quad + \frac{1 - p^{L-k-1} - q^{L-k-1} + (pq)^{L-k-1}}{(1 - p^{L-1})(1 - q^{L-1})} a^k b^k \tag{55}
 \end{aligned}$$

for  $k = 0, 1, \dots, L - 1$ . By using (6) and the relation

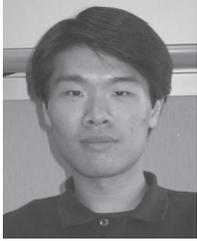
$$\begin{aligned}
 E[Z|\max(X, Y) \leq L-1] &= \sum_{k=1}^{L-1} \Pr(Z \geq k|\max(X, Y) \leq L-1), \tag{56}
 \end{aligned}$$

we can obtain (9) immediately.

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