

# Stackelberg Game for Distributed Resource Allocation over Multiuser Cooperative Communication Networks

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**Abstract**—In this paper, we propose a Stackelberg game theoretic framework for distributive resource allocation over multiuser cooperative communication networks to improve the system performance and stimulate cooperation. Two questions of who should relay and how much power for relaying are answered, by employing a two-level game to jointly consider the benefits of source nodes as buyers and relay nodes as sellers in cooperative communication. From the derived results, the proposed game not only helps the source smartly find relays at relatively better locations but also helps the competing relays ask reasonable prices to maximize their own utilities. From the simulation results, the relays in good locations or good channel conditions can play more important roles in increasing source node's utility, so the source would like to buy power from these preferred relays. On the other hand, because of competition from other relays and selections from the source, the relays have to set proper prices to attract the source's buying so as to optimize their utility values.

## I. INTRODUCTION

Recently, cooperative communications have gained many attentions as an emerging transmit strategy for future wireless networks [1] [2]. The basic idea is that the relay nodes can help the source node's transmission by relaying the replica of the information. The cooperative communications efficiently take advantage of the broadcasting nature of wireless networks, as well as exploit the inherent spatial and multiuser diversities.

Many recent works proposed various protocols for different layers of networks. The work in [3] analyzed with more complicated transmitter cooperative schemes involving dirty paper coding. The energy-efficient transmission was considered for broadcast networks in [4]. [5] considered the design of a cooperative relay strategy by exploiting the finite-alphabet property of the source. In [6], the relay assignment problem is solved for the multiuser cooperative communications. In [7], the cooperative resource allocation for OFDM is studied.

However, most existing work focuses on resource allocation by means of centralized fashion. To achieve the distributed implementation, game theory is a natural, flexible, and rich tool which studies how the autonomous nodes interact and cooperate with each other. For game theory literature in the wireless networking, in [8], the behaviors of selfish nodes in the case of random access and power control were examined. In [9] static pricing policies for multiple-service networks were proposed to offer the needed incentives for each node to choose the service that best matched her needs, thereby discouraging over-allocation of resources and improving social welfare. In [10], [11], and [12], the authors employed cooperative game, noncooperative game with a referee, and repeated game for single-cell OFDMA, multiple-cell OFDMA resource

allocation, and multiple access rate control, respectively.

In this paper, we consider how to employ game theory for the distributed nodes to optimize performances over cooperative communication paradigm. Two main resource allocation questions over cooperative multiuser wireless networks remain yet unanswered: First, among all the distributed nodes, who can help relay and improve the source's link quality better; Second, for the selected relay nodes, how much power they need to transmit. Both questions need to be answered in a distributed way.

To answer these questions, we employ a Stackelberg game [13] to jointly consider the benefits of source nodes and relay nodes in cooperative communication. The game is divided into two levels: the source node plays the *buyer-level* game and the relay nodes play the *seller-level* game. Each player is selfish and wants to maximize its own benefit. Specifically, the source can be viewed as a buyer and it aims to get most benefits at the least possible payment. Each relay can be seen as a seller and aims to earn the payment which can not only cover their forwarding cost but also gain as much extra profit as possible. Then we derive the expressions to the proposed game outcomes. We analyze how many relay nodes would be selected by the source to participate in the sale process after they announced their optimal prices. In addition, we optimize how much service amount the source should buy from each relay node. From the seller's point of view, the relay nodes set the corresponding optimal price per unit of the service such as relaying power so as to maximize its own benefit. From the simulations, because of competition from other relays and selections from the source, the relays have to set proper prices to attract the source's buying so as to optimize their utility values. The source optimally selects the relays and their relaying power, while the relays set the prices that can maximize their utilities.

This paper is organized as follows: Section II describes the system model. We construct distributed implementation of multiuser cooperation transmission, formulate the cooperative optimization as a Stackelberg game, and provide the solutions in Section III. Simulations are shown in Section IV. Finally, Section V concludes the paper.

## II. SYSTEM MODEL

In this paper, we use the amplify-and-forward (AF) cooperative protocol as an example. The relay nodes help the source node by relaying the received information to the destination. The receiver at the destination combines together the directly received signal from the source node and the relayed signals from the relay nodes, using techniques such as maximal ratio

combining (MRC). The above procedure can be described in two phases as the sequel.

At phase one, without relay node's help, the signal-to-noise ratio (SNR) that results from direct transmission from the source  $s$  to the destination  $d$  can be expressed by

$$\Gamma_{s,d} = P_s G_{s,d} / \sigma^2, \quad (1)$$

where  $P_s$  represents the transmit power,  $G_{s,d}$  is the channel gain, and  $\sigma^2$  is the noise power. The rate at the output of noncooperative transmission is

$$R_{s,d}^{nc} = W \log_2 \left( 1 + \frac{P_s G_{s,d} / \sigma^2}{\Gamma} \right), \quad (2)$$

where  $\Gamma$  is a constant for the capacity gap. Without loss of generality, we assume that the noise power is the same for all links. We also assume the channels are stable over each power control interval.

At phase two, the relayed SNR for the source  $s$ , which is helped by relay  $r_i$ , is given by [7]:

$$\Gamma_{s,r_i,d} = \frac{P_{r_i} P_s G_{r_i,d} G_{s,r_i}}{\sigma^2 (P_{r_i} G_{r_i,d} + P_s G_{s,r_i} + \sigma^2)}, \quad (3)$$

where  $P_{r_i}$  is the transmit power of  $r_i$ ,  $\sigma^2$  is the noise power,  $G_{s,r_i}$  and  $G_{r_i,d}$  are the channel gains from the source to  $r_i$  and from  $r_i$  to the destination respectively. Therefore, by (1) and (3), if there are  $N$  relays helping the source, then

$$R_{s,r,d}^{AF} = W \log_2 \left[ 1 + \frac{P_s G_{s,d}}{\Gamma \sigma^2} + \sum_{i=1}^N \frac{P_{r_i} P_s G_{r_i,d} G_{s,r_i}}{\Gamma \sigma^2 (P_{r_i} G_{r_i,d} + P_s G_{s,r_i} + \sigma^2)} \right]. \quad (4)$$

Since the source has paid on the transmission slot and relay power to the relays, the formulation is different from the conventional cooperative communication literatures where bandwidth and power need fair comparison.

### III. PROBLEM FORMULATION AND ANALYSIS

To explore the cooperative diversity for multiuser system, from (4), two fundamental questions need to be answered: First, which relay nodes should be included; second, what is the optimal power  $P_{r_i}$ . To answer the questions, we employ Stackelberg game for buyers and sellers as the following formulated problem.

(1) Source/Buyer: the source can be modeled as a buyer and it aims to get most benefits at the least possible payment. So the utility function of the source can be defined as

$$U_s = a \Delta R_{tot} - M, \quad (5)$$

where

$$\Delta R_{tot} = R_{s,r,d}^{AF} - R_{s,d}^{nc} \quad (6)$$

denotes the total rate increment with the relay nodes helping transmission,  $a$  denotes the gain per unit of rate increment at the MRC output, and

$$M = p_1 P_{r_1} + p_2 P_{r_2} + \dots + p_N P_{r_N} \quad (7)$$

represents the total payment paid by the source to the relay nodes. In (7),  $p_i$  represents the price per unit of power selling from relay node  $i$  to the source  $s$ , and  $P_{r_i}$  denotes how much power the source would like to buy from relay  $r_i$  when the prices are announced from the relays.

Assume the number of relay nodes is  $N$ , then the optimization problem for the source or buyer's game can be formulated as:

$$\max_{\{P_{r_i}\}} U_s = a \Delta R_{tot} - M, \quad \text{s.t. } \{P_{r_i}\}_{i=1}^N \geq 0. \quad (8)$$

It's worth noticing here that  $a$  reflects how much power the source would buy from the relays. For example, if  $a$  is large, meaning the gain of rate increment overwhelms the payments, then it is profitable for the source to buy more power so as to get more utility and larger rate.

(2) Relays/Seller: Each relay  $r_i$  can be seen as a seller and aims to earn the payment which not only covers their forwarding cost but also gain as much extra profit as possible. We introduce one parameter  $c_i$ , 'the cost of power for relaying data', in our formulation to correctly reflect relays' consideration about whether they can actually get profit by the sale. Then relay  $r_i$ 's utility function can be defined as

$$U_{r_i} = p_i P_{r_i} - c_i P_{r_i} = (p_i - c_i) P_{r_i}, \quad (9)$$

where  $c_i$  is the cost per unit of power in relaying data,  $p_i$  has the same meaning as in (7), and  $P_{r_i}$  is the source's decision by optimizing  $U_s$  described in (8). It is obvious that to determine the optimal  $p_i$  depends not only on each relay's own channel condition to the destination but also on its counterpart relays' prices. So in the sellers' competition, if one relay asks a higher price than what the source expects about it after jointly considering all relays' prices, the source will buy less from that relay or even disregard that relay. On the other hand, if the price is too low, the profit obtained by (9) will be unnecessarily low. So there is a tradeoff for setting the price. Moreover,  $c_i$  will also affect the relay's asking price. If  $c_i$  is large, relay  $r_i$  has to increase  $p_i$  to cover the cost, leading the source to buy less power and get lower  $U_s$  and achievable rate.

Then the optimization problem for relay  $r_i$  or the seller's game is:

$$\max_{\{p_i\} > 0} U_{r_i} = (p_i - c_i) P_{r_i}, \quad \forall i. \quad (10)$$

Therefore, the ultimate goal of the above two games is to decide the optimal pricing  $p_i$  to maximize relays' profits  $U_{r_i}$ , the actual number of relays who will finally get selected by the source and the corresponding optimal power consumption  $P_{r_i}$  to maximize  $U_s$ . Notice that the only signalings required to exchange between the source and relays are the price  $p_i$  and the information about how much power  $P_{r_i}$  to buy. Consequently, the proposed two-level game approach can have distributed resource allocation for the cooperative communication networks. The outcome of the games will be shown in details in the following two subsections.

#### A. Source/Buyer Level Analysis

We will give some observation of the  $U_s$  function with respect to  $\{P_{r_i}\}$ . When  $P_{r_i}$  is close to 0, few help is got from the relay, so  $U_s$  should be close to 0. As  $P_{r_i}$  increases, relays sell more power to the source hence more rate increment is obtained, and  $U_s$  increases. If  $P_{r_i}$  further increases, by the properties of the logarithmic functions, the gain of rate increment grows slower than the payment does, hence the utility of the source  $U_s$  begins to decrease. Assume the selling price of the relays' power  $p_i$ ,  $i = 1, 2, \dots, N$  has been announced, then from the first order optimality condition, the following equations must hold at the optimal point:

$$\frac{\partial U_s}{\partial P_{r_i}} = 0, \quad i = 1, 2, \dots, N. \quad (11)$$

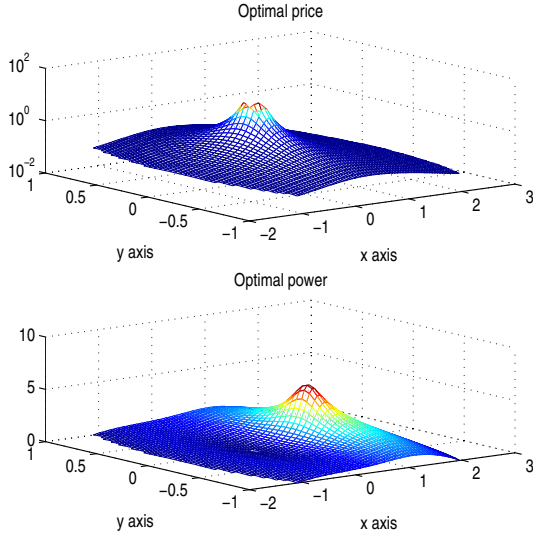


Fig. 1. Optimal price and power of the relay in different locations

For simplicity, define  $C = 1 + \frac{P_s G_{s,d}}{\sigma^2 \Gamma}$ ,  $W' = \frac{aW}{\ln 2}$ , then by (4) and (2) we get the first term of  $U_s$  as

$$\begin{aligned} a\Delta R_{tot} &= aW \log_2 \left[ 1 + \frac{1}{\Gamma C} \sum_{i=1}^N \frac{P_{r_i} P_s G_{r_i,d} G_{s,r_i}}{\sigma^2 (P_{r_i} G_{r_i,d} + P_s G_{s,r_i} + \sigma^2)} \right] \\ &= W' \ln \left( 1 + \sum_{i=1}^N \Gamma'_{s,r_i,d} \right) = W' \ln (1 + \Delta SNR'_{tot}), \end{aligned} \quad (12)$$

where  $\Gamma'_{s,r_i,d} = \frac{\Gamma_{s,r_i,d}}{\Gamma C} = \frac{A_i}{1 + \frac{B_i}{P_{r_i}}} = \frac{A_i P_{r_i}}{P_{r_i} + B_i}$ , (13)

with  $A_i = \frac{P_s G_{s,r_i}}{(\Gamma \sigma^2 + P_s G_{s,d})}$  and  $B_i = \frac{P_s G_{s,r_i} + \sigma^2}{G_{r_i,d}}$ .

Substituting (7) and (12) into (11), we have

$$\frac{\partial U_s}{\partial P_{r_i}} = \frac{W'}{A_i B_i} \frac{1}{\left( 1 + \sum_{k=1}^N \frac{A_k P_{r_k}}{P_{r_k} + B_k} \right) (P_{r_i} + B_i)^2} - p_i = 0, \quad (14)$$

i.e., 
$$\frac{p_i}{A_i B_i} (P_{r_i} + B_i)^2 = \frac{W'}{\left( 1 + \sum_{k=1}^N \frac{A_k P_{r_k}}{P_{r_k} + B_k} \right)}. \quad (15)$$

Since the L.H.S. of (15) is the same for any relay  $i$  on the R.H.S., it follows that

$$\frac{p_i}{A_i B_i} (P_{r_i} + B_i)^2 = \frac{p_j}{A_j B_j} (P_{r_j} + B_j)^2, \quad (16)$$

then

$$P_{r_j} = \sqrt{\frac{p_i A_j B_j}{p_j A_i B_i}} (P_{r_i} + B_i) - B_j. \quad (17)$$

Substitute the above  $P_{r_j}$  into (13) and simplify,

$$\Gamma'_{s,r_j,d} = A_j - \sqrt{\frac{p_j A_i B_i}{p_i A_j B_j}} \frac{A_j B_j}{(P_{r_i} + B_i)}, \quad (18)$$

and (15) can be reorganized as a quadratic equation of  $P_{r_i}$ ,

$$\begin{aligned} \left( 1 + \sum_{j=1}^N A_j \right) \left[ \sqrt{\frac{p_i}{A_i B_i}} (P_{r_i} + B_i) \right]^2 \\ - \sum_{j=1}^N \sqrt{p_j A_j B_j} \left[ \sqrt{\frac{p_i}{A_i B_i}} (P_{r_i} + B_i) \right] - W' = 0. \end{aligned} \quad (19)$$

Define  $X = 1 + \sum_{j=1}^N A_j$ ,  $Y = \sum_{j=1}^N \sqrt{p_j A_j B_j}$ , then we can solve the optimal power consumption of each relay as

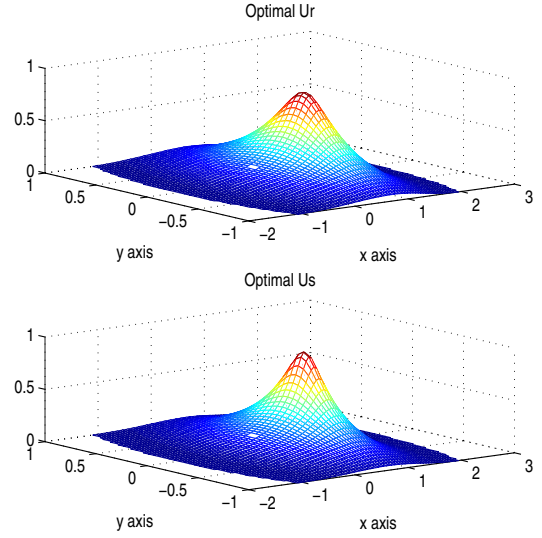


Fig. 2. Optimal utilities of the relay and the source in different locations

$$P_{r_i} = \sqrt{\frac{A_i B_i}{p_i} \frac{Y + \sqrt{Y^2 + 4XW'}}{2X}} - B_i. \quad (20)$$

However, the solution above may be negative for some relay's high price or bad location. Therefore the optimal power consumption should be modified as follows,

$$P_{r_i}^* = \max(P_{r_i}, 0) = (P_{r_i})^+, \quad (21)$$

where  $P_{r_i}$  is solved in (20).

### B. Relay/Seller Level Analysis

Substituting (21) into (10), we have

$$\max_{\{p_i\} > 0} U_{r_i} = (p_i - c_i) P_{r_i}^*(p_1, \dots, p_i, \dots, p_N). \quad (22)$$

Note this is a noncooperative game by the relay, and there exists a tradeoff between the price  $p_i$  and relay's utility  $U_{r_i}$ . If the relay asks for a relatively lower price  $p_i$  at first, the source would be glad to buy more power from the cheaper seller and  $U_{r_i}$  will increase as  $p_i$  grows. When  $p_i$  keeps growing, the source would think it is no longer profitable to buy power from the relay and  $P_{r_i}$  will shrink hence result in a decrement of  $U_{r_i}$ . Therefore there is an optimal price for each relay to ask for, and the optimal price is also affected by other relays' prices since the source only chooses the most beneficial relays among all the relays.

From the analysis above, by the first order optimality condition, it follows that

$$\begin{aligned} P_{r_i}^*(p_1, \dots, p_i, \dots, p_N) \\ + (p_i - c_i) \frac{\partial P_{r_i}^*(p_1, \dots, p_i, \dots, p_N)}{\partial p_i} = 0, \quad i = 1, 2, \dots, N. \end{aligned} \quad (23)$$

Solving (23) for  $N$  unknowns  $p_i$ , we have

$$p_i^* = p_i^*(\sigma^2, \{G_{s,r_i}\}, \{G_{r_i,d}\}), \quad i = 1, 2, \dots, N. \quad (24)$$

As we have mentioned at the beginning of this section, the source needs to select more beneficial relays, so we can substitute (24) into (20) to see whether  $P_{r_i}$  is positive. If not so, then the source will disregard the relay with negative  $P_{r_i}$  and only the remaining relays constitute the actual relaying

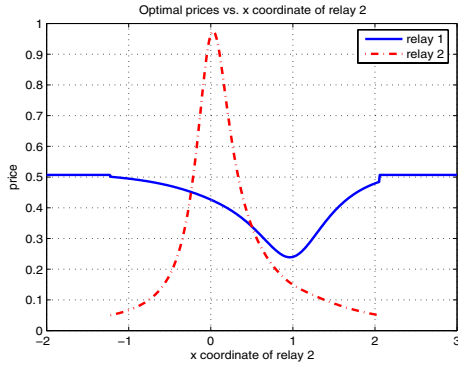


Fig. 3. Optimal relays' prices when relay 2 moves

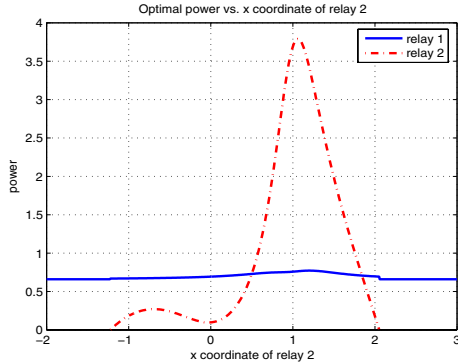


Fig. 4. Optimal power consumptions of two relays when relay 2 moves

subset. Re-solve (14) by changing the set of relay nodes to the subset solved above, and re-solve the new corresponding  $p_i^*$  then check the  $P_{r_i}$  until all  $P_{r_i}$  are positive. Then we can get the final optimal pricing  $p_i^*$  to maximize relays' utilities  $U_{r_i}$ , the actual number of relays which will get selected by the source and the corresponding optimal power consumption  $P_{r_i}^*$  to maximize  $U_s$ . The convergence of the problem can be proved in a similar way as in [14]. Due to the limit of the length, we omit the proof here.

#### IV. SIMULATION RESULTS AND ANALYSIS

To evaluate the performances of the proposed scheme and decide what price each relay should ask for and how much power the source should buy from each relay, we performed simulations for multiple relay systems. In what follows, the simulation results for a 1-relay case, for a 2-relay case, and for a multiple-relay case are shown.

##### A. 1-Relay Case

We set simulations of the first part as follows. There are 1 source-destination pair and 1 relay in the network. The destination was located at coordinates (0,0), the source was fixed at coordinates (1,0), and the relay was randomly located within the range of  $[-2, 3]$  in x-axis and  $[-1, 1]$  in y-axis. The propagation loss factor was set to 2. The noise level was  $\sigma^2 = 10^{-4}$  and we selected the capacity gap  $\Gamma = 1$ ,  $W = 1$ , the gain per unit of rate increment  $a = 1$  and cost per unit of power  $c_i = 0.05, \forall i$ . In Figure 1, we show the optimal price the relay should ask for and the optimal power bought by the source. We observe that when the relay is close to the source

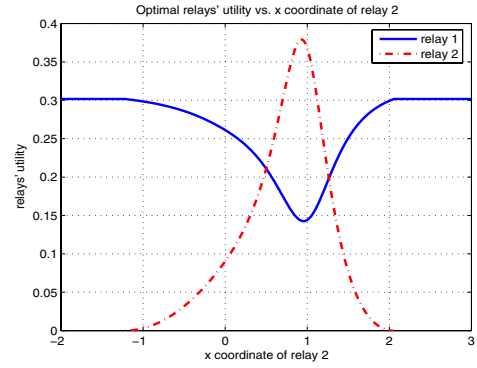


Fig. 5. Optimal relays' utilities when relay 2 moves

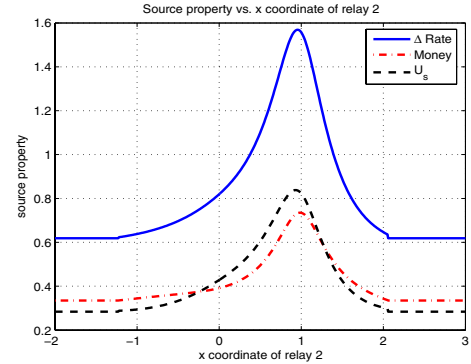


Fig. 6.  $U_s, \Delta Rate, M$  when relay 2 moves

at (1,0), it can more efficiently help the source transmit, so the relay would reduce the price to attract the source to buy more service. When the relay moves close to the destination at (0,0), it can use very small amount of power to relay the source's data, so it will set a very high price in order to get more profit by selling this small power. When the relay keeps moving away from the destination or the source, the source would stop buying service because the relay is in a very bad location.

In Figure 2, we show the optimal utility the source and the relay can get using the proposed scheme. When the relay is located close to the source, both the relay and the source can get the maximal utility. The reason is that around this location, the relay can most efficiently help the source increase its utility, and the optimal price of the relay is very low compared with that when the relay is at other locations. So the source would like to buy more power, resulting in high utility to the relay.

##### B. 2-Relay Case

We set up 2-relay simulations to test the proposed scheme. In our simulations, relay 1 is fixed at the coordinates (0.5,0.25) and relay 2 moves along the line from  $(-2, 0.25)$  to  $(3, 0.25)$ . Other settings are the same as the 1-Relay case.

In Figure 3 we show the optimal price that each relay should ask to maximize its profit. We can observe that even though only relay 2 moves, the prices of both relays change accordingly. This fact is because two relays compete and influence each other in the game. When relay 2 is close to



the destination at  $(0, 0)$ , it can use very small power to relay the source's information. So relay 2 can set very high price hoping to get more profit by selling small power. When relay 2 is close to the source at  $(1, 0)$ , relay 2 is more suitable to help the source transmit. Consequently, in order to attract the source to buy its service, relay 1 has to reduce the price. When relay 2 is faraway, its price will drop because it is less competitive compared to relay 1 at location  $(0.5, 0.25)$ . When the utility is less than 0, relay 2 will quit the competition. At that moment, relay 1 can slightly increase the price since there is no competition. But it cannot increase too much, otherwise relay 2 will rejoin the competition.

As shown in Figure 4, the source will smartly buy different amount of power from the two relays. When relay 2 moves away from the source,  $P_{r_2}^*$  gradually decreases. When relay 2 moves too far away from the source or the destination, the source will not choose relay 2. When relay 2 is close to the destination, its price shown in Figure 3 is too high, so that the source would not buy much power from relay 2. When relay 2 quits the competition, relay 1 will increase its price, but the source will buy slightly less. This fact also suppresses the incentive of relay 1 to ask for arbitrarily high price in the absence of competition. Note that when relay 2 moves to  $(0.5, 0.25)$ , the same location as relay 1, the power consumptions and prices of both relays are the same. This is because the source is indifferent for the two relays locating together and treats them equally.

In Figure 5, we show the optimal utility of two relays. When relay 2 is close to the source, its utility is high, while relay 1's utility drops. The utility of relay 2 is zero after it quits the competition, while the utility of relay 1 is smooth at the transition points. In Figure 6, we show the optimal utility of the source, the optimal rate increment and total payment to the relays. When relay 2 is close to the source, the channel conditions are the best in relaying the source's data, and therefore the relays should get the highest profits and all the three values reach their maxima.

### C. Multiple-Relay Case

We then set up multiple-relay simulations to test the proposed scheme. In these simulations, relays are randomly located within the range of  $[-2, 3]$  in  $x$ -axis and  $[-2, 2]$  in  $y$ -axis. From Figure 7, we can observe that as the total number of available relays increases, the source will get a higher utility. However, in this way, the competitions among relays become more severe, which leads to less average payment from the source.

## V. CONCLUSION

In this paper, we proposed the game theory approach for distributive resource allocation over multiuser cooperative communication networks. We target to answer two questions: who will be the relays and how much power for relaying in the amplify-and-forward cooperative scenario. We employ Stackelberg game to jointly consider the benefits of different types of nodes. The proposed scheme can not only help the source smartly choose relays at better locations but can also help the competing relays ask a reasonable price to maximize

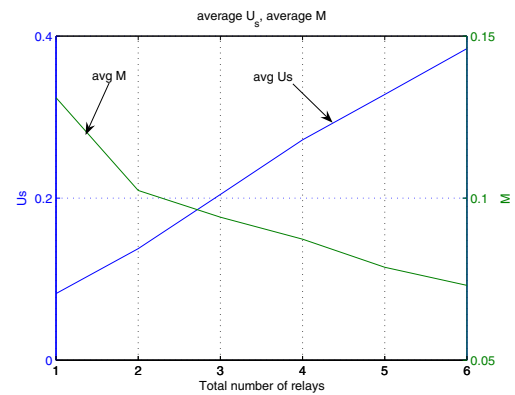


Fig. 7. Optimal source utility and average money transfer vs. number of relay nodes

their utilities. From the simulation results, relays close to the source can play a more important role in increasing source utility, so the source would like to buy power from these preferred relays. In order to attract more consumption from the source, the relay might adopt 'low-price, high-market' policy to further increase its utility value. It is also easy to use current structures as building blocks in large-scale wireless ad hoc networks to stimulate cooperation among nodes.

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