On Analysis of Wireless Uplink using Analog Network Coding with Non-Coherent Modulations

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***Abstract*— Analog network coding (ANC) has been widely used in wireless uplink to improve throughput and provide spatial diversity. However, the receiver has to estimate the channel coefficients of all users to perform coherent detection, thus the signaling overhead is sometimes formidable and may even outweigh the performance gain. To reduce the channel estimation overhead, we study non-coherent modulations in this work with emphasis on receiver design and performance analysis. Depending on channel state information, we first develop the coherent, partial coherent and non-coherent receivers based on maximum likelihood (ML) principle. As the ML non-coherent receiver has a non-tractable integral form, we further propose two suboptimum receivers depending on the relative quality of source-relay channel and relay-destination channel. We also study the pairwise error probability, and show that full diversity is still achievable at high signal-to-noise ratios using non-coherent modulations; however, the error rate decreases much slower than that of coherent systems due to the incapability to efficiently suppress multi-user interferences. Extensive simulations are also given to verify our analytical results.**

I. Introduction

User cooperation is a new communications paradigm in which some relay nodes are selected to help forward the source messages in order to provide spatial diversity, extend transmission coverage and save transmitted power [1]. Some early work in this area [2] has studied several repetition-coding based cooperation protocols, which are not bandwidth efficient in practice due to the half-duplex constraint of user devices. To be specific, the relay nodes have to serve each source individually using two orthogonal channels, thus lowering the bandwidth efficiency roughly by one half.

In a multi-user system, the relay nodes can actually serve multiple sources at the same time by use of wireless network coding [3][4]. The resulting throughput can be greatly enhanced at a cost of more complicated receiver design and increased signaling overhead. For example, special detection schemes have to be leveraged to address the error propagation issue associated with digital network coding [5][6], and multi-user interferences have to be suppressed if analog network coding (ANC) is used instead [7][8]. For both strategies, knowing global channel state information (CSI) is necessary for coherent detection, thus the channel estimation overhead increases linearly with the product of the number of users and the number of relays. Such signaling overhead is sometimes formidable and may even outweigh the performance gain. Besides, it is generally hard to track all the channels simultaneously in a fast-fading environment.

To mitigate the stringent needs of perfect CSI, non-coherent schemes such as differential modulations [9] and orthogonal signaling (e.g., ON-OFF keying (OOK) and frequency shift keying (FSK)) have been widely discussed. In [10], the maximum likelihood (ML) demodulator for amplify-and-forward system using FSK modulations is developed. Although the ML receiver generally has no closed form, the upper and lower bounds on the average bit error rate (BER) are obtained in [11], and it is observed that full diversity can only be achieved for FSK modulations but not for OOK scheme. In [12], a near-ML receiver is developed and simulation results show that this scheme can also achieve full diversity. Besides, it is demonstrated that the non-coherent AF subject to short-term power constraint performs the same as direct transmission. Two blind detection schemes are proposed in [13] and [14] based on maximum energy selection and generalized likelihood ratio test, respectively. The advantage of these blind detectors is that the receiver needs not to know the statistical information of the channels, which further reduces the signaling overhead.

Non-coherent modulations have also been studied in the context of two-way relay channel using network coding. In [15], a set of differential demodulators are developed, and the BER performance is studied in [16][17]. A relay selection strategy without requiring CSI is developed in [18]. For OOK modulations, a simple threshold-based energy detection scheme is developed in [19], and the optimum threshold is also obtained in closed form. In [20], the non-coherent relay detector is obtained for binary FSK modulations. The optimum/suboptimum receivers are developed in [21] and the bounds on BER are also obtained.

As mentioned earlier, reducing the channel estimation overhead is a critical issue for multi-user network-coded system. However, most of the literatures [9]-[14] focus only on single-user scenario. For [15]-[21], the considered two-way relay channel is basically a single-user system on each way, as each end node can suppress or even eliminate its self-interference. In the presence of multi-user interferences, how to design the non-coherent modulation scheme is an interesting issue that has not been properly discussed in the literatures, and such concerns motivate the current work. To be specific, we study the non-coherent receiver design problem for a two-user uplink channel using ANC and seek to quantify the error performances. As the ML receiver has an integral form, we

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develop two suboptimum receivers depending on the relative quality of source-relay channel and relay-destination channel. The pair-wise error probability (PEP) is then studied, and the scaling laws of different PEPs are derived at high signal-to-noise ratios (SNR). It is demonstrated that full diversity is still achievable; however, the error rate does not decrease as fast as that of coherent system.

Notations: $|\cdot|$, $(\cdot)^T$ and $(\cdot)^H$ stands for absolute value, transpose and conjugate transpose, respectively. The boldface lowercase letter $\mathbf{a}$ and the boldface uppercase letter $\mathbf{A}$ represents vector in column form and matrix, respectively. $\|\mathbf{a}\|$ and $\det \mathbf{A}$ denotes the Euclidean norm of a vector $\mathbf{a}$ and the determinant of a square matrix $\mathbf{A}$, respectively. We shall use abbreviation i.i.d. for independent and identically distributed. We denote $Z \sim \mathcal{CN}(\mathbf{u}, \mathbf{\Sigma})$ as a circularly symmetric complex Gaussian random variable vector with mean $\mathbf{u}$ and covariance matrix $\mathbf{\Sigma}$, and denote $Z \sim \chi_k^2$ as a chi-square random variable with the degree of freedom being k. The probability of an event $\mathcal{A}$ is denoted as $\Pr(\mathcal{A})$. The cumulative distribution function (CDF) and the probability density function (PDF) of a random variable Z is denoted as $F_Z(z)$ and $f_Z(z)$, respectively. Finally, we say $h(x) = \mathrm{O}(g(x))$ if $\limsup_{x\to\infty} \frac{h(x)}{g(x)} < \infty$.

II. SYSTEM MODEL

Consider a symmetric uplink channel with two source nodes sending data to a common destination with the help of a single relay node. Let $f_k \sim \mathcal{CN}(0,1)$ and $h_k \sim \mathcal{CN}(0,1)$ be the channel coefficients from the kth source for $k = 1, 2$ to the relay and to the destination, respectively, and $g \sim \mathcal{CN}(0,1)$ be the channel coefficient from the relay to the destination. All the channel coefficients are independent, and the additive noises on different channels are also i.i.d. $\mathcal{CN}(0,1)$. The path-loss coefficients are denoted by λ_{sr}, λ_{sd} and λ_{rd} for source-relay channel, source-destination channel and relay-destination channel, respectively. As these path-loss coefficients are second-order statistics which remain unchanged over a long time, we assume these coefficients are known to all the nodes in the network.

To reduce the channel estimation overhead, we focus on non-coherent M-ary FSK modulations in this work. Thus the source symbols are chosen from the set $\Omega = \{\mathbf{e}_1, \mathbf{e}_2, \cdots, \mathbf{e}_M\}$, where $\mathbf{e}_l$ is the unit vector with the lth element being 1 and the other elements being 0. The whole data transmission is completed in three phases. In the kth phase for $k = 1, 2$, the kth source node broadcasts its own messages to both the relay node and destination, and the received signal is

$$\mathbf{y}_{kr} = \sqrt{P\lambda_{sr}} f_k \mathbf{x}_k + \mathbf{n}_{kr}, \tag{1}$$

$$\mathbf{y}_{kd} = \sqrt{P\lambda_{sd}} h_k \mathbf{x}_k + \mathbf{n}_{kd}, \tag{2}$$

respectively. Here P is transmitted power, $\mathbf{x}_k$ is the kth source symbol with $\mathbf{x}_k \in \Omega$, $\mathbf{n}_{kr}$ and $\mathbf{n}_{kd}$ are the corresponding additive noises. Suppose ANC is used at the relay, the two received signals $\mathbf{y}_{kr}$ for $k = 1, 2$ are combined directly in the complex field with equal weights. Thus the relay symbol can be represented as $\mathbf{x}_r = \sqrt{\alpha}(\mathbf{y}_{1r} + \mathbf{y}_{2r})$, where

$$\alpha = \frac{1}{2(P\lambda_{sr} + M)} \tag{3}$$

is the amplification factor to normalize the relay power, i.e., $E\|\mathbf{x}_r\|^2 = 1$. Note that this factor is a constant that is independent of the instantaneous channel variations. Finally in the third phase, the relay node forwards its symbol to the destination while the two source nodes remain silent. The received signal is given by

$$\begin{aligned} \mathbf{y}_{rd} &= \sqrt{\alpha P \lambda_{rd}}\, g \sum_{k=1}^{2} \left(\sqrt{P\lambda_{sr}} f_k \mathbf{x}_k + \mathbf{n}_{kr} \right) + \mathbf{n}_{rd} \\ &= \sqrt{\alpha P^2 \lambda_{rd} \lambda_{sr}}\, g \sum_{k=1}^{2} f_k \mathbf{x}_k + \tilde{\mathbf{n}}_{rd}, \end{aligned} \tag{4}$$

where $\tilde{\mathbf{n}}_{rd} = \sqrt{\alpha P \lambda_{rd}}\, g \sum_{k=1}^{2} \mathbf{n}_{kr} + \mathbf{n}_{rd}$ is the equivalent noise vector.

Upon observing the signals $\mathbf{y}_{kd}$ for $k = 1, 2$ and $\mathbf{y}_{rd}$, the destination can perform ML detection to jointly decode the two source symbols as

$$(\mathbf{x}_{d,1}, \mathbf{x}_{d,2}) = \arg \max_{\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2 \in \Omega} L(\mathbf{y}_{rd} | \hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2) \times \prod_{k=1}^{2} L(\mathbf{y}_{kr} | \hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2), \tag{5}$$

where $L(\cdot)$ is the corresponding likelihood function. Clearly, the form of likelihood function depends on how much CSI (denoted by Ψ) is known at the receiver. If full CSI is available, we have $\Psi_C = \{f_1, f_2, h_1, h_2, g\}$, in which case the detection is coherent and we can obtain

$$L_C(\mathbf{y}_{kd} | \hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2) = p\left(\mathbf{y}_{kd} - \sqrt{P\lambda_{sd}} h_k \hat{\mathbf{x}}_k, \mathbf{I}\right), \tag{6}$$

$$L_C(\mathbf{y}_{rd} | \hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2) = p(\mathbf{y}_{rd} - \mathbf{u}_C, \mathbf{\Sigma}_C), \tag{7}$$

where

$$\mathbf{u}_C = \sqrt{\alpha P^2 \lambda_{rd} \lambda_{sr}}\, g \sum_{k=1}^{2} f_k \hat{\mathbf{x}}_k, \tag{8}$$

$$\mathbf{\Sigma}_C = \left(2\alpha P \lambda_{rd} |g|^2 + 1 \right) \mathbf{I}, \tag{9}$$

$$p(\mathbf{y}, \mathbf{\Sigma}) = \frac{1}{\pi^M |\mathbf{\Sigma}|} \exp\left(-\mathbf{y}^H \mathbf{\Sigma}^{-1} \mathbf{y}\right). \tag{10}$$

If only limited CSI is known at the receiver, i.e., $\Psi_P = \{g\}$, then the detection is partial coherent and we have

$$L_P(\mathbf{y}_{kd} | \hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2) = p(\mathbf{y}_{kd}, \mathbf{\Sigma}_{P,k}), \tag{11}$$

$$L_P(\mathbf{y}_{rd} | \hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2) = p(\mathbf{y}_{rd}, \mathbf{\Sigma}_P), \tag{12}$$

where

$$\mathbf{\Sigma}_{P,k} = P\lambda_{sd} \hat{\mathbf{x}}_k \hat{\mathbf{x}}_k^H + \mathbf{I}, \tag{13}$$

$$\mathbf{\Sigma}_P = \alpha P^2 \lambda_{rd} \lambda_{sr} |g|^2 \sum_{k=1}^{2} \hat{\mathbf{x}}_k \hat{\mathbf{x}}_k^H + \mathbf{\Sigma}_C. \tag{14}$$

If no instantaneous CSI is known at the receiver (i.e., $\Psi_N = \phi$), then the detection is non-coherent with the likelihood function being $L_N\left(\mathbf{y}_{kd}|\,\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2\right) = L_P\left(\mathbf{y}_{kd}|\,\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2\right)$ and

$$L_N\left(\mathbf{y}_{rd}|\,\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2\right) = E_g\left(p\left(\mathbf{y}_{rd}, \mathbf{\Sigma}_P\right)\right). \tag{15}$$

From (15), we observe that the likelihood function includes the average over the distribution of relay-destination channel g, which has an integral form without any closed-form solution and thus complicates the practical implementation. To simplify the receiver design, we first revisit the signal model (4). The aggregate scaling factor effective on the relaying signal component is given by

$$\sqrt{\alpha P \lambda_{rd}} g = \sqrt{\frac{P\lambda_{rd}}{2\left(P\lambda_{sr} + M\right)}} g \overset{P \gg 1}{\approx} \sqrt{\frac{\lambda_{rd}}{2\lambda_{sr}}} g. \tag{16}$$

When the source-relay channel is much better than the relay-destination channel (i.e., $\lambda_{rd} \ll \lambda_{sr}$), the above scaling coefficient remains small with large probability, whereas the noise power of $\mathbf{n}_{rd}$ is a constant. As a result, we can approximate g by its mean and obtain

$$L_N\left(\mathbf{y}_{rd}|\,\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2\right) \approx L_P\left(\mathbf{y}_{rd}|\,\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2, g = 1\right), \tag{17}$$

which is called *fading elimination receiver* (FER). The error performance is expected to remain similar because the channel fading only brings very limited effects when the scaling factor is small on average. On the other hand, if the source-relay channel is much worse than the relay-destination channel (i.e., $\lambda_{rd} \gg \lambda_{sr}$), then the noise power of $\mathbf{n}_{rd}$ is generally much smaller than that of $\mathbf{n}_{kr}$ after being amplified by the relay node, and we can approximately neglect $\mathbf{n}_{rd}$ and obtain $\mathbf{y}_{rd} \approx \tilde{\mathbf{y}}_{rd}$ with

$$\tilde{\mathbf{y}}_{rd} = \sqrt{\alpha P \lambda_{rd}} g \sum_{k=1}^{2} \left(\sqrt{P\lambda_{sr}} f_k \mathbf{x}_k + \mathbf{n}_{kr}\right). \tag{18}$$

To obtain the likelihood function of $\tilde{\mathbf{y}}_{rd}$, we first prove the following lemma.

Lemma 1: Suppose $\mathbf{v} \sim \mathcal{CN}\left(\mathbf{0}, diag\left\{\sigma_{v_i}^2\right\}_{i=1}^{M}\right)$ and $u \sim \mathcal{CN}\left(0, \sigma_u^2\right)$ are independent random variables, then the PDF of $\mathbf{z} = u\mathbf{v}$ is

$$f\left(\mathbf{z}\right) = \left(\prod_{i=1}^{M} \frac{2}{\pi \sigma_u^2 \sigma_{v_i}^2}\right) q\left(\frac{4}{\sigma_u^2} \sum_{i=1}^{M} \frac{\left|z_i\right|^2}{\sigma_{v_i}^2}\right), \tag{19}$$

where $q\left(x\right) = x^{-\frac{M-1}{2}} K_{M-1}\left(\sqrt{x}\right)$ and $K_M\left(x\right)$ is the Mth-order modified Bessel function of the second kind [22, 9.6.1].

Proof: Denote $z_i = r_i e^{j\theta_i}$ for $i = 1, 2, \cdots, M$, then it is easy to show that the phases $\{\theta_i\}$ are independent of the amplitudes $\{r_i\}$, and $\{\theta_i\}$ are i.i.d. and uniformly distributed on $[0, 2\pi)$. Therefore,

$$f\left(\mathbf{z}\right) = \frac{1}{|J|} f\left(\mathbf{r}, \boldsymbol{\theta}\right) = \prod_{i=1}^{M} \left(2\pi r_i\right)^{-1} f\left(\mathbf{r}\right), \tag{20}$$

where $|J| = \prod_{i=1}^{M} r_i$ is the Jacobian determinant. The CDF of $\mathbf{r}$ is given by

$$F\left(\mathbf{r}\right) = \int_0^\infty \frac{1}{\sigma_u^2} \exp\left(-\frac{x}{\sigma_u^2}\right) \prod_{i=1}^{M} \left(1 - \exp\left(-\frac{r_i^2}{\sigma_{v_i}^2 x}\right)\right) dx. \tag{21}$$

After taking derivatives, we can obtain

$$\begin{aligned} f\left(\mathbf{r}\right) &= \frac{1}{\sigma_u^2} \prod_{i=1}^{M} \frac{2r_i}{\sigma_{v_i}^2} \int_0^\infty x^{-M} \exp\left(-\frac{x}{\sigma_u^2} - \frac{1}{x}\sum_{i=1}^{M} \frac{r_i^2}{\sigma_{v_i}^2}\right) dx \\ &= \left(\prod_{i=1}^{M} \frac{4r_i}{\sigma_u^2 \sigma_{v_i}^2}\right) q\left(\frac{4}{\sigma_u^2} \sum_{i=1}^{M} \frac{r_i^2}{\sigma_{v_i}^2}\right), \end{aligned} \tag{22}$$

where we use [23, 3.478.4] in the last equality. Plugging (22) back into (20) completes the proof. ■

According to *Lemma 1*, the likelihood function of $\tilde{\mathbf{y}}_{rd}$ can be obtained after redefining the parameters in (19). To be specific, for $\hat{\mathbf{x}}_1 = \hat{\mathbf{x}}_2 = \mathbf{e}_l$ we have $\sigma_u^2 = \alpha P \lambda_{rd}$, $\sigma_{v_l}^2 = 2\left(P\lambda_{sr} + 1\right)$ and $\sigma_{v_i}^2 = 2$ for $i \neq l$, whereas for $\hat{\mathbf{x}}_1 = \mathbf{e}_k, \hat{\mathbf{x}}_2 = \mathbf{e}_l$ with $k \neq l$ we have $\sigma_u^2 = \alpha P \lambda_{rd}$, $\sigma_{v_k}^2 = \sigma_{v_l}^2 = P\lambda_{sr} + 2$ and $\sigma_{v_i}^2 = 2$ for $i \neq l, k$. In later sections, this suboptimum receiver is referred to as *noise elimination receiver* (NER).

TABLE I
FOUR TYPES OF PEPS

Notations	True Symbols	Trial Symbols	Error Rates
P_1	$(\mathbf{e}_1, \mathbf{e}_1)$	$(\mathbf{e}_2, \mathbf{e}_2)$	$\mathrm{O}\left(\frac{\log P}{P^3}\right)$
P_2	$(\mathbf{e}_1, \mathbf{e}_2)$	$(\mathbf{e}_2, \mathbf{e}_1)$	$\mathrm{O}\left(\frac{1}{P^2}\right)$
P_3	$(\mathbf{e}_1, \mathbf{e}_1)$	$(\mathbf{e}_1, \mathbf{e}_2)$	$\mathrm{O}\left(\frac{\log^2 P}{P^2}\right)$
P_4	$(\mathbf{e}_1, \mathbf{e}_2)$	$(\mathbf{e}_1, \mathbf{e}_1)$	$\mathrm{O}\left(\frac{\log^3 P}{P^2}\right)$

III. ERROR PERFORMANCE ANALYSIS

The objective of this section is to study the error performance of the considered multi-user uplink using non-coherent modulations. Since the non-coherent ML receiver has an integral form which is analytically intractable, we shall investigate the partial coherent receiver instead, the error rate of which serves as a tight lower bound on the error rate of non-coherent receiver. To simplify the notations, we focus only on the binary FSK modulations (i.e., $\Omega = \{\mathbf{e}_1, \mathbf{e}_2\}$). Our analytical framework can be easily extended to any higher-order modulations.

As an analytical tool, PEP is defined as the probability of mistaking the true symbols $(\mathbf{x}_1, \mathbf{x}_2)$ by another trial symbols $(\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2)$. It is well known that the real error rate is approximately characterized by the dominant PEPs [1]. As for the binary modulations, there is a total of four types of PEPs as listed in Table I, where we also briefly summarize the asymptotic error rates at high SNRs (i.e., $P \gg 1$) to be derived below.

A. Type I PEP

When both of the two source symbols are flipped at the receiver, all the three likelihood functions in (5) are different under the two hypotheses and we have

$$P_1 = \Pr\left\{\frac{\lambda_{sd}P}{1+\lambda_{sd}P}\left(U_2^I + V_2^I\right) + \frac{\lambda_{sr}PQ}{1+\lambda_{sr}PQ}W_2^I \geq \lambda_{sd}P\left(U_1^I + V_1^I\right) + \lambda_{sr}PQW_1^I\right\} \leq \Pr\left\{U_2^I + V_2^I + W_2^I \geq PY\right\}, \quad (23)$$

where $W_1^I = \frac{|y_{rd,1}|^2}{1+2\alpha P\lambda_{rd}|g|^2+2\alpha P^2\lambda_{rd}\lambda_{sr}|g|^2}$, $U_1^I = \frac{|y_{1d,1}|^2}{1+\lambda_{sd}P}$, $V_1^I = \frac{|y_{2d,1}|^2}{1+\lambda_{sd}P}$, $W_2^I = \frac{|y_{rd,2}|^2}{1+2\alpha P\lambda_{rd}|g|^2}$, $U_2^I = |y_{1d,2}|^2$, $V_2^I = |y_{2d,2}|^2$ are i.i.d. exponential random variables with unit mean, and

$$Q = \frac{2\alpha P\lambda_{rd}|g|^2}{1+2\alpha P\lambda_{rd}|g|^2}. \quad (24)$$

Since $U_2^I + V_2^I + W_2^I \sim \frac{1}{2}\chi_6^2$, the conditional PEP given $Y \triangleq \lambda_{sd}\left(U_1^I + V_1^I\right) + \lambda_{sr}QW_1^I = y$ is

$$P_1|_{Y=y} \leq \left(1 + Py + \frac{1}{2}P^2y^2\right)\exp\left(-Py\right). \quad (25)$$

Note that when $P \gg 1$, the above error rate decreases really fast with y. Therefore, the unconditional probability is roughly determined by the behavior of the distribution of $Y \ll 1$. Denoting $T \triangleq QW_1^I$, we can obtain

$$F_Y\left(y\right) \overset{y\ll 1}{\approx} \int_0^{\frac{y}{\lambda_{sr}}} f_T\left(t\right)\frac{\left(y-\lambda_{sr}t\right)^2}{\lambda_{sd}^2}dt, \quad (26)$$

Taking derivative with respect to y leads to

$$f_Y\left(y\right) \overset{y\ll 1}{\approx} \frac{2\lambda_{sr}}{\lambda_{sd}^2}\int_0^{\frac{y}{\lambda_{sr}}} F_T\left(t\right)dt = -\frac{y^2}{2\alpha P\lambda_{sr}\lambda_{rd}\lambda_{sd}^2}\left(\log\frac{y}{\lambda_{sr}} - \frac{1}{2}\right), \quad (27)$$

where we use $F_T\left(t\right) \overset{t\ll 1}{\approx} -\frac{1}{2\alpha P\lambda_{rd}}t\log t$ [7][8] in the second equality. Using the above PDF to average the conditional PEP in (25), we obtain

$$P_1 \overset{P\to\infty}{\leq} \frac{20\log P - 39 + 20\gamma + 20\log\left(\sqrt{e}\lambda_{sr}\right)}{2\alpha P^4\lambda_{sr}\lambda_{rd}\lambda_{sd}^2} \overset{P\to\infty}{\approx} \frac{10}{\alpha P\lambda_{sr}\lambda_{rd}\lambda_{sd}^2}\frac{\log P}{P^3}, \quad (28)$$

where γ is Euler constant [23, 4.352.2].

B. Type II PEP

For Type II PEP, the likelihood function of $\mathbf{y}_{rd}$ remains the same under both hypotheses, which greatly simplifies the computation. After some manipulations, the PEP is given by

$$P_2 = \Pr\left\{|y_{1d,2}|^2 + |y_{2d,1}|^2 \geq |y_{1d,1}|^2 + |y_{2d,2}|^2\right\} = \frac{4+3\lambda_{sd}P}{\left(2+\lambda_{sd}P\right)^3} \overset{P\to\infty}{\approx} \frac{3}{\lambda_{sd}^2P^2}. \quad (29)$$

C. Type III PEP

For Type III PEP, the likelihood function of $\mathbf{y}_{1d}$ remains the same under both hypotheses, and the PEP is given by

$$P_3 = \Pr\left\{\frac{\lambda_{sd}P}{1+\lambda_{sd}P}V_2^{III} + \frac{\lambda_{sr}PQ}{2+\lambda_{sr}PQ}W_2^{III} \geq \lambda_{sd}PV_1^{III} + \frac{\lambda_{sr}PQ}{2+\lambda_{sr}PQ}W_1^{III} + \log\frac{\left(2+\lambda_{sr}PQ\right)^2}{4\left(1+\lambda_{sr}PQ\right)}\right\} \leq \Pr\left\{W_2^{III} + V_2^{III} \geq \lambda_{sd}PV_1^{III} + \log Z\right\}, \quad (30)$$

where $W_1^{III} = \frac{|y_{rd,1}|^2}{1+2\alpha P\lambda_{rd}|g|^2+2\alpha P^2\lambda_{rd}\lambda_{sr}|g|^2}$, $V_1^{III} = \frac{|y_{2d,1}|^2}{1+\lambda_{sd}P}$, $W_2^{III} = \frac{|y_{rd,2}|^2}{1+2\alpha P\lambda_{rd}|g|^2}$, and $V_2^{III} = |y_{2d,2}|^2$ are i.i.d. exponential random variables with unit mean, and

$$\frac{2+\lambda_{sr}PQ}{4} \triangleq Z \leq \frac{1}{2} + \frac{P\lambda_{sr}}{4} \overset{P\to\infty}{\approx} \frac{P\lambda_{sr}}{4}. \quad (31)$$

After some manipulations, the conditional PEP given $Z = z$ can be obtained as

$$P_3|_{Z=z} \leq \frac{\left(1+2\lambda_{sd}P\right) + \left(1+\lambda_{sd}P\right)\log\left(\frac{1}{2}+\frac{P\lambda_{sr}}{4}\right)}{\left(1+\lambda_{sd}P\right)^2z} \overset{P\to\infty}{\approx} \frac{1}{\lambda_{sd}z}\frac{\log P}{P}. \quad (32)$$

By defining $\eta = \alpha P\lambda_{rd}\left(2+\lambda_{sr}P\right)$, we can further obtain

$$E\left(\frac{1}{Z}\right) \leq \frac{1}{\eta}\left(4\alpha P\lambda_{rd} + \frac{2\lambda_{sr}P}{2+\lambda_{sr}P}\log\left(1+\eta\right)\right) \overset{P\to\infty}{\approx} \frac{2}{\alpha P\lambda_{sr}\lambda_{rd}}\frac{\log P}{P}. \quad (33)$$

Plugging the above result back into the conditional PEP (32), we can obtain

$$P_3 \leq \frac{2}{\alpha P\lambda_{sd}\lambda_{sr}\lambda_{rd}}\frac{\log^2 P}{P^2}, P\to\infty. \quad (34)$$

D. Type IV PEP

Similar to Type III PEP, the likelihood function of $\mathbf{y}_{1d}$ remains the same under both hypotheses, and the PEP is given by

$$P_4 = \Pr\left\{\frac{\lambda_{sd}P}{1+\lambda_{sd}P}V_1^{IV} + \frac{\lambda_{sr}PQ}{2\left(1+\lambda_{sr}PQ\right)}W_1^{IV} \geq \lambda_{sd}PV_2^{IV} + \frac{1}{2}\lambda_{sr}PQW_2^{IV} - \log\frac{\left(2+\lambda_{sr}PQ\right)^2}{4\left(1+\lambda_{sr}PQ\right)}\right\}, \quad (35)$$

where $W_1^{IV} = \frac{|y_{rd,1}|^2}{1+2\alpha P\lambda_{rd}|g|^2+\alpha P^2\lambda_{rd}\lambda_{sr}|g|^2}$, $V_1^{IV} = |y_{2d,1}|^2$, $W_2^{IV} = \frac{|y_{rd,2}|^2}{1+2\alpha P\lambda_{rd}|g|^2+\alpha P^2\lambda_{rd}\lambda_{sr}|g|^2}$, $V_2^{IV} = \frac{|y_{2d,2}|^2}{1+\lambda_{sd}P}$ are i.i.d. exponential random variables with unit mean. As the logarithmic term is asymptotically upper bounded by $\log\frac{\lambda_{sr}P}{4}$ when $P\to\infty$, we can upper bound the PEP by

$$P_4 \leq \underbrace{\Pr\left\{W_1^{IV} + V_1^{IV} \geq \max\left(\frac{1}{2}\lambda_{sd}PV_2^{IV}, \log\frac{\lambda_{sr}P}{4}\right)\right\}}_{\triangleq P_4^{U_1}} + \underbrace{\Pr\left\{2\log\frac{\lambda_{sr}P}{4} \geq \lambda_{sd}PV_2^{IV} + \frac{1}{2}\lambda_{sr}PQW_2^{IV}\right\}}_{\triangleq P_4^{U_2}}. \quad (36)$$

$$P_4^{U_1} = \frac{4\left(1+\log\frac{\lambda_{sr}P}{4}\right)}{\lambda_{sr}P} - \left(\frac{\lambda_{sd}P}{2+\lambda_{sd}P}\right)^2\left(1+\frac{2+\lambda_{sd}P}{\lambda_{sd}P}\log\frac{\lambda_{sr}P}{4}\right)\exp\left(-\frac{2+\lambda_{sd}P}{\lambda_{sd}P}\log\frac{\lambda_{sr}P}{4}\right) \overset{P\to\infty}{\approx} \frac{8}{\lambda_{sr}\lambda_{sd}}\frac{\log^2 P}{P^2}. \tag{37}$$

$$P_4^{U_2} \overset{P\to\infty}{\approx} -\frac{\lambda_{sr}}{8\alpha P\lambda_{sd}\lambda_{rd}}\left(\frac{4}{\lambda_{sr}P}\log\frac{\lambda_{sr}P}{4}\right)^2\left(\log\left(\frac{4}{\lambda_{sr}P}\log\frac{\lambda_{sr}P}{4}\right)-\frac{1}{2}\right) \overset{P\to\infty}{\approx} \frac{2}{\alpha P\lambda_{sr}\lambda_{sd}\lambda_{rd}}\frac{\log^3 P}{P^2}. \tag{38}$$

$$P_4^L\big|_{Q=q} = 1 - \frac{2\lambda_{sd}}{2\lambda_{sd}-q\lambda_{sr}}\exp\left(-\frac{1}{\lambda_{sd}P}\log\frac{1}{2}\left(1+\frac{q\lambda_{sr}P}{2}\right)\right) + \frac{q\lambda_{sr}}{2\lambda_{sd}-q\lambda_{sr}}\exp\left(-\frac{2}{q\lambda_{sr}P}\log\frac{1}{2}\left(1+\frac{q\lambda_{sr}P}{2}\right)\right) \tag{40}$$

The two terms can be obtained after some lengthy algebra and are shown in (37) and (38) on the top of this page. Therefore, Type IV PEP is upper bounded by $\mathrm{O}\left(\frac{\log^3 P}{P^2}\right)$, which appears to dominate all types of PEP. To make the argument rigorous, yet we still need to show that this is the exact scaling law of Type IV PEP by finding a proper lower bound P_4^L of P_4. This can be done by neglecting the first two terms on the left-hand side of the inequality in (35), which leads to

$$P_4^L = \Pr\left\{\log\frac{2+\lambda_{sr}PQ}{4} \geq \lambda_{sd}PV_2^{IV} + \frac{\lambda_{sr}PQW_2^{IV}}{2}\right\}. \tag{39}$$

After some manipulations, the conditional probability of P_4^L given $Q = q$ is shown in (40) on the top of this page. When $q \geq P^{-\beta}$ for any constant $\beta \in (0,1)$, we have $qP \geq P^{1-\beta} \overset{P\to\infty}{\to} \infty$ and thus

$$P_4^L\big|_{Q=q} \overset{P\to\infty}{\approx} \frac{1}{q\lambda_{sr}\lambda_{sd}P^2}\left(\log\frac{1}{2}\left(1+\frac{q\lambda_{sr}P}{2}\right)\right)^2. \tag{41}$$

The final step is to average the above expression over the distribution of Q, which is given by

$$f_Q(q) = \frac{1}{2\alpha P\lambda_{rd}(1-q)^2}\exp\left(-\frac{q}{2\alpha P\lambda_{rd}(1-q)}\right) \tag{42}$$

for $0 \leq q \leq 1$. It is easy to see that $f_Q(q)$ is a continuous function with $f_Q(0) = \frac{1}{2\alpha P\lambda_{rd}}$ and $f_Q(1) = 0$, therefore it is lower bounded by some constant C on the region $q \in [0, b]$ with $b < 1$ being some fixed number. Using the above facts, we can obtain

$$\begin{aligned} P_4^L &\geq \frac{C\log\left(bP^{\beta}\right)}{\lambda_{sr}\lambda_{sd}P^2}\left(\log\frac{1}{2}\left(1+\frac{\lambda_{sr}P^{1-\beta}}{2}\right)\right)^2 \\ &\overset{P\to\infty}{\approx} \frac{C\beta(1-\beta)^2}{\lambda_{sr}\lambda_{sd}}\frac{\log^3 P}{P^2}. \end{aligned} \tag{43}$$

As a result, the upper bound and lower bound on P_4 have exactly the same scaling law of $\mathrm{O}\left(\frac{\log^3 P}{P^2}\right)$ when $P \to \infty$.

E. Discussions

So far, we have shown that Type IV PEP dominates the error rate of partial coherent detection and has a scaling law of $\mathrm{O}\left(\frac{\log^3 P}{P^2}\right)$. By comparison, the error rate of coherent detection has a scaling law of $\mathrm{O}\left(\frac{\log P}{P^2}\right)$ [7][8]. Therefore, although partial coherent detection can still achieve full diversity when system SNR is extremely high, the lack of perfect CSI introduces additional logarithmic terms in the error rate expression, which makes the error rate decay really slow compared to coherent detection. Note that in the single-user system, the non-coherent detection only brings 3dB SNR loss, and the scaling law remains the same as that of coherent detection. The additional loss in the multi-user scenario is mainly due to the incapability to efficiently suppress multi-user interferences when perfect CSI is unavailable.

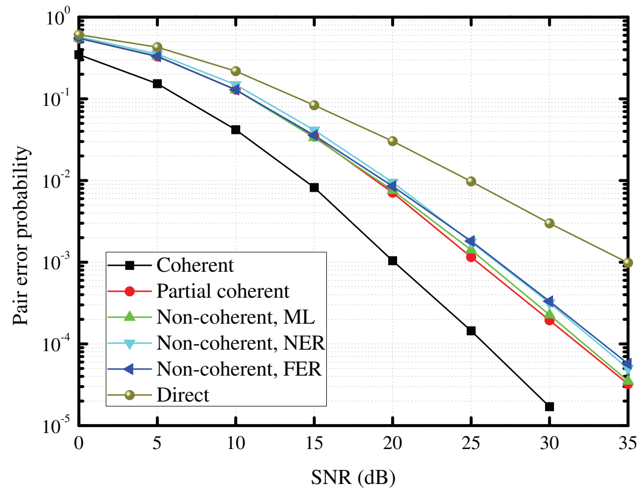

Fig. 1. Error performances of a symmetric network.

IV. SIMULATIONS

In this section, we present simulation results to validate our analysis. Throughout simulations, we use the path loss model $\lambda = D^{-3}$, where λ is the channel gain and D is the distance between two terminals. Pair error probability is used as the performance metric, i.e., the probability that at least one of the source symbols is decoded incorrectly at the destination. To simplify the simulation settings, only binary FSK modulation is considered and D_{sd} is always normalized to 1.

In Fig. 1, we compare the error rates of different detection schemes in a symmetric network where all the inter-node distances are normalized to 1. It is observed that the partial coherent detection performs almost the same as non-coherent

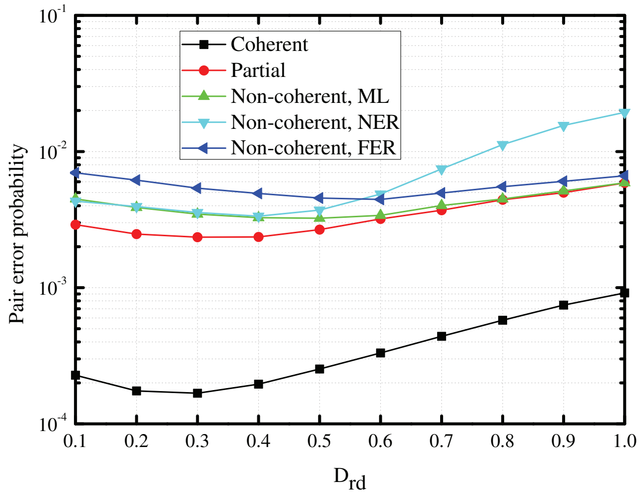

Fig. 2. Error performances with different relay positions.

detection. Besides, the two suboptimum non-coherent receivers also perform reasonably well with about 1dB SNR loss. Compared to direct transmission without user cooperation, both coherent and non-coherent network-coded cooperation can provide a diversity order of 2. However, the error curves of partial/non-coherent schemes decay much slower than the error curve of coherent scheme due to the additional logarithmic term loss. For example, non-coherent detection incurs about 6dB SNR loss when the error rate is 10^{-4}. Note that such performance gap is much larger than that in the single-user systems. This is because the two source signals are randomly combined in the air and the receiver is unable to efficiently suppress the multi-user interferences.

Then we investigate the error performances with different relay positions in Fig. 2. For the network topology, we place the destination at $(0,0)$, and locate the two source nodes at $(\frac{\sqrt{3}}{2}, \pm\frac{1}{2})$, respectively. The relay node shall move along the x-axis from $(0.1,0)$ to $(1,0)$. Simulation results confirm that NER is nearly optimum when the relay node is close to the destination. The performance of FER gets closer to that of non-coherent ML receiver as the relay node moves away from the destination and to the two source nodes. Although the error performance analysis in Section III is focused only on partial coherent detection, we conjecture that the error rate of non-coherent detection should have the same scaling law, since these two schemes seem to perform very close to each other in all simulations. A rigorous proof shall be deferred to future work.

V. Conclusions and Future Work

We have studied the two-user uplink using analog network coding when the receiver has non-perfect CSI. Both the optimum and suboptimum detection schemes are developed under different CSI assumptions. We also obtained the scaling law of the error rate of partial coherent detection and quantified the performance loss compared to coherent detection. Future work may focus on the same applications using digital network coding.

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