# Distributed Relay Selection and Power Control for Multiuser Cooperative Communication Networks Using Stackelberg Game

Beibei Wang, *Student Member*, *IEEE*, Zhu Han, *Member*, *IEEE*, and K.J. Ray Liu, *Fellow*, *IEEE*

**Abstract**—The performance in cooperative communication depends on careful resource allocation such as relay selection and power control, but the traditional centralized resource allocation requires precise measurements of channel state information (CSI). In this paper, we propose a distributed game-theoretical framework over multiuser cooperative communication networks to achieve optimal relay selection and power allocation without knowledge of CSI. A two-level Stackelberg game is employed to jointly consider the benefits of the source node and the relay nodes in which the source node is modeled as a buyer and the relay nodes are modeled as sellers, respectively. The proposed approach not only helps the source find the relays at relatively better locations and "buy" an optimal amount of power from the relays, but also helps the competing relays maximize their own utilities by asking the optimal prices. The game is proved to converge to a unique optimal equilibrium. Moreover, the proposed resource allocation scheme with the distributed game can achieve comparable performance to that employing centralized schemes.

**Index Terms**—Cooperative communication networks, relay selection, distributed power allocation, game theory.

## 1 Introduction

Recently, cooperative communications [1] have gained much attention as an emerging transmit strategy for future wireless networks. The basic idea is that relay nodes can act as a virtual antenna array to help the source node forward its information to the destination. In this way, cooperative communication efficiently takes advantage of the broadcasting nature of wireless networks. Besides, it exploits the inherent spatial and multiuser diversities.

The performance in cooperative communication depends on careful resource allocations such as relay placement, relay selection, and power control [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17]. In [2], the power allocation is optimized to satisfy the outage probability criterion. The authors in [3] provide the analysis on symbol error rates and optimum power allocations for the decode-and-forward cooperation protocol in wireless networks. The energy-efficient broadcast problem in wireless networks is considered in [4]. The work in [5] evaluates the cooperative diversity performance when the best relay is chosen according to the average signal-to-noise ratio (SNR) and the outage probability of relay selection based on instantaneous SNRs. In [6], the authors propose a distributed relay selection scheme that requires limited network knowledge with instantaneous SNRs. In [7], the relay assignment problem is solved for the multiuser cooperative communications. In [8], the cooperative resource allocation for OFDM is studied. The authors of [9] and [10] investigate the relay selection problem with focus on when to cooperate and which relay to cooperate with, which requires channel state information (CSI). In [11], centralized power allocation schemes are presented by assuming that all the relay nodes help. In order to further minimize the system outage behaviors and improve the average throughput, a selection forward protocol is proposed to choose only one "best" relay node to assist the transmission. A centralized resource allocation algorithm for power control, bandwidth allocation, relay selection, and relay strategy choice in an OFDMA-based relay network is proposed in [12]. The work in [13] develops distributed power control strategies for multihop cooperative transmission schemes. Lifetime extension for wireless sensor networks with the aid of relay selection and power management schemes is investigated in [14]. The authors of [15] study the optimal power allocation problem in the high-SNR regime for different relaying protocols. Relay station placement and relay time allocation in IEEE 802.16j networks is investigated in [16].

However, most existing work focuses on resource allocation in cooperative communications by means of a centralized fashion. Such schemes require that complete and precise CSI be available in order to optimize the system performance, which are generally neither scalable nor robust to channel estimation errors. This fact motivates the research on distributed resource allocation without requiring CSI. For distributed resource allocation, there are two main questions over multiuser cooperative wireless networks: 1) among all the distributed nodes, who can help relay and improve the source node's link quality better and 2) for the selected relay nodes, how much power do they need to transmit?

- *B. Wang and K.J.R. Liu are with the Department of Electrical and Computer Engineering, University of Maryland, College Park, MD 20742. E-mail: {bebewang, kjrliu}@umd.edu.*
- *Z. Han is with the Electrical and Computer Engineering Department, University of Houston, N324 Engineering Building 1, Houston, TX 77004. E-mail: zhan2@mail.uh.edu.*

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To answer these two questions, game theory is a natural and flexible tool that studies how the autonomous nodes interact and cooperate with each other. In game theory literature of wireless networking, in [18], the behaviors of selfish nodes in the case of random access and power control are examined. In [19], static pricing policies for multiple-service networks are proposed. Such policies can offer incentives for each node to choose the service that best matches its needs so as to discourage overallocation of resources and improve social welfare. The work in [20] presents a power control solution for wireless data in the analytical setting of a game-theoretical framework. Pricing of transmit powers is introduced to improve user utilities that reflect the quality of service a wireless terminal receives. A pricing game that stimulates cooperation via reimbursements to the relay is proposed in [21], but there was no detailed analysis on how to select the best relays and how to achieve the equilibrium distributively. In [22], the authors employ a cooperative game for the single-cell OFDMA resource allocation.

In general, in multiuser cooperative wireless networks with selfish nodes, nodes may not serve a common goal or belong to a single authority. Therefore, a mechanism of reimbursement to relay nodes is needed such that relay nodes can earn benefits from spending their own transmission power in helping the source node forward its information. On the other hand, if the source node reimburses relay nodes for their help, it needs to choose the most beneficial relay nodes. According to such characteristics, in this paper, we employ a Stackelberg game [25] to jointly consider the benefits of the source node and relay nodes in cooperative communications. The game is divided into two levels. The source node plays the *buyer-level* game, since it aims to achieve the best performance with the relay nodes' help with the least reimbursements to them. We analyze how many and which relay nodes are selected by the source node to participate in relaying after they announce their optimal prices. In addition, we optimize how much service (such as power) the source node will buy from each relay node. On the other hand, each relay node plays the *seller-level* game, in which it aims to earn the payment that not only covers its forwarding cost but also gains as many extra profits as possible. Therefore, the relay node needs to set the optimal price per unit for the service so as to maximize its own benefit. To study the game outcomes, we analyze several properties of the proposed game. Then, we develop a distributed algorithm that can converge to the optimal game equilibrium.

From the simulations, the relay nodes close to the source node play an important role in increasing the source node's utility, so the source node likes to buy power from these preferred relay nodes. On the other hand, in order to attract more buying from the source, the relay adopts a "low-price, high-market" policy to further increase its benefit. If the total number of available relay nodes increases, the source node will obtain a larger utility value while the average payment to the relay nodes will decrease. We finally show that the proposed resource allocation scheme with distributed game achieves comparable performance to those of the centralized scheme [11].

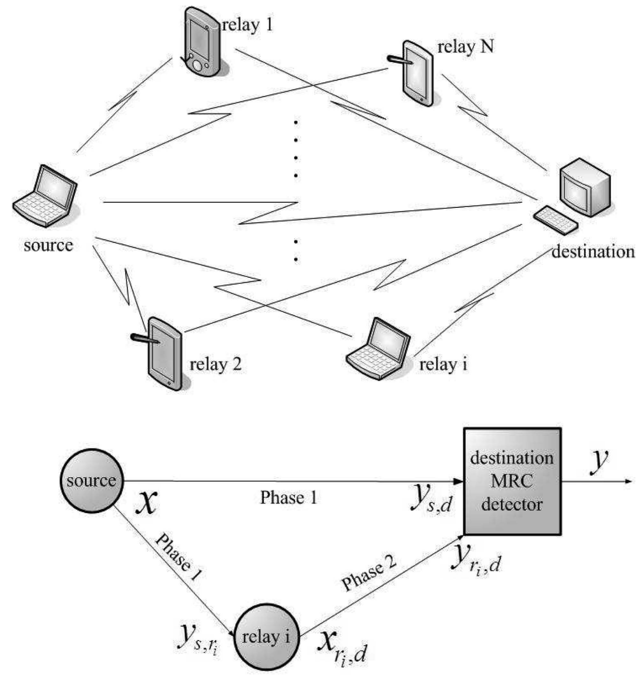



Fig. 1. System diagrams.

The rest of the paper is organized as follows: In Section 2, we describe the system model and formulate the cooperative optimization as a Stackelberg game. We construct the distributed implementation of multiuser cooperation transmissions and provide the solutions in Section 3. Simulation results are shown in Section 4. Finally, conclusions are drawn in Section 5.

## 2 System Model and Problem Formulation

In this section, we first derive the expression of the maximal achievable rate in cooperative transmission with the relay nodes' help. Then, we formulate the optimization problem of relay selection and power control using a Stackelberg game.

### 2.1 System Model

In the sequel, we employ the amplify-and-forward (AF) cooperation protocol [1] as our system model; other cooperation protocols [1] can be considered in a similar way. The system diagrams are shown in Fig. 1, in which there are in total $N$ relay nodes, one source node $s$, and one destination node $d$. The cooperative transmission consists of two phases.

In Phase 1, source node $s$ broadcasts its information to both destination node $d$ and each relay node $r_i$. The received signals $y_{s,d}$ and $y_{s,r_i}$ at node $d$ and node $r_i$ can be expressed as

$$y_{s,d} = \sqrt{P_s G_{s,d}}x + \eta_{s,d} \tag{1}$$

and

$$y_{s,r_i} = \sqrt{P_s G_{s,r_i}}x + \eta_{s,r_i}, \tag{2}$$

where $P_s$ represents the transmit power at node $s$, $x$ is the broadcast information symbol with unit energy from node $s$ to node $d$ and node $r_i$, $G_{s,d}$ and $G_{s,r_i}$ are the channel gains from node $s$ to node $d$ and node $r_i$, respectively, and $\eta_{s,d}$ and $\eta_{s,r_i}$ are the additive white Gaussian noises (AWGNs). Without loss of generality, we assume that the noise power is the same for all the links, denoted by $\sigma^2$. We also assume that the channels are stable over each transmission frame.

Without the relay nodes' help, the SNR that results from the direct transmission from node $s$ to node $d$ can be expressed by

$$\Gamma_{s,d} = \frac{P_s G_{s,d}}{\sigma^2}, \tag{3}$$

and the rate of the direct transmission is

$$R_{s,d} = W \log_2\left(1 + \frac{\Gamma_{s,d}}{\Gamma}\right), \tag{4}$$

where $W$ is the bandwidth for transmission, and $\Gamma$ is a constant representing the capacity gap.

In Phase 2, relay node $r_i$ amplifies $y_{s,r_i}$ and forwards it to destination $d$ with transmitted power $P_{r_i}$. The received signal at destination node $d$ is

$$y_{r_i,d} = \sqrt{P_{r_i} G_{r_i,d}} x_{r_i,d} + \eta_{r_i,d}, \tag{5}$$

where

$$x_{r_i,d} = \frac{y_{s,r_i}}{|y_{s,r_i}|} \tag{6}$$

is the transmitted signal from node $r_i$ to node $d$ that is normalized to have unit energy, $G_{r_i,d}$ is the channel gain from node $r_i$ to node $d$, and $\eta_{r_i,d}$ is the received noise. Substituting (2) into (6), we can rewrite (5) as

$$Y_{r_i,d} = \frac{\sqrt{P_{r_i} G_{r_i,d}}\left(\sqrt{P_s G_{s,r_i}} X + \eta_{s,r_i}\right)}{\sqrt{P_s G_{s,r_i} + \sigma^2}} + \eta_{r_i,d}. \tag{7}$$

Using (7), the relayed SNR for source node $s$, which is helped by relay node $r_i$, is given by

$$\Gamma_{s,r_i,d} = \frac{P_{r_i} P_s G_{r_i,d} G_{s,r_i}}{\sigma^2 (P_{r_i} G_{r_i,d} + P_s G_{s,r_i} + \sigma^2)}. \tag{8}$$

Therefore, by (4) and (8), we have the rate at the output of the maximal-ratio combining (MRC) detector with one relay node $r_i$ helping as

$$R_{s,r_i,d} = \frac{W}{2} \log_2\left(1 + \frac{\Gamma_{s,d} + \Gamma_{s,r_i,d}}{\Gamma}\right). \tag{9}$$

If the relay nodes available to help source node $s$ at a certain time constitute a set, denoted by $L = \{r_1, \ldots, r_N\}$, then we have

$$R_{s,r,d} = \gamma_L W \log_2\left(1 + \frac{\Gamma_{s,d} + \sum_{r_i \in L} \Gamma_{s,r_i,d}}{\Gamma}\right), \tag{10}$$

where $\gamma_L$ denotes a bandwidth factor.

According to different network applications, $\gamma_L$ can have different definitions. For the energy-constrained networks, $\gamma_L$ is set to 1. For the network with a limited bandwidth, the bandwidth should be divided for the source node and relay nodes, and $\gamma_L$ depends on the number of relay nodes that actually help forwarding, since not all the relay nodes will contribute to a better performance for the source node. If $N'$ out of $N$ relay nodes are selected by the source node, $N' \leq N$, then $\gamma_L = \frac{1}{N'+1}$.[1] We will study the energy constrained scenario first, then we show the effects of the varying $\gamma_L$ in the simulation part.

## 2.2 Problem Formulation

To exploit the cooperative diversity for multiuser systems, from (10), two fundamental questions on resource allocation need to be answered: 1) which relay nodes will be included, and 2) what is the optimal power $P_{r_i}$? However, solving these issues in a centralized manner requires accurate and complete CSI, bringing considerable overheads and signaling of information about channel estimations. In contrast, the distributed resource allocation only needs local knowledge about channel information. Moreover, in general, nodes in multiuser cooperative wireless networks may belong to different authorities and act selfishly. Incentives need to be provided by the source node to the relay nodes for relaying the information. Consequently, the source node needs to choose the most beneficial relay nodes. According to the behaviors of the source node and the relay nodes, we employ a distributed resource allocation using a Stackelberg-game-based scheme as the following formulated problem:

1. **Source node/buyer.** The source node $s$ can be modeled as a buyer and aims to obtain the most benefits at least possible payments. The utility function of source node $s$ can be defined as

$$U_s = aR_{s,r,d} - M, \tag{11}$$

where $R_{s,r,d}$ denotes the achievable rate with the relay nodes' help, $a$ denotes the gain per unit of rate at the MRC output, and

$$M = \sum_{r_i \in L} p_i P_{r_i} = p_1 P_{r_1} + p_2 P_{r_2} + \cdots + p_N P_{r_N} \tag{12}$$

represents the total payments paid by source node $s$ to the relay nodes. In (12), $p_i$ represents the price per unit of power selling from relay node $r_i$ to source node $s$, and $P_{r_i}$ denotes how much power node $s$ will buy from node $r_i$.

The relay nodes helping source node $s$ constitute a set, still denoted by $L$; then, the optimization problem for source node $s$ or the *buyer-level* game can be formulated as

$$\max_{\{P_{r_i}\}} U_s = aR_{s,r,d} - M, \quad \text{s.t. } P_{r_i} \geq 0, r_i \in L. \tag{13}$$

2. **Relay node/seller.** Each relay node $r_i$ can be seen as a seller and aims to not only earn the payment

1. The source node can know the number of available relay nodes by broadcasting its signal and listening to the relay nodes' feedback on whether to help forward the source node's information.

that covers its forwarding cost but also gain as many extra profits as possible. We introduce a parameter $c_i$, the cost of power for relaying data, in our formulation. Then, relay node $r_i$'s utility function can be defined as

$$U_{r_i} = p_i P_{r_i} - c_i P_{r_i} = (p_i - c_i) P_{r_i}, \qquad (14)$$

where $P_{r_i}$ is the source node's power consumption by optimizing $U_s$ described in (13). The optimization problem for relay node $r_i$ or the *seller-level* game is

$$\max_{p_i > 0} U_{r_i} = (p_i - c_i) P_{r_i}, \quad \forall i. \qquad (15)$$

The choice of the optimal price $p_i$ is affected not only by each relay node's own channel conditions to the source node and the destination node but also by the other relay nodes' prices. This is because the seller-level game is noncooperative, and the relay nodes compete to get selected by source node $s$. If a certain relay node $r_j$ asks such a high price that makes it less beneficial than the other relay nodes to source node $s$, then source node $s$ will buy less from relay node $r_j$ or even discard it. On the other hand, if the price is too low, the profit obtained by (14) will be unnecessarily low. Overall, there is a trade-off for setting the price. If under the optimal price, denoted by $p_i^*$, the resulting utility of relay node $r_i$ is negative, i.e., $U_{r_i}^* \leq 0$, then node $r_i$ will quit the *seller-level* game since it cannot cover the basic cost by selling power to the source node.

It is worth noticing that the only signaling required to exchange between the source node and the relay nodes are the price $p_i$ and the information about how much power $P_{r_i}$ to buy. Consequently, the proposed two-level game-theoretical approach can be implemented in a distributed way. The outcome of the proposed games will be shown in detail in the following section.

## 3 Analysis of the Proposed Games

First, we obtain closed-form solutions to the outcomes of the proposed games. Then, we prove that these solutions are the global optima. Furthermore, we show that the set of the solutions is a unique fixed point and the proposed distributed game converges to that point. Finally, we compare the performance of the proposed distributed scheme to that of a centralized scheme.

### 3.1 Analysis of the Buyer-Level Game for the Source Node

#### 3.1.1 Relay Selection by the Source Node

As relay nodes are located in different places and ask different prices for helping the source node, it may not be good for source node $s$ to choose all relay nodes, especially those with bad channel conditions but asking a high price. Moreover, if the source node will exclude the less beneficial relay nodes sooner or later during the buyer-level game, it is better to reject them at the beginning so as to reduce the signaling overhead. Because source node $s$ aims at maximizing utility $U_s$ through buying an optimal amount of power $P_{r_i}$, then a natural way of relay selection for source node $s$ is to observe how $U_s$ varies with $P_{r_i}$, i.e., observe the sign of $\frac{\partial U_s}{\partial P_{r_i}}$. Since source node $s$ gradually increases the amount of power bought from the relay nodes to approach the optimum, by observing the sign of $\frac{\partial U_s}{\partial P_{r_i}}$ when $P_{r_i} = 0$, node $s$ can exclude (or select) those less (or more) beneficial relay nodes.

From the definition in (11), we know that

$$\frac{\partial U_s}{\partial P_{r_i}} = a \frac{\partial R_{s,r,d}}{\partial P_{r_i}} - p_i, \quad i = 1, \ldots, N. \qquad (16)$$

When $P_{r_j} = 0$, $j = 1, \ldots, N$, if $p_i$ satisfies $p_i < a \frac{\partial R_{s,r,d}}{\partial P_{r_i}}$ for relay node $r_i$, then we have $\frac{\partial U_s}{\partial P_{r_i}} > 0$, meaning that the source node will obtain a larger $U_s$ by increasing $P_{r_i}$. Otherwise, relay node $r_i$ should be excluded.

Then, a question is how each relay node $r_i$ asks its price $p_i$ at the beginning. Since in a distributed implementation, each relay node does not know the other relay nodes' prices, it is natural to first tentatively set $p_i = c_i$. If the initial price $p_i$ is lower than $c_i$, utility $U_{r_i}$ will be negative and, hence, impractical; on the other hand, if the initial price is greater than $c_i$, relay node $r_i$ may be at the risk of being excluded by the source node. If under these lowest initial prices, the source node would choose not to buy any power from some relay node $r_i$, then $r_i$ will not participate in the *seller-level* game because $U_{r_i} = 0$.

To summarize the analysis above, the *relay rejection criteria* of the source node are described as follows: Assume that the total number of the relay nodes is $N$. At first, the source node tentatively chooses $P_{r_i} = 0$, $i = 1, \ldots, N$, and all the relay nodes set their initial prices as $p_i = c_i$, $\forall i$. For relay node $r_j$, if $c_j \geq (a \frac{\partial R_{s,r,d}}{\partial P_{r_j}})$, then $r_j$ is rejected by the source node with correspondingly $P_{r_j} = 0$. It will be shown later that this rejection is fixed and will not change after the game is played.

With the proposed *relay rejection criteria*, source node $s$ can exclude the least beneficial relay nodes at the very beginning. In this way, the signaling overhead can be further reduced, because the source node and the rejected relay nodes no longer need to exchange their information about the purchased power and prices.

#### 3.1.2 Optimal Power Allocation for the Selected Relay Nodes

After the selection, for the selected relay nodes that constitute a set $L_h = \{r_1, \ldots, r_{N'}\}$, we can solve the optimal power $P_{r_i}$ by taking the derivative of $U_s$ in (11) with respect to $P_{r_i}$ as

$$\frac{\partial U_s}{\partial P_{r_i}} = a \frac{\partial R_{s,r,d}}{\partial P_{r_i}} - p_i = 0, \quad r_i \in L_h. \qquad (17)$$

For simplicity, define $C = 1 + \frac{\Gamma_{s,d}}{\Gamma}$ and $W' = \frac{aW}{\ln 2}$. By (10), we get the first term of $U_s$ as

$$\begin{aligned} aR_{s,r,d} &= aW \log_2 \left( C + \frac{1}{\Gamma} \sum_{r_i \in L_h} \Gamma_{s,r_i,d} \right) \\ &= W' \ln\left(1 + \Delta SNR'_{tot}\right) + W' \ln C, \end{aligned} \qquad (18)$$

where

$$\Delta SNR'_{tot} = \sum_{r_i \in L_h} \Gamma'_{s,r_i,d} = \frac{1}{\Gamma C} \sum_{r_i \in L_h} \Gamma_{s,r_i,d}, \qquad (19)$$

and

$$\Gamma'_{s,r_i,d} = \frac{\Gamma_{s,r_i,d}}{\Gamma C} = \frac{A_i}{1 + \frac{B_i}{P_{r_i}}} = \frac{A_i P_{r_i}}{P_{r_i} + B_i}, \tag{20}$$

with $A_i = \frac{P_s G_{s,r_i}}{(\Gamma\sigma^2 + P_s G_{s,d})}$ and $B_i = \frac{P_s G_{s,r_i} + \sigma^2}{G_{r_i,d}}$.

Substituting (12) and (18) into (17), we have

$$\frac{W'}{\left(1 + \sum_{r_k \in L_h} \frac{A_k P_{r_k}}{P_{r_k} + B_k}\right)} = \frac{p_i}{A_i B_i}(P_{r_i} + B_i)^2. \tag{21}$$

Since the left-hand side (LHS) of (21) is the same for any relay node on the right-hand side (RHS), by equating the RHS of (21) for relay nodes $r_i$ and $r_j$, we get

$$P_{r_j} = \sqrt{\frac{p_i A_j B_j}{p_j A_i B_i}}(P_{r_i} + B_i) - B_j. \tag{22}$$

Substituting the above $P_{r_j}$ into (20) and simplifying, we have

$$\Gamma'_{s,r_j,d} = \frac{A_j}{1 + \frac{B_j}{P_{r_j}}} = A_j - \sqrt{\frac{p_j A_i B_i}{p_i A_j B_j}} \frac{A_j B_j}{(P_{r_i} + B_i)}. \tag{23}$$

Then, (19) can be reorganized as

$$\begin{aligned}\Delta SNR'_{tot} &= \left[A_1 - \sqrt{\frac{p_1 A_i B_i}{p_i A_1 B_1}} \frac{A_1 B_1}{(P_{r_i} + B_i)}\right] + \cdots + \left[A_i - \frac{A_i B_i}{P_{r_i} + B_i}\right] \\ &\quad + \cdots + \left[A_{N'} - \sqrt{\frac{p_{N'} A_i B_i}{p_i A_{N'} B_{N'}}} \frac{A_{N'} B_{N'}}{(P_{r_i} + B_i)}\right] \\ &= \sum_{r_j \in L_h} A_j - \sqrt{\frac{A_i B_i}{p_i}} \frac{1}{P_{r_i} + B_i} \sum_{r_j \in L_h} \sqrt{p_j A_j B_j}.\end{aligned} \tag{24}$$

Substituting (24) into (21), after some manipulations, we can have a quadratic equation of $P_{r_i}$. The optimal power consumption is

$$P^*_{r_i} = \sqrt{\frac{A_i B_i}{p_i}} \frac{Y + \sqrt{Y^2 + 4XW'}}{2X} - B_i, \tag{25}$$

where $X = 1 + \sum_{r_j \in L_h} A_j$, and $Y = \sum_{r_j \in L_h} \sqrt{p_j A_j B_j}$.

The solution in (25) can also be verified by the Karush-Kuhn-Tucker (KKT) condition [27] to be the global optimum to problem (13), since the $U_s$ function is concave in $\{P_{r_i}\}_{i=1}^N$ and the supporting set $\{P_{r_i} | P_{r_i} \geq 0, i = 1, \ldots, N\}$ is convex.

### 3.2 Analysis of the Seller-Level Game for the Relay Nodes

Substituting (25) into (15), we have

$$\max_{\{p_i\}>0} U_{r_i} = (p_i - c_i) P^*_{r_i}(p_1, \ldots, p_i, \ldots, p_{N'}). \tag{26}$$

We can note that (26) is a noncooperative game by the relay nodes, and there exists a trade-off between the price $p_i$ and the relay node's utility $U_{r_i}$. If relay node $r_i$ in good channel conditions asks for a relatively low price $p_i$ at first, source node $s$ will buy more power from relay node $r_i$, and $U_{r_i}$ will increase as $p_i$ grows. When $p_i$ keeps growing and exceeds a certain value, it is no longer beneficial for source $s$ to buy power from relay $r_i$, even though relay $r_i$ may be in very good channel conditions. In this way, $P_{r_i}$ will shrink and hence results in a decrement of $U_{r_i}$. Therefore, there is an optimal price for each relay node to ask for, depending on the relay node's channel conditions. Besides, the optimal price is also affected by the other relay nodes' prices since the source node only chooses the most beneficial relay nodes.

From the analysis above, by taking the derivative of $U_{r_i}$ to $p_i$ and equating it to zero, we have

$$\frac{\partial U_{r_i}}{\partial p_i} = P^*_{r_i} + (p_i - c_i)\frac{\partial P^*_{r_i}}{\partial p_i} = 0, \quad r_i \in L_h. \tag{27}$$

Solving the above equations of $p_i$, we denote the optimal prices as

$$p^*_i = p^*_i\left(\sigma^2, \{G_{s,r_i}\}, \{G_{r_i,d}\}\right), \quad r_i \in L_h. \tag{28}$$

In Section 3.1, we assume that the source node transmits with a constant power. However, if the source node has a lower transmission power, it is willing to buy more power from the relay nodes in order to obtain a high data rate, and hence, the relay nodes can ask higher prices for helping the source node. On the other hand, if the source has a higher transmission power, it will buy less power from the relay nodes and also pay less to them.

### 3.3 Existence of the Equilibrium for the Proposed Game

In this section, we prove that the solutions $P^*_{r_i}$ in (25) and $p^*_i$ in (28) are the Stackelberg Equilibrium (SE) for the proposed game and show the conditions for the SE to be optimal by the following properties, proposition, and theorem.

We first define the SE of the proposed game as follows:

**Definition 1.** *$P^{SE}_{r_i}$ and $p^{SE}_i$ are the SE of the proposed game if for every $r_i \in L$, when $p_i$ is fixed*

$$U_s\left(\left\{P^{SE}_{r_i}\right\}\right) = \sup_{\{P_{r_i}\}\geq \mathbf{0}} U_s(\{P_{r_i}\}), \forall r_i \in L, \tag{29}$$

*and for every $r_i \in L_h$, when $P_{r_i}$ is fixed*

$$U_{r_i}\left(p^{SE}_i\right) = \sup_{p_i > c_i} U_{r_i}(p_i), \quad \forall r_i \in L_h. \tag{30}$$

Then, we show that the optimizer $P^*_{r_i}$ of (13) can be solved by equating $\frac{\partial U_s}{\partial P_{r_i}}$ to zero by the following property.

**Property 1.** *The utility function $U_s$ of the source node is jointly concave in $\{P_{r_i}\}_{i=1}^N$, with $P_{r_i} \geq 0$, and $p_i$ is fixed, $\forall i$.*

**Proof.** See Appendix A.1. □

Due to *Property 1*, $P^*_{r_i}$ in (25) is the global optimum that maximizes the source node's utility $U_s$. Therefore, $P^*_{r_i}$ satisfies (29) and is the SE $P^{SE}_{r_i}$. Moreover, in the practical implementation of the game, the source node can find the optimal power amount by gradually increasing the purchased power from each relay node until $U_s$ reaches its maximum without knowing CSI.

In the following two properties, we show that the relay nodes cannot infinitely increase $U_{r_i}$ by asking arbitrarily high prices:

**Property 2.** *The optimal power consumption $P^*_{r_i}$ for relay node $r_i$ is decreasing with its price $p_i$ when other relay nodes' prices are fixed.*

**Proof.** See Appendix A.2. □

Consequently, there is a trade-off for each relay node to ask a proper price, and we can solve the optimal price by equating $\frac{\partial U_{r_i}}{\partial p_i} = 0$, the reason of which is shown as follows:

**Property 3.** *The utility function $U_{r_i}$ of each relay node is concave in its own price $p_i$ when its power consumption is the optimized purchase amount from the source node as calculated in (25) and the other relay nodes' prices are fixed.*

**Proof.** See Appendix A.3. □

Based on *Properties 1, 2, and 3*, we can show that the *relay rejection criteria* stated in Section 3.1.1 help the source node reject the least beneficial relay nodes in the following proposition:

**Proposition 1.** *The* relay rejection criteria *described in Section 3.1.1 are* necessary *and* sufficient *to exclude the least beneficial relay nodes to the source node. By* necessary, *it means that any $r_i$ in $L_h$ cannot get further discarded in the following $U_{r_i}$ maximization process; While by* sufficient, *it means that even if we keep $r_j$ that satisfies the rejection criteria in $L_h$, it is still discarded in the following $U_{r_j}$ maximization process.*

**Proof.** We first prove the *sufficient* part. Assume that the relay rejection criteria apply to some relay node $r_j$, i.e., $(\frac{\partial U_s}{\partial P_{r_j}}) < 0$, when $P_{r_i} = 0$ and $p_i = c_i$, $\forall i$. Since $U_s$ is concave in $\{P_{r_i}\}_{i=1}^N$, $r_j$'s optimal power allocation $P^*_{r_j} < 0$. Suppose source $s$ does not exclude relay $r_j$ and in the following price update process, all remaining relay nodes gradually increase their prices to get more utilities. To prove that the new resulting $P^{*new}_{r_j} < 0$, it suffices to prove that $\Delta P^*_{r_j} < 0$, where $\Delta P^*_{r_j}$ denotes the increase of $P^*_{r_j}$ when each relay node $r_i$ increases $p_i$ by a very small positive amount from the cost $c_i$. This can be verified equivalent by proving

$$\sum_{i\neq j} \frac{\partial P^*_{r_j}}{\partial p_i} + \frac{\partial P^*_{r_j}}{\partial p_j}\bigg|_{\{p_i=c_i,\forall i\}} < 0. \quad (31)$$

We know that

$$\frac{\partial P^*_{r_j}}{\partial p_i} = \sqrt{\frac{A_j B_j}{p_j}} \frac{Y + \sqrt{Y^2 + 4XW'}}{2X} \left( \frac{1}{2p_i} \frac{\sqrt{p_i A_i B_i}}{\sqrt{Y^2 + 4XW'}} \right) > 0 \quad (32)$$

and

$$\frac{\partial P^*_{r_j}}{\partial p_j} = \sqrt{\frac{A_j B_j}{p_j}} \frac{Y + \sqrt{Y^2 + 4XW'}}{2X} \times \left(-\frac{1}{2p_j}\right) \left(1 - \frac{\sqrt{p_j A_j B_j}}{\sqrt{Y^2 + 4XW'}}\right) < 0, \quad (33)$$

so it suffices to prove (31) by proving the following:

$$\sum_{i\neq j} \left( \frac{1}{2p_i} \frac{\sqrt{p_i A_i B_i}}{\sqrt{Y^2 + 4XW'}} \right) < \left(\frac{1}{2p_j}\right) \times \left(1 - \frac{\sqrt{p_j A_j B_j}}{\sqrt{Y^2 + 4XW'}}\right)\bigg|_{\{p_i=c_i,\forall i\}}. \quad (34)$$

Without loss of generality, assuming that the selected relay nodes generally share similar properties, i.e., $c_i = c_j = c$, $\forall i \neq j$, we can prove (34) by the following inequality:

$$\sum_{i\neq j} \frac{1}{2p_i} \frac{\sqrt{p_i A_i B_i}}{\sqrt{Y^2 + 4XW'}} < \frac{1}{2c} \sum_{i\neq j} \frac{\sqrt{p_i A_i B_i}}{Y} = \frac{1}{2p_j}\left(1 - \frac{\sqrt{p_j A_j B_j}}{Y}\right) < \frac{1}{2p_j}\left(1 - \frac{\sqrt{p_j A_j B_j}}{\sqrt{Y^2 + 4XW'}}\right). \quad (35)$$

Therefore, in the following price increasing process, $r_j$ is still discarded by the source node by observing $P^{*new}_{r_j} < 0$.

Next, we prove the *necessary* part. In each round, any two relay nodes $r_k$ and $r_i$ update their prices in two consecutive steps. First, $r_k$ increases its price $p^*_k$ to the new optimal $p^{*new}_k$, and then, by (32), the resulting $P^{*new}_{r_i}$ is larger than $P^*_{r_i}$, where $P^*_{r_i} > 0$. Thus, $P^{*new}_{r_i} > 0$, which means that $r_i$ will not be discarded if $r_k$ increases $p_k$. Second, after $p_k$ is increased, $r_i$ increases its own price $p_i$. In (54), assuming that $\bar{p}_i$ is the price for $r_i$ such that $P^*_{r_i} = 0$ when the other relay nodes' prices are fixed, we have

$$\frac{\partial U_{r_i}}{\partial p_i}\bigg|_{p_i \to \bar{p}_i} < -B_i + \sqrt{\frac{A_i B_i}{\bar{p}_i}} \frac{Y + \sqrt{Y^2 + 4XW'}}{2X} \to 0. \quad (36)$$

By *Property 3*, the optimal price $p^*_i$ such that $\frac{\partial U_{r_i}}{\partial p_i} = 0$ must satisfy $c_i < p^*_i < \bar{p}_i$. This means that to maximize $U_{r_i}$, $r_i$ asks a lower price than $\bar{p}_i$ to avoid being rejected by the source node. □

If relay node $r_i$ gets selected by the source node, due to the concavity of $U_{r_i}$ proved in *Property 3*, $r_i$ can always find its optimal price $p^*_i \in (c_i, \infty)$, and thus, $U_{r_i}(p^*_i) \geq U_{r_i}(p_i)$, $\forall r_i \in L_h$. Together with *Property 1*, we conclude the following theorem:

**Theorem 1.** *The pair of $\{P^*_{r_i}\}_{i=1}^N$ in (25) and $\{p^*_i\}_{i=1}^{N'}$ in (28) is the SE for the proposed game, where the SE is defined in (29) and (30).*

In the next section, we will show that the SE is unique, and the proposed game converges to the unique SE when each relay node updates its price according to a simple function.

### 3.4 Convergence of the Distributed Price Updating Function

From the previous section, one relay node needs to modify its own price after the other relay nodes change their prices. Consequently, for every $r_i \in L_h$, relay node $r_i$ updates $p_i$ so that its utility $U_{r_i}$ satisfies the following equality:

$$\frac{\partial U_{r_i}}{\partial p_i} = \frac{\partial}{\partial p_i}\left[(p_i - c_i)P^*_{r_i}\right] = P^*_{r_i} + (p_i - c_i)\frac{\partial P^*_{r_i}}{\partial p_i} = 0, \quad (37)$$

with the equality holding if and only if $p_i$ reaches the optimum.

After rearranging (37), we have

$$p_i = I_i(\boldsymbol{p}) \triangleq c_i - \frac{P^*_{r_i}}{\partial P^*_{r_i}/\partial p_i}. \quad (38)$$

In order to calculate $p_i$ in (38), each relay node $r_i$ listens to the instantaneous feedback information about $P^*_{r_i}$ and $\partial P^*_{r_i}/\partial p_i$ from the source node, which is similar to the needed information exchange in iterative power control [28]. Then, the updating of the relay nodes' prices can be described by a vector equality of the form

$$\boldsymbol{p} = \boldsymbol{I}(\boldsymbol{p}), \quad (39)$$

where $\boldsymbol{p} = (p_1, \dots, p_{N'})$, with $p_i$ denoting relay node $r_i$'s price, and $\boldsymbol{I}(\boldsymbol{p}) = (I_1(\boldsymbol{p}), \dots, I_{N'}(\boldsymbol{p}))$, with $I_i(\boldsymbol{p})$ representing the price competition constraint to $r_i$ from the other relay nodes. Therefore, for the $N'$ relay nodes in set $L_h$ with the competition constraints in (39), the iterations of the price updating can be expressed as follows:

$$\boldsymbol{p}(t+1) = \boldsymbol{I}(\boldsymbol{p}(t)). \quad (40)$$

**Remark.** If $K$ source nodes, denoted by $\mathcal{S} = \{s_1, s_2, \dots, s_K\}$, exist in the network, assuming that the price of relay node $r_i$ when it helps source node $s_k$ is $p_i^{s_k}$ with corresponding power $P_{r_i}^{s_k}$, then the buyer-level game for each source node $s_k$ is essentially the same as the single-buyer case. However, the seller-level game becomes more complicated, because now, relay node $r_i$ needs to choose $K$ prices, $\{p_i^{s_k}\}_{s_k \in \mathcal{S}}$, in order to maximize its utility:

$$U_{r_i} = \sum_{s_k \in \mathcal{S}} \left(p_i^{s_k} - c_i\right) P_{r_i}^{s_k}. \quad (41)$$

If the relay nodes treat all source nodes equally with $p_i^{s_k} = p_i, \forall s_k \in \mathcal{S}$, i.e., relay node $r_i$ asks a uniform price $p_i$ no matter which source node it helps, then utility $U_{r_i}$ is simplified as

$$U_{r_i} = (p_i - c_i) \sum_{s_k \in \mathcal{S}} P_{r_i}^{s_k}, \quad (42)$$

and the proposed algorithm is still applicable, with the following modified price updating function:

$$p_i = I_i(\boldsymbol{p}) \triangleq c_i - \frac{\sum_{s_k \in \mathcal{S}} P_{r_i}^{s_k^*}}{\sum_{s_k \in \mathcal{S}} \partial P_{r_i}^{s_k^*}/\partial p_i}. \quad (43)$$

However, if the relay nodes treat the source nodes differently, then each relay node $r_i$ needs to update $K$ prices, $\{p_i^{s_k}\}_{s_k \in \mathcal{S}}$, using the following updating function:

$$p_i^{s_k} = I_i(\boldsymbol{p}^{s_k}) \triangleq c_i - \frac{P_{r_i}^{s_k^*}}{\partial P_{r_i}^{s_k^*}/\partial p_i^k}. \quad (44)$$

Therefore, if there are multiple source nodes in the network, the proposed algorithm is still applicable: the buyer-level game of each source node is essentially the same as the single-source case; the only change is in the seller-level game of the relay nodes, where the price updating function is modified as in (43) or (44).

We show next the convergence of the iterations in (40) by proving that the price updating function $\boldsymbol{I}(\boldsymbol{p})$ is a *standard function* [28].

**Definition 2.** *A function* $\boldsymbol{I}(\boldsymbol{p})$ *is* standard *if for all* $\boldsymbol{p} \geq 0$, *the following properties are satisfied [28]:*

- Positivity. $\boldsymbol{I}(\boldsymbol{p}) > 0$.
- Monotonicity. *If* $\boldsymbol{p} \geq \boldsymbol{p}'$, *then* $\boldsymbol{I}(\boldsymbol{p}) \geq \boldsymbol{I}(\boldsymbol{p}')$.
- Scalability. *For all* $\alpha > 1$, $\alpha \boldsymbol{I}(\boldsymbol{p}) > \boldsymbol{I}(\alpha \boldsymbol{p})$.

**Proposition 2.** *The price updating function* $\boldsymbol{I}(\boldsymbol{p})$ *is* standard.

**Proof.** See Appendix A.4. □

In [28], a proof has been given that starting from any feasible initial power vector $\boldsymbol{p}$, the power vector $\boldsymbol{I}^n(\boldsymbol{p})$ produced after $n$ iterations of the standard power control algorithm gradually converges to a unique fixed point. As we have discussed in Section 3.1.1, it is natural for the relay nodes to initialize the prices as $p_i = c_i$, because lowering $p_i$ below $c_i$ will result in a negative utility $U_{r_i}$, while by setting $p_i$ above $c_i$, relay node $r_i$ may be at the risk of being excluded by the source node at the very beginning. Therefore, we assume that the initial price vector is $\boldsymbol{c} = (c_1, \dots, c_{N'})$, where $c_i$ is the cost per unit of power for relay node $r_i$, as introduced in (14). Therefore, we can conclude that starting from the feasible initial price vector $\boldsymbol{c} = (c_1, \dots, c_{N'})$, the iteration of the *standard* price updating produces a nondecreasing sequence of price vectors $\boldsymbol{I}^n(\boldsymbol{c})$ that converges to a unique fixed point $\boldsymbol{p}^*$.

From (37), we know that for relay node $r_i \in L_h$, its utility $U_{r_i}$ satisfies $\frac{\partial U_{r_i}}{\partial p_i} = 0$ every time after $r_i$ updates its price $p_i$ given the feedback of $\frac{\partial P^*_{r_i}}{\partial p_i}$ from the source. After the vector $\boldsymbol{I}^n(\boldsymbol{p})$ converges to $\boldsymbol{p}^*$, no relay can gain a higher utility by further varying its price, meaning that $\frac{\partial U_{r_i}}{\partial p_i} = 0 \; \forall r_i \in L_h$. From (27) and (28), we know that $\boldsymbol{p}^*$ is exactly the optimal price vector. As *Property 1* shows, $U_s$ is concave in $P_{r_i}$, so the source node can gradually increase the power from 0 and find the optimal $P^*_{r_i}$. Thus, if the prices of all the selected relay nodes converge to their optima, then the source node will correspondingly buy the optimal power. Therefore, once $\boldsymbol{I}^n(\boldsymbol{p})$ converges to $\boldsymbol{p}^*$, $P_{r_i}$ and $p_i$ converge to the SE. It is worth mentioning that although the closed-form solutions $\{P^*_{r_i}\}_{i=1}^N$ in (25) and $\{p_i^*\}_{i=1}^{N'}$ in (28) are functions of the CSI, in the practical implementation of the game, the source node can find the optimal power amount by gradually increasing the purchased power from each relay node until $U_s$ reaches its maximum due to *Property 1*. Actually, the reason why we express the closed-form solution $\{P^*_{r_i}\}_{i=1}^N$ as a function of CSI is just to show that the relay node's utility $U_{r_i}$ is concave in $p_i$ (*Property 3*) and hence to prove that the relay nodes can utilize the proposed price updating algorithm and gradually converge to the optimal price $\{p_i^*\}_{i=1}^{N'}$ (*Proposition 2*). Hence, the only signalings between an individual relay node and the source node are the instant price and corresponding power, and no CSI is needed. Moreover, there is no price information exchange between the relay nodes. Therefore, the proposed game achieves its equilibrium in a distributed way with local information.

### 3.5 Comparison with the Centralized Optimal Scheme

In order to demonstrate the performance of our proposed game-theoretical scheme, we first investigate a centralized optimal power allocation problem with closed-form solutions. Then, we illustrate the numerical comparison of the performance in Section 4.

Suppose the system resources are shared by all available $N$ relay nodes. From [11], we can model the centralized optimal power allocation problem as follows:

$$\begin{aligned} \max_{P_{r_i}} \quad & \frac{W}{N+1}\log_2\left(1+\frac{\Gamma_{s,d}+\sum_{i=1}^{N}\Gamma_{s,r_i,d}}{\Gamma}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{N} P_{r_i} \leq P_r^{tot},\ 0 \leq P_{r_i} \leq P_{r_i}^{max} \quad \forall i, \end{aligned} \tag{45}$$

where $\Gamma_{s,d}$ and $\Gamma_{s,r_i,d}$ are defined in (3) and (8), respectively.

Because $\log_2(1+x)$ is a strictly increasing function of $x$, reorganizing the objective function of (45), we can get an equivalent optimization problem as in [11]

$$\begin{aligned} \min \quad & \sum_{i=1}^{N}\frac{P_s^2 a_i^2 + P_s a_i}{P_s a_i + P_{r_i} b_i + 1} \\ \text{s.t.} \quad & \sum_{i=1}^{N} P_{r_i} \leq P_r^{tot},\ 0 \leq P_{r_i} \leq P_{r_i}^{max} \quad \forall i, \end{aligned} \tag{46}$$

where $a_i = \frac{G_{s,r_i}}{\sigma^2}$, and $b_i = \frac{G_{r_i,d}}{\sigma^2}$.

The solution of (46) can be solved as

$$P_{r_i} = \left(\sqrt{\frac{P_s^2 a_i^2 + P_s a_i}{b_i}\lambda} - \frac{P_s a_i + 1}{b_i}\right)_0^{P_{r_i}^{max}}, \tag{47}$$

where $\lambda$ is a constant chosen to meet the total power constraint, and $(x)_l^u$ is defined as

$$(x)_l^u = \begin{cases} l, & x < l, \\ x, & l \leq x \leq u, \\ u, & u < x. \end{cases} \tag{48}$$

In order to make a fair comparison, in the proposed game-theoretical scheme, we can change $a$, the gain per unit of the rate, to equivalently reflect different $P_r^{tot}$ constraints as in the centralized scheme. The reason is explained as follows: When $a$ is so large that the total payment $M$ in $U_s$ is negligible, $U_s \approx aR_{s,r,d}$, then the optimal power consumption of the problem in (13) will be $P_{r_i}^* \to \infty$. It is equivalent to have $P_r^{tot} \to \infty$ in the centralized scheme. On the contrary, when $a$ is so small that the total gain of the rate $aR_{s,r,d}$ in $U_s$ is negligible, $U_s \approx -M = -\sum_i p_i P_{r_i}$, then in this case, we get $P_{r_i}^* = 0$. It is equivalent to have $P_r^{tot} = 0$ in the centralized scheme. Therefore, by varying $a$ in a large range, we can get the optimal achievable rates corresponding to different total power consumptions and fairly compare the performance with that of the centralized scheme.[2] For more detailed discussions, please see Appendix A.5.

2. We do not include explicitly the constraints on the relay nodes' power in the proposed game for ease of analysis. From the simulation in the next section and the analytical proof in Appendix A.5, it will be shown that the game will achieve comparable performance when we consider the constraints on relay nodes' power.

However, the centralized optimal power allocation scheme needs considerable overheads and signaling, because it requires that the complete CSI, i.e., $G_{s,d}$, $G_{s,r_i}$, and $G_{r_i,d}$, is available. In Section 4, we show that our proposed distributed scheme can achieve comparable performance while the needed signaling between the source node and the relay nodes is only the information about the prices and the power consumptions.

## 4 Simulation Results and Analysis

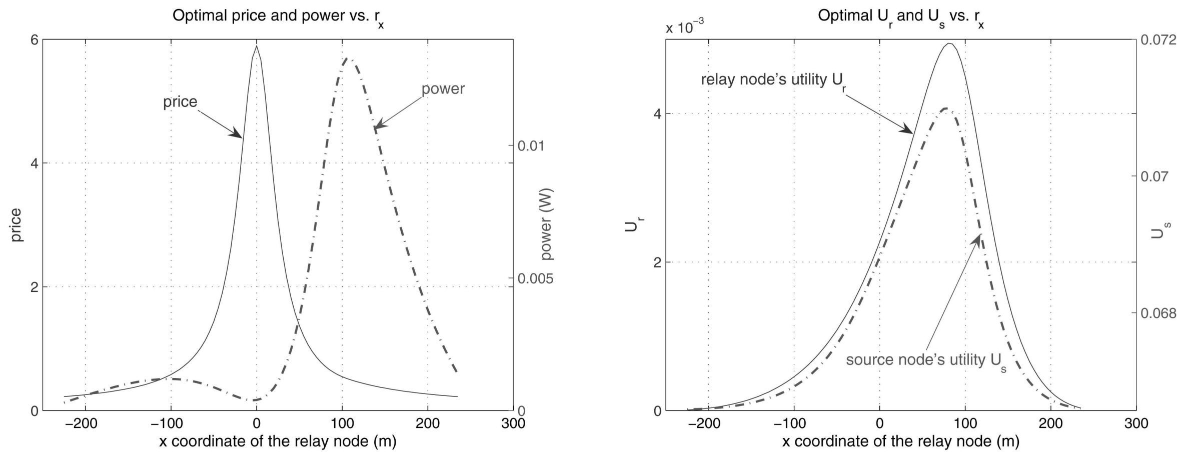
To evaluate the performance of the proposed scheme, in what follows, the simulation results for a one-relay case, for a two-relay case, and for a multiple-relay case are to be shown. Then, we provide the performance comparisons of the proposed approach with the centralized optimal scheme. Finally, we discuss the effect of the bandwidth factor.

### 4.1 One-Relay Case

There are one source-destination node pair $(s, d)$ and one relay node $r$ in the network. Destination node $d$ is located at coordinate (0 m, 0 m), and source node $s$ is located at coordinate (100 m, 0 m). We fix the $y$-coordinate of relay node $r$ at 25 m, and its $x$-coordinate varies within the range of [−250 m, 300 m]. The propagation loss factor is set to two. The transmit power $P_s = 10$ mW, the noise level is $\sigma^2 = 10^{-8}$ W, and we select the capacity gap $\Gamma = 1$, $W = 1$ MHz, the gain per unit of rate $a = 0.01$, and the cost per unit of power $c = 0.2$.

In Fig. 2a, we show the optimal price for relay node $r$ and the optimal power bought by source node $s$, respectively. In this simulation, relay node $r$ moves along a line. We observe that when relay node $r$ is close to source node $s$ at (100 m, 0 m), the source can gain a higher $U_s$ in the game, so the relay can more efficiently help source node $s$. However, the relay cannot arbitrarily select its price in order to improve its utility. As we have shown in *Property 2* and *Property 3*, the optimal power $P_r^*$ the source buys from relay node $r$ is decreasing with $p$, and node $r_i$'s utility $U_{r_i}$ is concave in $p$. Since the objective of the relay node is to maximize its utility $U_r$, the price $p$ should be carefully selected instead of an arbitrarily large value. As decreasing price $p$ can attract more buying from the source, relay node $r$ reduces its price to enhance its utility $U_r$. When relay node $r$ moves close to destination node $d$ at (0 m, 0 m), relay node $r$ can use a very small amount of power to relay source node $s$'s data, so relay node $r$ sets a very high price in order to get more profits by selling this small amount of power. However, even the price is higher than that when $r$ is closer to the source, the utility $U_r$ is still lower when the relay is close to the destination. When relay node $r$ keeps moving away from destination node $d$, source node $s$ stops buying services because asking for relay node $r$'s help is no longer beneficial to source node $s$. Similarly, when relay node $r$ moves in the opposite direction and locates far away from source node $s$, $s$ would not buy services either.

In Fig. 2b, we show, respectively, the optimal utilities relay node $r$ and source node $s$ can obtain using the proposed game. When relay node $r$ is close to source node $s$, both $r$ and $s$ can get their maximal utilities. The reason is




(a) (b)

Fig. 2. One-relay case with the relay node at different locations. (a) Optimal price and power of the relay node. (b) Optimal utility of the relay and the source node.

that around this location, relay node $r$ can most efficiently help source node $s$ increase its utility, and the optimal price of relay node $r$ is lower than that when $r$ is at other locations. Therefore, source node $s$ buys more power, resulting in a higher utility to relay node $r$.

### 4.2 Two-Relay Case

We also set up two-relay simulations to test the proposed game. In the simulations, the coordinates of $s$ and $d$ are (100 m, 0 m) and (0 m, 0 m), respectively. Relay node $r_1$ is fixed at the coordinate (50 m, 25 m), and relay node $r_2$ moves along the line from ($-250$ m, 25 m) to (300 m, 25 m). For each $r_i$, we set $c_i = 0.1$. Other settings are the same as those of the one-relay case.

In Fig. 3, we can observe that even though only $r_2$ moves, the prices of both the relay nodes change accordingly, and $s$ buys different amounts of power from them. This fact is because the relay nodes influence and compete with each other in the proposed game. When relay node $r_2$ is close to $d$ at (0 m, 0 m), it sets a very high price as explained in the one-relay case. Accordingly, $r_1$ increases its price and $P^*_{r_1}$ slightly decreases. When $r_2$ is close to $s$ at (100 m, 0 m), $r_2$ is more suitable to help $s$ than $r_1$, and $U^*_{r_2}$ is very high. Hence, in order to attract source $s$ to buy its service, $r_1$ reduces its price a lot, but $U^*_{r_1}$ still drops. Because $r_2$ close to $s$ results in the most efficient help to $s$ from the relay nodes, both $U_s$ and $M$ reach their maxima around this location. As $r_2$ moves far away from $s$ or $d$, $r_2$'s price drops because $r_2$ is less competitive than $r_1$. When its utility is less than 0, $r_2$ quits the competition, and $P^*_{r_2} = 0\ \mathrm{mW}$. At that moment, $r_1$ can slightly increase its price since there is no competition. However, source node $s$ will buy slightly less power from $r_1$. This fact suppresses the incentive of $r_1$ to ask an arbitrarily high price in the absence of competition; otherwise, $r_2$ will rejoin the competition. At the transition point when $r_2$ quits, $U_{r_1}$ is smooth. Note that when $r_2$ moves to (50 m, 25 m), which is the same location as $r_1$, the power consumptions, the prices, and the utilities of both relay nodes are the same. This is because the source node is indifferent for the two relay nodes located together and treats them equally.

### 4.3 Multiple-Relay Case

We then set up multiple-relay simulations to test the proposed game. The coordinates of the source node and the destination node are (100 m, 0 m) and (0 m, 0 m), respectively, and the relay nodes are uniformly located within the range of [$-50$ m, 150 m] in the $x$-axis and [0 m, 20 m] in the $y$-axis. In Fig. 4, we can observe that as the total number of the available relay nodes increases, the competitions among the relay nodes become more severe, so the average price per relay node decreases. The source node increases the amount of average power purchase when the number of the relay nodes is not so large (less than three), because the average price is decreased. When the number of the relay nodes becomes larger (greater than three), the source node decreases the amount of average power purchase, because it buys power from more relay nodes. Correspondingly, the total payments are shared by more relay nodes, which leads to less average payment from the source node. Thus, the source node obtains an increasing utility.

### 4.4 Convergence Speed of the Proposed Game

As described in Section 3.4, the relay nodes start increasing their price $p_i$ from $c_i$ after the $N'$ more beneficial relay nodes have been selected by the source node. Denote the price vector at time $t$ as $\mathbf{p}(t) = (p_1(t), p_2(t), \dots, p_{N'}(t))$. From (25), the optimal power purchased by the source node at time $t$ can be denoted as

$$P^*_{r_i}(t) := P^*_{r_i}(\mathbf{p}(t)) = P^*_{r_i}(p_1(t), p_2(t), \dots, p_{N'}(t)). \quad (49)$$

In order to obtain $\partial P^*_{r_i}/\partial p_i$ and update their prices by (38), the selected relay nodes will simultaneously increase each $p_i(t)$ by a small amount $\delta_i$. The source node receives this price updating and calculates $\partial P^*_{r_i}/\partial p_i$ using the following approximation:

$$\frac{\partial P^*_{r_i}}{\partial p_i} \simeq \frac{P^*_{r_i}(p_1(t), \dots, p_i(t) + \delta_i, \dots, p_{N'}(t)) - P^*_{r_i}(\mathbf{p}(t))}{\delta_i}. \quad (50)$$

Substituting the above approximation signaled from the source node into (38), where the numerator $P^*_{r_i}(t)$ is as defined in (49), the relay nodes can obtain $\mathbf{p}(t+1) = \mathbf{I}(\mathbf{p}(t))$.

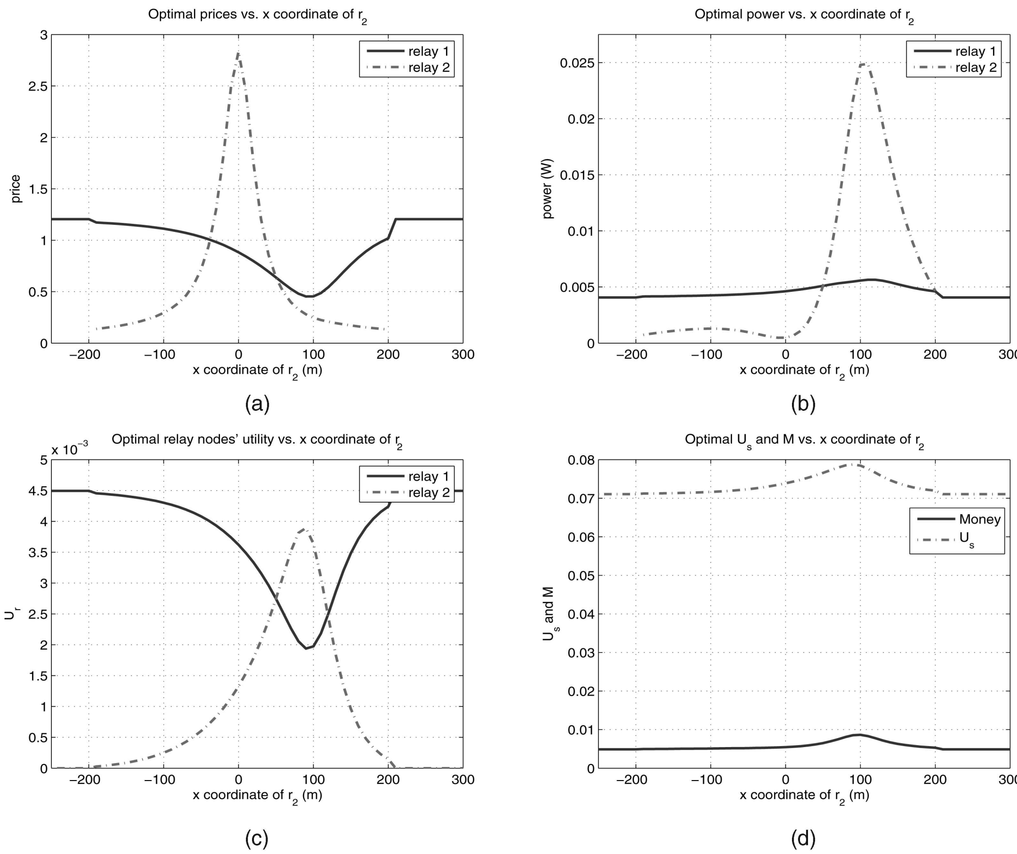



Fig. 3. Two-relay case with relay node $r_2$ at different locations. (a) Optimal prices of the relay nodes. (b) Optimal power consumptions of the relay nodes. (c) Optimal utilities of the relay nodes. (d) $U_s$ and $M$ of the source node.

In the above updating process, the source node can signal the approximated derivatives calculated by (50) to all the relay nodes at one time and need not interact with them one by one. Therefore, this process can be viewed as one iteration and does not depend on the number of relay nodes. Then, we conducted simulations when two to four relay nodes are available to help the source node and observe the convergence behavior of the proposed game. In Fig. 5a, it is seen that the proposed scheme has fast convergence to the SE $\mathbf{p}^*$. It takes less than 15 iterations until the price vector $\mathbf{p}$ converges to the optimum when there are two relay nodes in the system for $a = 1$, where $a$ denotes the gain per unit of rate as defined in (11), and less than 10 iterations for $a = 0.2$. In addition, in Fig. 5b, the convergence behavior of $R_{s,r,d}$ to the optimized transmission rate using $\mathbf{P}_{\mathbf{r}}^*$ and $\mathbf{p}^*$ appears to be exponentially fast. Finally, we keep $a = 1$, increase the number of relay nodes to three and four, and show the convergence behavior of

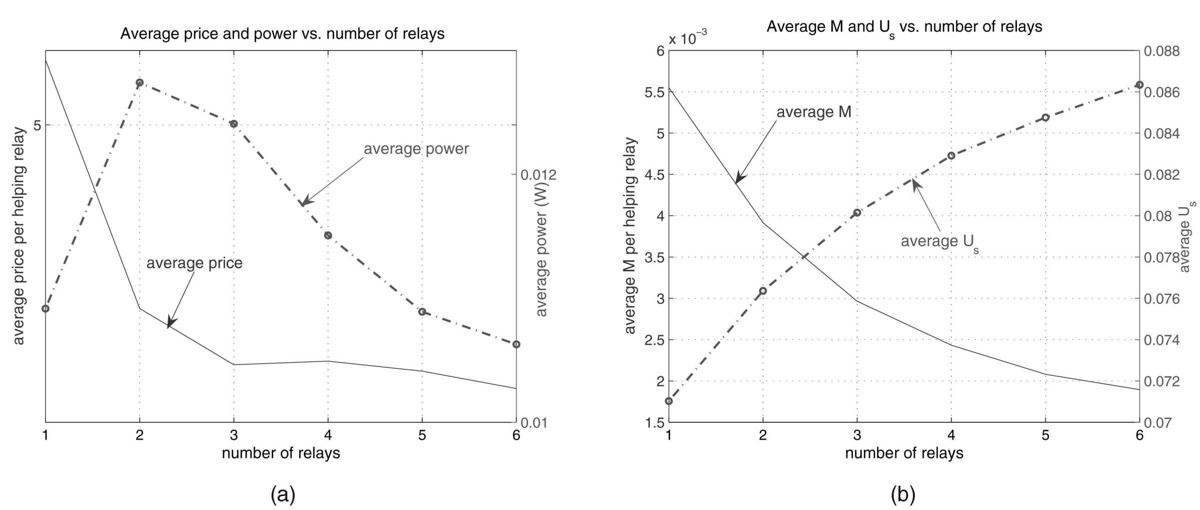



Fig. 4. Multiple-relay case with different numbers of relay nodes. (a) Average price and power versus number of relay nodes. (b) Average $U_s$ and $M$ versus number of relay nodes.

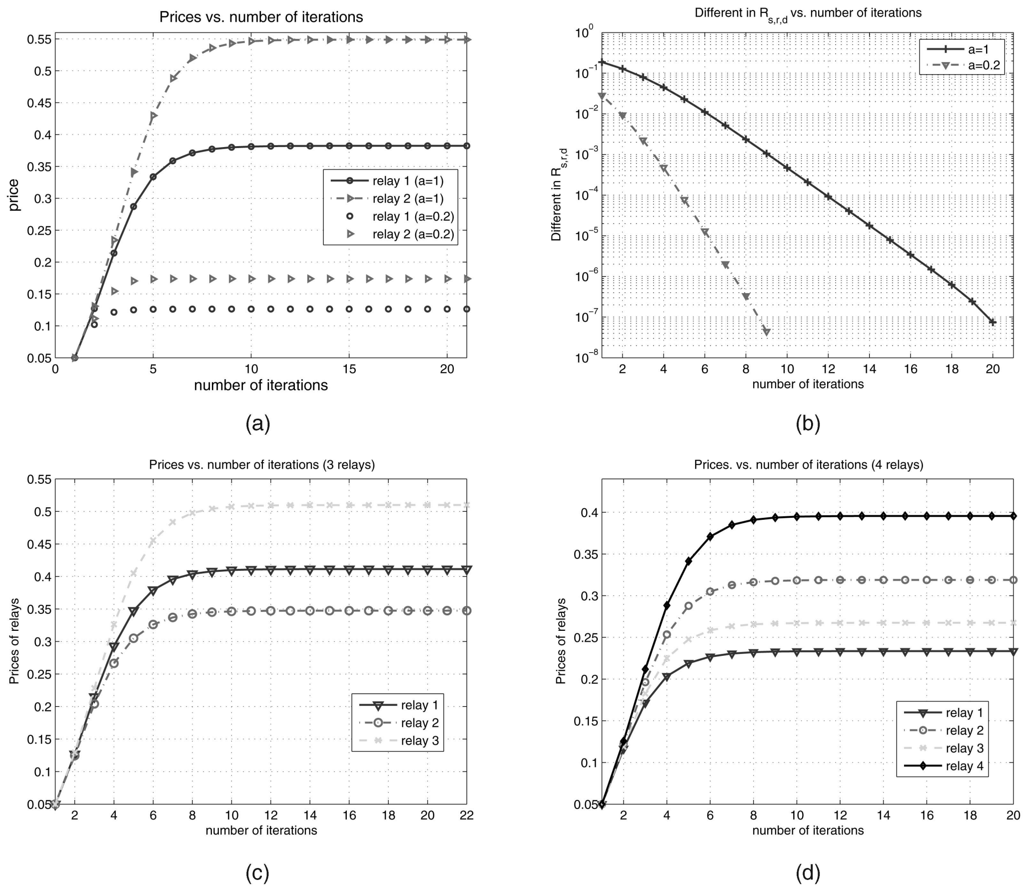



Fig. 5. Observation of convergence speed. (a) Prices of relay nodes versus iteration index. (b) Difference in $R_{s,r,d}$ versus iteration index. (c) Prices of relay nodes versus iteration index (three relays). (d) Prices of relay nodes versus iteration index (four relays).

the prices in Figs. 5c and 5d, respectively. We can see that the number of iterations until convergence happens almost keeps the same as there are more relay nodes existing in the system.

### 4.5 Comparison with the Centralized Optimal Scheme

To compare the performance of the proposed game with the centralized scheme, we set up two simulations as follows: There are two relay nodes and one $(s, d)$ pair. One of the relay nodes is fixed at coordinate (50 m, 25 m) and the other node is fixed at (60 m, 25 m) and (40 m, 25 m) in the two simulations, respectively. For the centralized scheme defined in (45), we set $P_{r_i}^{max} = 10\,\mathrm{mW}$ and let $P_r^{tot}$ vary within the range of [10, 20] mW. Then, we can obtain a curve of the maximal rates versus different total power consumption constraints. For the distributed scheme, as explained in Section 3.5, by varying $a$ and including the same constraint $P_{r_i}^{max} = 10\,\mathrm{mW}$ on $P_{r_i}$, we can also get different total power consumptions and corresponding maximal rates. In Figs. 6a and 6b, we observe that the proposed game achieves almost equal rates as the centralized scheme under the same total power consumptions.

### 4.6 Effect of the Bandwidth Factor

As explained in Section 2.1, for the network with a limited bandwidth, the bandwidth should be divided for the source node and the relay nodes. If $N'$ out of the total $N$ available relay nodes are selected by the source node, where $N' \leq N$, then $\gamma_L = \frac{1}{N'+1}$ in (10), indicating that the bandwidth factor decreases as more relay nodes help the source node. Thus, using fewer relay nodes among the selected $N'$ relay nodes may further increase $U_s$ for the source node. Therefore, for the networks with a limited bandwidth, it is not sufficient for the source node to implement only one round of relay selection. Instead, after source node $s$ selects $N'$ relay nodes using the *relay rejection criteria*, $s$ continues to try different subsets of the $N'$ selected relay nodes, get the corresponding optimal utility $U_s^*$ for each trial, and choose the subset of relay nodes that results in the largest $U_s^*$. In this section, we set up simulations to observe the effect of the varying bandwidth factor.

We set $a = 0.85$, relay node $r_1$ is at (100 m, 5 m), and $r_2$ moves along the line between points (−250 m, 5 m) and (300 m, 5 m). In Fig. 7, we show the optimal $U_s^*$ obtained by the source node under four scenarios, i.e., when no relay node, only $r_1$, only $r_2$, and both relay nodes are available to help, respectively. We see that when $r_2$ moves close to $r_1$ and the source node $s$, i.e., the $x$-coordinate of $r_2$ lies in the interval of (85 m, 115 m), both $r_2$ and $r_1$ are beneficial to node $s$. Moreover, as explained in the multiple-relay case in Section 4.3, since there is competition between two relay nodes, the average power bought from the relay node is much greater, while the average payment is lower, compared with the one-relay case. Hence, although $\gamma_L$ is only 1/3, $U_s(r_1, r_2)$ is still greater than $U_s$ ($r_i$ only), for

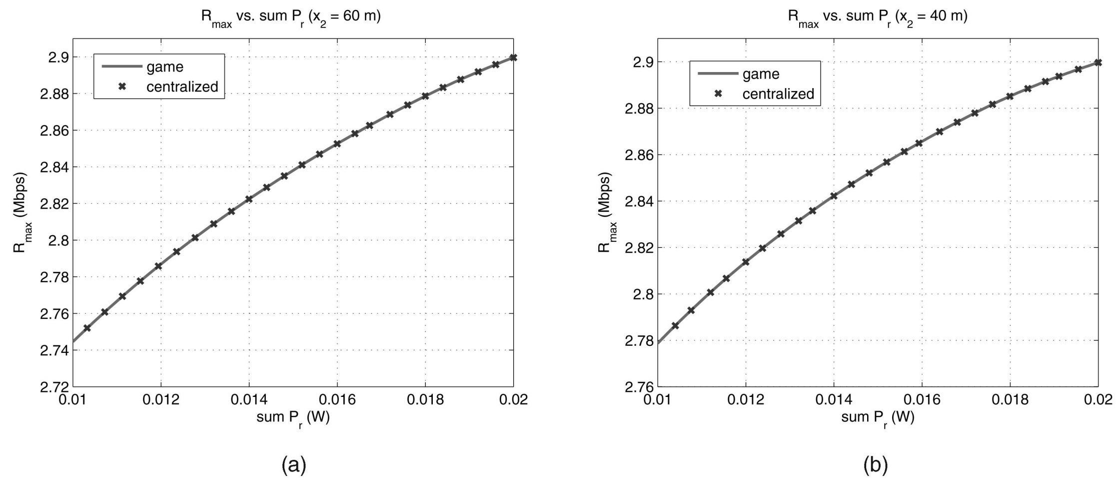



Fig. 6. Optimal rate in distributed and centralized schemes. (a) $x$-coordinate of $r_2 = 60\ \mathrm{m}$. (b) $x$-coordinate of $r_2 = 40\ \mathrm{m}$.

$i = 1, 2$, and both relay nodes are selected. When $r_2$ moves farther away from $s$, $r_2$ is less beneficial and asks a higher price, and $r_1$ is also influenced to ask a higher price. Therefore, $U_s(r_1, r_2)$ decreases and becomes smaller than $U_s$ ($r_1$ only) where $\gamma_L$ is 1/2. Thus, choosing $r_1$ only is better than choosing both relay nodes. When $r_2$ keeps moving away from $s$, it is no longer beneficial for the source node $s$ to select it to help. Hence, $r_2$ will be rejected, and the bandwidth factor jumps from 1/3 to 1/2. Therefore, there are two bumps of $U_s$ $(r_1, r_2)$ when the $x$-coordinate of $r_2$ is about $-70$ m and 140 m.

## 5 Conclusions

In this paper, we propose a game-theoretical approach for the distributed resource allocation over multiuser cooperative communication networks. We target to answer two questions: Who will be the relays, and how much power for the relays to transmit for the cooperative transmission? We employ a Stackelberg (buyer/seller) game to jointly consider the benefits of the source node and the relay nodes. The proposed scheme not only helps the source node optimally choose the relay nodes at better locations but also helps the competing relay nodes ask optimal prices to maximize their utilities. From the simulation results, relay nodes closer to the source node can play a more important role in increasing the source node's utility, so the source node buys more power from these preferred relay nodes. If the total number of the available relay nodes increases, the source node can obtain a larger utility value, and the average payment to the relay nodes shrinks, due to more severe competitions among the relay nodes. It is also shown that the distributed resource allocation can achieve a comparable performance to that of the centralized scheme, without requiring knowledge of CSI. The proposed Stackelberg-game-based framework can be extended as a building block in large-scale wireless ad hoc networks to stimulate cooperation among distributed nodes.

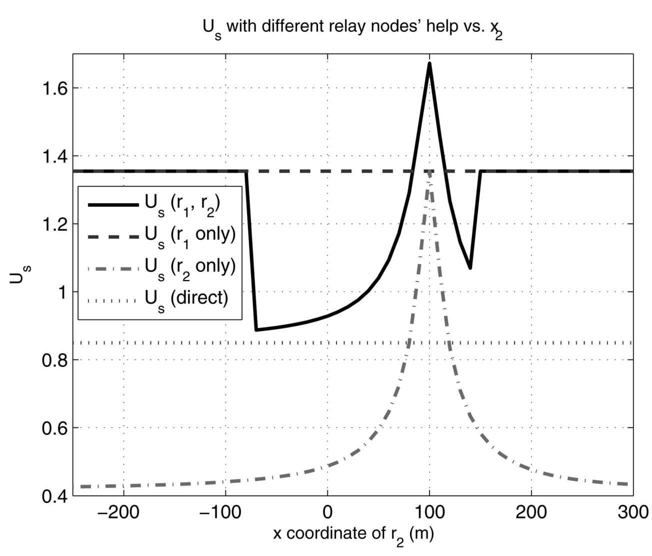



Fig. 7. Optimal $U_s$ including the bandwidth-factor effect, with different relay nodes' help, $a = 0.85$.

## Appendix A

### A.1 Proof of Property 1

Taking the second-order derivatives of the source node's utility $U_s$, we can get

$$\frac{\partial^2 U_s}{\partial P_{r_i}^2} = -\frac{W'}{\left(1+\sum_{k=1}^{N}\frac{A_k P_{r_k}}{P_{r_k}+B_k}\right)^2}\left[\frac{A_i B_i}{(P_{r_i}+B_i)^2}\right]^2 - 2\frac{W'}{\left(1+\sum_{k=1}^{N}\frac{A_k P_{r_k}}{P_{r_k}+B_k}\right)}\frac{A_i B_i}{(P_{r_i}+B_i)^3} \tag{51}$$

and

$$\frac{\partial^2 U_s}{\partial P_{r_i}\partial P_{r_j}} = -\frac{W'}{\left(1+\sum_{k=1}^{N}\frac{A_k P_{r_k}}{P_{r_k}+B_k}\right)^2} \times \frac{A_i B_i}{(P_{r_i}+B_i)^2}\frac{A_j B_j}{\left(P_{r_j}+B_j\right)^2}. \tag{52}$$

For each relay, by definition, $W'>0$, $A_i>0$, $B_i>0$, and $P_{r_i}\geq 0$. As a result, $\frac{\partial^2 U_s}{\partial P_{r_i}^2}<0$, and $\frac{\partial^2 U_s}{\partial P_{r_i}\partial P_{r_j}}<0$. It is straightforward to verify that $\frac{\partial^2 U_s}{\partial P_{r_i}^2}\frac{\partial^2 U_s}{\partial P_{r_j}^2}-(\frac{\partial^2 U_s}{\partial P_{r_i}\partial P_{r_j}})^2>0, \forall i\neq j$. Moreover, $U_s$ is continuous in $P_{r_i}$, so when $P_{r_i}\geq 0$, $U_s$ is strictly concave in each $P_{r_i}$, $\forall i$, and jointly concave over $\{P_{r_i}\}_{i=1}^N$ as well.

### A.2 Proof of Property 2

Taking the first-order derivative of the optimal power consumption $P_{r_i}^*$, we have

$$\frac{\partial P_{r_i}^*}{\partial p_i}=\sqrt{\frac{A_iB_i}{p_i}\frac{Y+\sqrt{Y^2+4XW'}}{2X}} \times\left[-\frac{1}{2p_i}\left(1-\frac{\sqrt{p_iA_iB_i}}{\sqrt{Y^2+4XW'}}\right)\right]<0. \tag{53}$$

Therefore, $P_{r_i}^*$ is decreasing with $p_i$. This is because when some relay node individually increases its price while the others keep the same prices as before, the source node will buy less from that relay node.

### A.3 Proof of Property 3

$P_{r_i}^*$ is a continuous function of $p_i$, so $U_{r_i}$ is continuous in $p_i$ too. Taking the derivatives of the relay node's utility $U_{r_i}$ results in

$$\frac{\partial U_{r_i}}{\partial p_i}=-B_i+\sqrt{\frac{A_iB_i}{p_i}\frac{Y+\sqrt{Y^2+4XW'}}{2X}} \times\left[1-\frac{p_i-c_i}{2p_i}\left(1-\frac{\sqrt{p_iA_iB_i}}{\sqrt{Y^2+4XW'}}\right)\right] \tag{54}$$

and, further,

$$\begin{aligned}\frac{\partial^2 U_{r_i}}{\partial p_i^2}=&\sqrt{\frac{A_iB_i}{p_i}}\frac{Y_i}{2X}\left(1-\frac{\sqrt{p_iA_iB_i}}{\sqrt{Y^2+4XW'}}\right)\\ &\times\left(\frac{-p_i-3c_i}{4p_i^2}\right)+\frac{\sqrt{\frac{A_iB_i}{p_i}}}{8Xp_i^2\left(\sqrt{Y^2+4XW'}\right)^3}\\ &\times\Big[\left(Y_i^2+2Y_i\sqrt{p_iA_iB_i}+4XW'\right)^2(-p_i-3c_i)\\ &+p_iA_iB_i\left(Y_i^2+2Y_i\sqrt{p_iA_iB_i}\right)(-p_i-3c_i)\\ &+p_iA_iB_i4XW'(-4c_i)\Big],\end{aligned} \tag{55}$$

where $Y_i=Y-\sqrt{p_iA_iB_i}$. Since $A_i$, $B_i$, $p_i$, $Y_i$, $c_i$, $X$, and $W'>0$, we have $\frac{\partial^2 U_{r_i}}{\partial p_i^2}<0$. Therefore, $U_{r_i}$ is concave with respect to $p_i$.

### A.4 Proof of Proposition 2

*Positivity.* By *Property 2*, $\frac{\partial P_{r_i}^*}{\partial p_i}<0$. Moreover, if $c_i>0$ and $P_{r_i}\geq 0$, then by the definition of (38), $I_i(\boldsymbol{p})\geq c_i>0$. Therefore, in a real price updating process, each relay node starts increasing its price from $c_i$.

*Scalability.* Comparing $\alpha\boldsymbol{I}(\boldsymbol{p})$ and $\boldsymbol{I}(\alpha\boldsymbol{p})$ in an element-wise manner, we have

$$\alpha I_i(\boldsymbol{p})-I_i(\alpha\boldsymbol{p})=(\alpha-1)c_i +\alpha\left[\frac{P_{r_i}(\alpha\boldsymbol{p})}{\partial P_{r_i}(\alpha\boldsymbol{p})/\partial p_i}-\frac{P_{r_i}(\boldsymbol{p})}{\partial P_{r_i}(\boldsymbol{p})/\partial p_i}\right]. \tag{56}$$

Since $\alpha>1$, $(\alpha-1)c_i>0$. Then, the problem reduces to proving that the second term in the RHS of (56) is positive.

If we define $F_i(W')$ as follows:

$$F_i(W')=\frac{P_{r_i}(\boldsymbol{p})}{\partial P_{r_i}(\boldsymbol{p})/\partial p_i}=\left(1-\frac{B_i}{\sqrt{\frac{A_iB_i}{p_i}\frac{Y+\sqrt{Y^2+4XW'}}{2X}}}\right) \times\left[-\frac{1}{2p_i}\left(1-\frac{\sqrt{p_iA_iB_i}}{\sqrt{Y^2+4XW'}}\right)\right]^{-1}. \tag{57}$$

Then, we can get

$$\begin{aligned}\frac{P_{r_i}(\alpha\boldsymbol{p})}{\partial P_{r_i}(\alpha\boldsymbol{p})/\partial p_i}=&\left(1-\frac{B_i}{\sqrt{\frac{A_iB_i}{p_i}\frac{Y+\sqrt{Y^2+4XW'/\alpha}}{2X}}}\right)\\ &\times\left[-\frac{1}{2p_i}\left(1-\frac{\sqrt{p_iA_iB_i}}{\sqrt{Y^2+4XW'/\alpha}}\right)\right]^{-1}\\ =&F_i(W'/\alpha).\end{aligned} \tag{58}$$

Therefore, to prove the positivity of the second term of the RHS of (56) is equivalent to prove $F_i(\frac{W'}{\alpha})>F_i(W')$, where $\frac{W'}{\alpha}<W'$. Since $F_i(W')$ is continuous and differentiable in $W'$, we only need to prove that $\frac{\partial F_i}{\partial W'}<0$. Expanding $\frac{\partial F_i}{\partial W'}$, we get

$$\begin{aligned}\frac{\partial F_i}{\partial W'}=&8Xp_i\times\left(\sqrt{\frac{A_iB_i}{p_i}}\sqrt{Y^2+4XW'}\right)^{-1}\\ &\times\left(\sqrt{Y^2+4XW'}-\sqrt{p_iA_iB_i}\right)^{-2}\\ &\times\left(Y+\sqrt{Y^2+4XW'}\right)^{-2}\\ &\times\Big[-XB_i\left(Y^2+4XW'+Y\sqrt{p_iA_iB_i}\right)\\ &+\frac{1}{2}A_iB_i\left(Y+\sqrt{Y^2+4XW'}\right)^2\Big].\end{aligned} \tag{59}$$

The first four terms of the RHS of (59) are all positive. After extensive numerical tests for a wide range of parameters when the nodes are randomly located, we observe that the last term in the square brackets is negative. Then, the $\frac{\partial F_i}{\partial W'}$ in (59) is less than zero. Therefore, we can claim that $\alpha I(\boldsymbol{p})>I(\alpha\boldsymbol{p})$.

*Monotonicity.* Suppose $\boldsymbol{p}$ and $\boldsymbol{p}'$ are different price vectors, and the vector inequality $\boldsymbol{p}\geq\boldsymbol{p}'$ means that $p_i\geq p_i'$, $\forall i\in\{1,\dots,N'\}$. If $\forall i\neq j$, $i$, $j\in\{1,\dots,N'\}$,

$$I_j([p_1,\dots,p_i,\dots,p_j,\dots,p_{N'}])\geq I_j([p_1,\dots,p_i',\dots,p_j,\dots,p_{N'}]),$$

and

$$I_i([p_1,\dots,p_i,\dots,p_j,\dots,p_{N'}])\geq I_i([p_1,\dots,p_i',\dots,p_j,\dots,p_{N'}]),$$

then monotonicity can be shown to hold. Therefore, the problem reduces to proving $\partial I_j(\boldsymbol{p})/\partial p_i\geq 0$ and $\partial I_i(\boldsymbol{p})/\partial p_i\geq 0$. Expanding and reorganizing $\partial I_j(\boldsymbol{p})/\partial p_i$ to

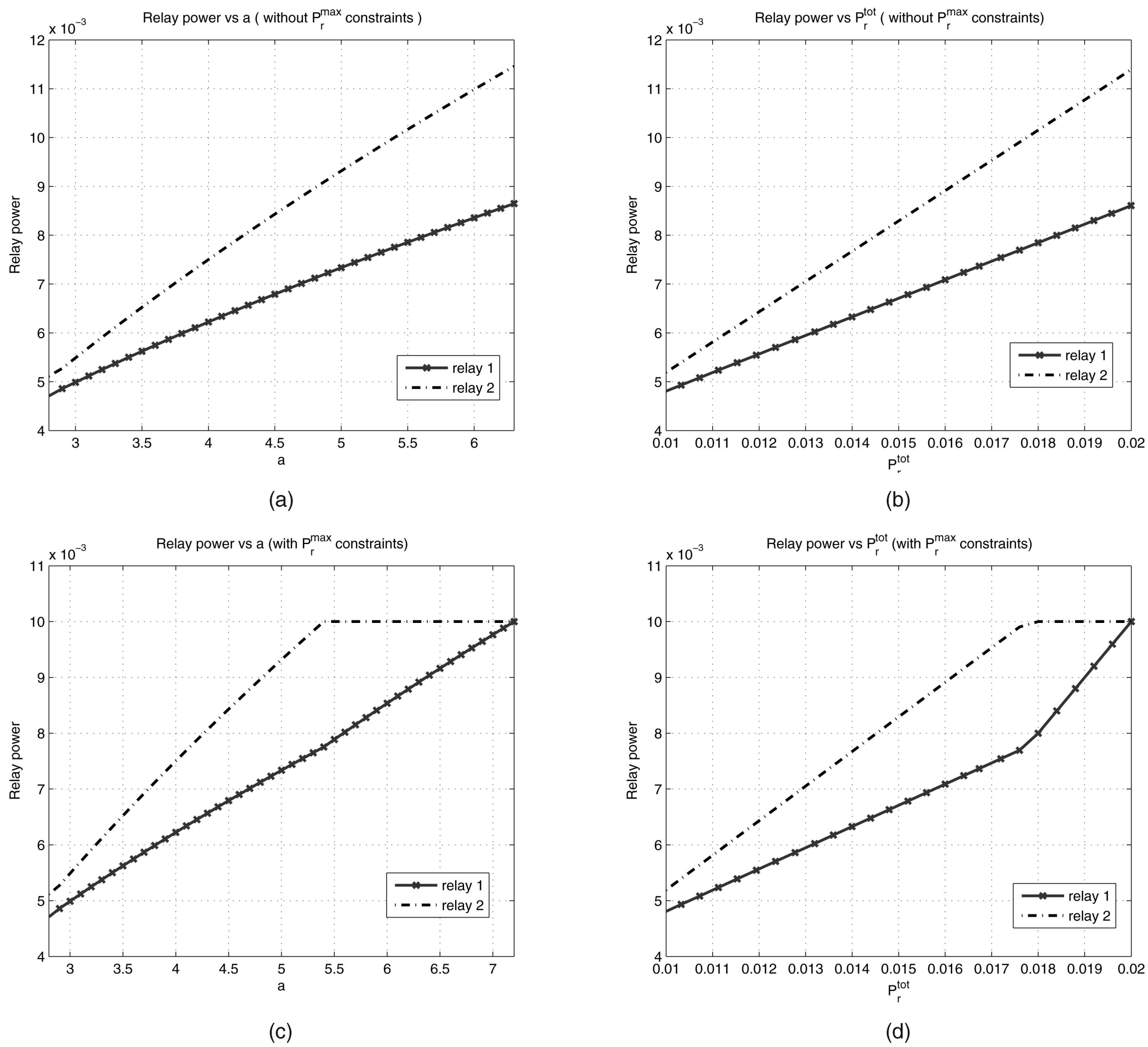



Fig. 8. Comparison of optimal relay power of the game and the centralized scheme. (a) Optimal power in game. (b) Optimal power in centralized scheme. (c) Optimal power in game ($P_{r_i} \leq P_{r_i}^{max}$). (d) Optimal power in centralized scheme ($P_{r_i} \leq P_{r_i}^{max}$).

express it as a product of a positive term and a second term, we get

$$\frac{\partial I_j(\boldsymbol{p})}{\partial p_i} = \frac{\frac{1}{p_i}\frac{\sqrt{p_iA_iB_i}}{\sqrt{Y^2+4XW'}}}{\frac{1}{p_j}\left(1-\frac{\sqrt{p_jA_jB_j}}{\sqrt{Y^2+4XW'}}\right)} \times \left[1-\frac{\left(\sqrt{\frac{A_jB_j}{p_j}}\frac{Y+\sqrt{Y^2+4XW'}}{2X}-B_j\right)}{\left(\sqrt{\frac{A_jB_j}{p_j}}\frac{Y+\sqrt{Y^2+4XW'}}{2X}\right)} \times \frac{\left(1-\frac{\sqrt{p_jA_jB_j}}{\sqrt{Y^2+4XW'}}+\frac{Y\sqrt{p_jA_jB_j}}{Y^2+4XW'}\right)}{\left(1-\frac{\sqrt{p_jA_jB_j}}{\sqrt{Y^2+4XW'}}\right)}\right]. \tag{60}$$

The first term of the RHS of (60) is positive; to decide the sign of the second term, it suffices to compare the difference of the denominator and numerator of the fraction inside the square brackets, which are both positive. By using $\sqrt{\frac{A_iB_i}{p_i}} < \frac{XB_i}{Y}$, proved in the scalability property, we can finally show that

$$\begin{aligned}
&\left(\sqrt{\frac{A_jB_j}{p_j}}\frac{Y+\sqrt{Y^2+4XW'}}{2X}\right)\left(1-\frac{\sqrt{p_jA_jB_j}}{\sqrt{Y^2+4XW'}}\right) \\
&\quad-\left(\sqrt{\frac{A_jB_j}{p_j}}\frac{Y+\sqrt{Y^2+4XW'}}{2X}-B_j\right) \\
&\quad\times\left(1-\frac{\sqrt{p_jA_jB_j}}{\sqrt{Y^2+4XW'}}+\frac{Y\sqrt{p_jA_jB_j}}{Y^2+4XW'}\right) \\
&\quad> B_j\left(1-\frac{2\sqrt{p_jA_jB_j}}{\sqrt{Y^2+4XW'}}+\frac{\sqrt{p_jA_jB_j}\sqrt{p_jA_jB_j}}{Y^2+4XW'}\right) \\
&\quad= B_j\left(1-\frac{\sqrt{p_jA_jB_j}}{\sqrt{Y^2+4XW'}}\right)^2 > 0,
\end{aligned} \tag{61}$$

so $\frac{\partial I_j(\boldsymbol{p})}{\partial p_i} > 0$. Similarly, we can also prove that $\frac{\partial I_i(\boldsymbol{p})}{\partial p_i} > 0$, so monotonicity holds for the price updating function. Finally, from the above three parts, we prove that the price updating function is *standard*.

### A.5 Analytical Comparison between the Centralized Scheme and the Proposed Game

In this Appendix, we sketch the analytical comparison between the centralized optimization scheme in Section 3.5 and the proposed distributed game. First, according to (45), we can represent the Lagrangian of the centralized optimal scheme as follows:

$$L_{cen}(\mathbf{P_r}, \lambda, \nu) = R_{s,r,d} + \sum_{i=1}^{N} \nu_i(-P_{r_i}) + \sum_{i=1}^{N} \lambda_i \left(P_{r_i} - P_{r_i}^{max}\right) + \lambda_{N+1}\left(\sum_{i=1}^{N} P_{r_i} - P_r^{tot}\right), \quad (62)$$

where the Lagrangian multipliers are $\lambda = (\lambda_1, \ldots, \lambda_{N+1})$ and $\nu = (\nu_1, \ldots, \nu_N)$, with $\lambda_i$, $\nu_i \geq 0$. In the proposed game, each node maximizes its own utility, defined in (13) and (15), so we can equivalently view the objective as a vector optimization, and the scalarization can be represented in the following:

$$\max \quad U_s + \sum_{i=1}^{N} w_i U_{r_i} \quad (63)$$

$$\text{s.t.} \quad 0 \leq P_{r_i} \leq P_{r_i}^{max}, \quad i = 1, \ldots, N, \quad (64)$$

$$p_i \geq 0, \quad i = 1, \ldots, N, \quad (65)$$

where $\mathbf{w} = (w_1, \ldots, w_N)$ is any weight vector, and $w_i > 0$, $\forall i$. Similarly, we can express the Lagrangian for the scalarized optimization as

$$\tilde{L}_{game}(\mathbf{Pr}, \mathbf{p}, \tilde{\lambda}, \tilde{\nu}, \tilde{\mu}) = U_s + \sum_{i=1}^{N} w_i U_{r_i} + \sum_{i=1}^{N} \tilde{\mu}_i(-p_i) + \sum_{i=1}^{N} \tilde{\nu}_i(-P_{r_i}) + \sum_{i=1}^{N} \tilde{\lambda}_i \left(P_{r_i} - P_{r_i}^{max}\right), \quad (66)$$

where the Lagrangian multipliers are $\tilde{\lambda} = (\tilde{\lambda}_1, \ldots, \tilde{\lambda}_N)$, $\tilde{\mu} = (\tilde{\mu}_1, \ldots, \tilde{\mu}_N)$, and $\tilde{\nu} = (\tilde{\nu}_1, \ldots, \tilde{\nu}_N)$, with $\tilde{\lambda}_i$, $\tilde{\mu}_i$, $\tilde{\nu}_i \geq 0$, $\forall i$.

Substituting (13) and (15) into (66), after some manipulation, $\tilde{L}_{game}(\mathbf{Pr}, \mathbf{p}, \tilde{\lambda}, \tilde{\nu}, \tilde{\mu})$ becomes

$$\tilde{L}_{game}(\mathbf{Pr}, \mathbf{p}, \tilde{\lambda}, \tilde{\nu}, \tilde{\mu}) = aR_{s,r,d} + \sum_{i=1}^{N} [w_i(p_i - c_i) - p_i] P_{r_i} - \sum_{i=1}^{N} \tilde{\mu}_i p_i + \sum_{i=1}^{N} \tilde{\nu}_i(-P_{r_i}) + \sum_{i=1}^{N} \tilde{\lambda}_i \left(P_{r_i} - P_{r_i}^{max}\right). \quad (67)$$

Since $a > 0$ and for simplicity, the above Lagrangian can be further converted to

$$\tilde{L}'_{game}(\mathbf{Pr}, \mathbf{p}, \lambda, \nu, \mu) = R_{s,r,d} + \sum_{i=1}^{N} \frac{[w_i(p_i - c_i) - p_i]}{a} P_{r_i} - \sum_{i=1}^{N} \frac{\tilde{\mu}_i}{a} p_i + \sum_{i=1}^{N} \frac{\tilde{\nu}_i}{a}(-P_{r_i}) + \sum_{i=1}^{N} \frac{\tilde{\lambda}_i}{a} \left(P_{r_i} - P_{r_i}^{max}\right). \quad (68)$$

Comparing (62) and (68), we can find that they have similar terms, which can be viewed as one-to-one mappings, i.e., $\lambda_i \leftrightarrow \frac{\tilde{\lambda}_i}{a}$, $\nu_i \leftrightarrow \frac{\tilde{\nu}_i}{a}$, and

$$\lambda_{N+1}\left(\sum_{i=1}^{N} P_{r_i} - P_r^{tot}\right) \leftrightarrow \frac{\sum_{i=1}^{N} [w_i(p_i - c_i) - p_i] P_{r_i} - \sum_{i=1}^{N} \tilde{\mu}_i p_i}{a}.$$

Without loss of generality, let us view $a$ as a parameter in the proposed game and, correspondingly, $P_r^{tot}$ as a parameter in the centralized optimal scheme. When $a$ increases,

$$\frac{\sum_{i=1}^{N} [w_i(p_i - c_i) - p_i] P_{r_i} - \sum_{i=1}^{N} \tilde{\mu}_i p_i}{a}$$

decreases. In order to map $\lambda_{N+1}(\sum_{i=1}^{N} P_{r_i} - P_r^{tot})$ to it, $P_r^{tot}$ should increase. That is the reason why varying the parameter $a$ in the proposed game is equivalent to varying $P_r^{tot}$ in the centralized optimization. To justify our claim, we show the optimal powers versus $P_r^{tot}$ and $a$ of the two schemes in Fig. 8, with or without the $P_{r_i}^{max}$ constraints, respectively. From both the simulation and the above analysis, we can see that due to the equivalence of the Lagrangian in the two approaches, the proposed game can achieve comparable performance to that in the centralized optimal scheme.

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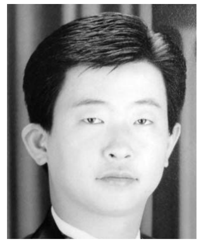
**Beibei Wang** received the BS degree in electrical engineering (with highest honors) from the University of Science and Technology of China, Hefei, in July 2004. She is currently a PhD candidate in the Department of Electrical and Computer Engineering, University of Maryland, College Park. Her research interests include resource allocation and management in cognitive radio systems, wireless communications and networking, game theory, wireless multimedia, and network security. She was the recipient of both the Graduate School Fellowship and the Future Faculty Fellowship from the University of Maryland. She is a student member of the IEEE.



**Zhu Han** received the BS degree in electronic engineering from Tsinghua University in 1997 and the MS and PhD degrees in electrical engineering from the University of Maryland, College Park, in 1999 and 2003, respectively. From 2000 to 2002, he was an R&D engineer of JDSU, Germantown, Maryland. From 2002 to 2003, he was a graduate research assistant, and from 2003 to 2006, a research associate, at the University of Maryland. From 2006 to 2008, he was an assistant professor at Boise State University, Idaho. Currently, he is an assistant professor in the Electrical and Computer Engineering Department, University of Houston, Texas. From June to August 2006, he was a visiting scholar at Princeton University. From May to August 2007, he was a visiting professor at Stanford University. From May to August 2008, he was a visiting professor at the University of Oslo, Norway, and Supelec, Paris, France. His research interests include wireless resource allocation and management, wireless communications and networking, game theory, wireless multimedia, and security. He is the MAC Symposium vice chair of the IEEE Wireless Communications and Networking Conference in 2008. He is the guest editor for the special issue on fairness of radio resource management techniques in wireless networks of the *EURASIP Journal on Wireless Communications and Networking* and the special issue on game theory of the *EURASIP Journal on Advances in Signal Processing.* He is a member of the technical programming committee for the IEEE International Conference on Communications, the IEEE Vehicular Technology Conference, the IEEE Consumer Communications and Networking Conference, the IEEE Wireless Communications and Networking Conference, and the IEEE Globe Communication Conference. He is a member of the IEEE.

**K.J. Ray Liu** is a Distinguished Scholar-Teacher at the University of Maryland, College Park. He is the associate chair of graduate studies and research in the Electrical and Computer Engineering Department. He also leads the Maryland Signals and Information Group, conducting research encompassing broad aspects of information technology including communications and networking, information forensics and security, multimedia signal processing, and biomedical technology. He is the recipient of numerous honors and awards, including the IEEE Signal Processing Society Distinguished Lecturer Award, the EURASIP Meritorious Service Award, the National Science Foundation Young Investigator Award, and best paper awards from the IEEE Signal Processing Society (twice), the IEEE Vehicular Technology Society, and EURASIP. He has also received various teaching and research recognitions from the University of Maryland, including the university-level Invention of the Year Award, the Outstanding Faculty Research Award, and the Poole and Kent Senior Faculty Teaching Award from the A. James Clark School of Engineering. He is the vice president for publications and is on the Board of Governors of the IEEE Signal Processing Society. He was the editor-in-chief of the *IEEE Signal Processing Magazine* and the founding editor-in-chief of the *EURASIP Journal on Applied Signal Processing.* His recent books include *Cooperative Communications and Networking* (Cambridge University Press, 2008), *Resource Allocation for Wireless Networks: Basics, Techniques, and Applications* (Cambridge University Press, 2008), *Ultra-Wideband Communication Systems: The Multiband OFDM Approach* (IEEE-John Wiley & Sons, 2007), *Network-Aware Security for Group Communications* (Springer, 2007), *Multimedia Fingerprinting Forensics for Traitor Tracing* (Hindawi, 2005), and *Handbook on Array Processing and Sensor Networks* (IEEE-John Wiley & Sons, 2009). He is a fellow of the IEEE and AAAS.