Variable Rate Space-Time Trellis Codes

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***Abstract*—Emerging wireless applications have created a strong motivation to increase the throughput of wireless communication systems. Space-time coding has become a promising method to achieve high data rates. In this work, we propose a space-time trellis code construction method that can provide variable protection against the adverse effects of the radio channel. The method is based on the multiple trellis coded modulation construction: the variable data protection is realized by changing the number of output symbols per state transition. The simulation results show the achievable tradeoffs between spectral efficiency, diversity and decoding complexity.**

I. INTRODUCTION

Space-Time (ST) trellis codes have been developed to improve the performance of wireless communication systems. The performance criteria for the fast Rayleigh fading channel model were developed in [1], characterizing the ST codes with two quantities: the diversity advantage, which describes the asymptotic error rate decrease as a function of the signal to noise ratio (SNR), and the coding advantage, which determines the vertical shift of the error performance curve.

Traditionally, the objective of ST code design has been to find good codes for a fixed source data rate and error correction capability [1], [2], [3], [4], [5], [6], [7]. However, in certain applications, it is desirable to have a more flexible channel code construction scheme. In mobile radio propagation environment, the channel may vary considerably with time, so if the transmitter has the ability to receive SNR measurement values from the receiver through a low data rate feedback channel, the throughput of the system could be adopted to the channel conditions. Moreover, in most transmission scenarios, the transmitted data can be decomposed into layers of different importance. Examples include control and data bits, or quantizer step sizes, filter coefficients and compressed data in some voice encoding schemes. Emerging multimedia applications also manifest this layered structure: for instance, the source encoded video data stream can be decomposed into header information, motion vectors and DCT coefficients, each having different sensitivity to transmission errors. These applications could benefit from variable rate channel coding methods that provide unequal error protection (UEP).

The MTCM construction for multi-antenna systems was introduced in [1], where the authors presented hand-crafted ST codes for a few number of transmit antennas and for fixed data rates.

In this paper, we show that in case of the fast Rayleigh fading channel model, multiplicity acts as diversity in ST coded multiple antenna communication systems, similar to the traditional multiple trellis coded modulation (MTCM) construction [8]. The number of output channel symbols per state transition can influence the asymptotic behavior of the bit error rate curve. Moreover, we present a simple and computationally efficient method to design ST trellis codes that can achieve different diversity levels based on the MTCM scheme. Our method can be used to design ST trellis codes for an arbitrary number of transmit antennas and any memoryless constellation. Finally, we describe a variable rate ST trellis code construction procedure that offers the possibility of variable protection against channel impairments. The variable data protection is achieved by changing the number of output channel symbols per state transition.

The paper is organized as follows. Section II will introduce the notation and the mathematical model of the communication system employing multiple ST trellis codes. Section III will discuss the achievable diversity order using the MTCM construction. The code construction method will be developed in Section IV. Section V will describe the possible tradeoffs between source rate (spectral efficiency) and diversity advantage. Section VI will provide the simulation results, and some conclusions will be drawn in the last section.

II. SYSTEM MODEL

Consider a wireless communication system with K transmit and L receive antennas. The input bit stream is divided into b_s bit long blocks, forming B-ary ($B = 2^{b_s}$) source symbols. The ST encoder works as a finite state machine: it takes the current source symbol, b_n ($b_n \in \{0, 1, \ldots, B-1\}$), at state transition n, and, governed by this input and the current state, moves to the next state. During this state transition, the encoder outputs M B-ary channel symbol indices for each transmit antenna. These channel symbol indices are mapped to channel symbols (or constellation points) by the modulators and transmitted through the transmit antennas. We denote the mth ($m = 0, 1, \ldots, M-1$) channel symbol generated during state transition n for transmit antenna k ($k = 0, 1, \ldots, K-1$) at discrete time $t = Mn + m$ by $c_n^k(m)$. All the constellations are assumed to be normalized so that the average energy of the constellation is unity (if the channel symbols are equally likely). We will also use the channel symbol vector, defined as:

$$\mathbf{c}_n(m) = [\, c_n^0(m), c_n^1(m), \ldots, c_n^{K-1}(m) \,]^T.$$

The transmission medium is assumed to be a flat (frequency nonselective), fast Rayleigh fading channel. At each discrete time instant, the path gains between the transmit and the receive antennas are modeled as independent, complex, zero mean, circularly symmetric Gaussian random variables with unit variance. These path gains are assumed to be known by the receiver.

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At the receiver side, the received signals at each receive antenna are demodulated, and the ST decoder produces the decoded bit stream. The receiver noise is modeled as independent, complex, zero mean, circularly symmetric Gaussian random variables.

III. Diversity Order

Assume that N B-ary source symbols are sent to the receiver. The encoder encodes the source symbol sequence $\{b_n\}$ and produces $\mathbf{C}$, the K by NM channel symbol matrix:

$$\mathbf{C} = [\ \mathbf{c}_0(0), \mathbf{c}_0(1), ..., \mathbf{c}_0(M-1), \mathbf{c}_1(0), ..., \mathbf{c}_{N-1}(M-1)\ ].$$

The decoder, due to decoding errors, decodes a different channel symbol matrix, $\mathbf{C}'$:

$$\mathbf{C}' = [\ \mathbf{c}'_0(0), \mathbf{c}'_0(1), ..., \mathbf{c}'_0(M-1), \mathbf{c}'_1(0), ..., \mathbf{c}'_{N-1}(M-1)\ ].$$

Define $\mathbf{D}$, the channel symbol difference matrix, as $\mathbf{D} = \mathbf{C} - \mathbf{C}'$. Assume that at state transition n_i, M_i columns of $\mathbf{D}$ ($1 \leq M_i \leq M$) are nonzero, i.e.

$$\mathbf{c}_{n_i}(m_l) - \mathbf{c}'_{n_i}(m_l) \neq \mathbf{0},$$

for $i = 0, 1, \ldots, \nu - 1$ and $l = 0, 1, \ldots, M_i - 1$. Using the results of [1], the probability that the decoder erroneously decodes $\mathbf{C}'$ if $\mathbf{C}$ was sent can be upper bounded as:

$$P(\mathbf{C}'|\mathbf{C}) \leq \prod_{i=0}^{\nu-1} \left(\frac{E_0}{4KMN_0} \right)^{-M_i L} \cdot \prod_{l=0}^{M_i-1} \|\mathbf{c}_{n_i}(m_l) - \mathbf{c}'_{n_i}(m_l)\|^{-2L}, \qquad (1)$$

where E_0/N_0 is the average signal to noise ratio per source symbol at the receive antennas (i.e. each transmit antenna transmits the channel symbols with $E_0/(KM)$ average transmit energy), and $\|\mathbf{x}\| = \sqrt{\mathbf{x}^H \mathbf{x}}$.

From (1), we can conclude that multiple ST trellis codes inherit some properties of trellis coded modulation [8]. Assuming fast (independent) Rayleigh fading channel, multiplicity acts as diversity. Changing the number of output channel symbols per state transition changes the achievable diversity order of the system, even if the number of encoder states stays the same.

IV. Multiple Space-Time Code Construction

To improve the performance of the communication system, the diversity advantage and the coding advantage of the ST code must be maximized. However, by introducing multiplicity, the computational complexity of computer search is considerably increased compared to that of the traditional ST codes. Therefore, for larger number of transmit antennas and larger constellation sizes, the exhaustive search for good codes becomes impractical.

We propose a simple approach to exploit the diversity available in the MTCM construction for an arbitrary number of transmit antennas and any memoryless modulation. The proposed method does not guarantee optimality, but it offers a low complexity code construction procedure by reducing the multiple ST code design problem to the traditional ST code design problem. The basic idea behind the method is to use repetition coding. At each state transition, the same channel symbols are transmitted M times from a certain transmit antenna. Then we have:

$$\mathbf{c}_n(m) = \mathbf{c}_n(m'),$$

for $m, m' \in \{0, 1, \ldots, M-1\}$. This simplification allows us to use the code construction method of [7], developed for traditional ST codes ($M = 1$ case) and encoders having B^{K-1} states. The design method, which assigns channel symbol indices to state transitions, can be briefly described as follows:

Since the ST encoder has B^{K-1} states, any state S ($S \in \{0, 1, \ldots, B^{K-1} - 1\}$) can be uniquely represented as a $K-1$ digit B-ary number with digits $l_1, l_2, \ldots, l_{K-1}$ ($l_k \in \{0, 1, \ldots, B-1\}$):

$$S = B^{K-2} l_{K-1} + B^{K-3} l_{K-2} + \ldots + B l_2 + l_1.$$

The design rules are [7]:

1. The channel symbol index for the 0th transmit antenna corresponding to input b ($b \in \{0, 1, \ldots, B-1\}$) at state S is determined as:

$$i^0(S, b) = (b + l_{K-1} + l_{K-2} + \ldots + l_1) \bmod B. \qquad (2)$$

2. The channel symbol index for the kth transmit antenna, $k = 1, 2, \ldots, K-1$, corresponding to input b at state S is calculated as:

$$i^k(S, b) = (i^0(S, b) + l_k) \bmod B. \qquad (3)$$

This design method ensures that any ν-long error event will achieve a diversity advantage of νL by producing ν nonzero columns in the channel symbol difference matrix.

As an example, consider the 16 state ST code shown in Figure 1 designed for 3 antennas and any 4-ary constellation ($K = 3, b = 2, M = 1$). Since $B = 4$, state $S = 6$ can be represented as $S = Bl_2 + l_1$ with $l_2 = 1$ and $l_1 = 2$. The channel symbol indices corresponding to the 2nd branch ($b = 2$) emanating from state 6 are determined as:

$$i^0(S, b) = (b + l_2 + l_1) \bmod B = 1,$$
$$i^1(S, b) = (i^0(S, b) + l_1) \bmod B = 3,$$
$$i^2(S, b) = (i^0(S, b) + l_2) \bmod B = 2.$$

Therefore, the channel symbol index vector assigned to this state transition will be $[1, 3, 2]^T$.

In case of the repetition coded multiple ST code construction, the number of nonzero columns is multiplied by M (i.e. we have $M_i = M$ in (1) for $i = 0, 1, \ldots, \nu - 1$). Since the shortest error event is K state transitions long, the minimum value of ν is K. As a consequence, the multiple ST codes are guaranteed to achieve a diversity advantage of KLM. Given the trellis complexity (B^{K-1} states), this is the maximum achievable diversity advantage.

However, in the case of the quasi-static channel model, increasing the number of sent channel symbols per state transition does not increase the diversity of the system. Intuitively,

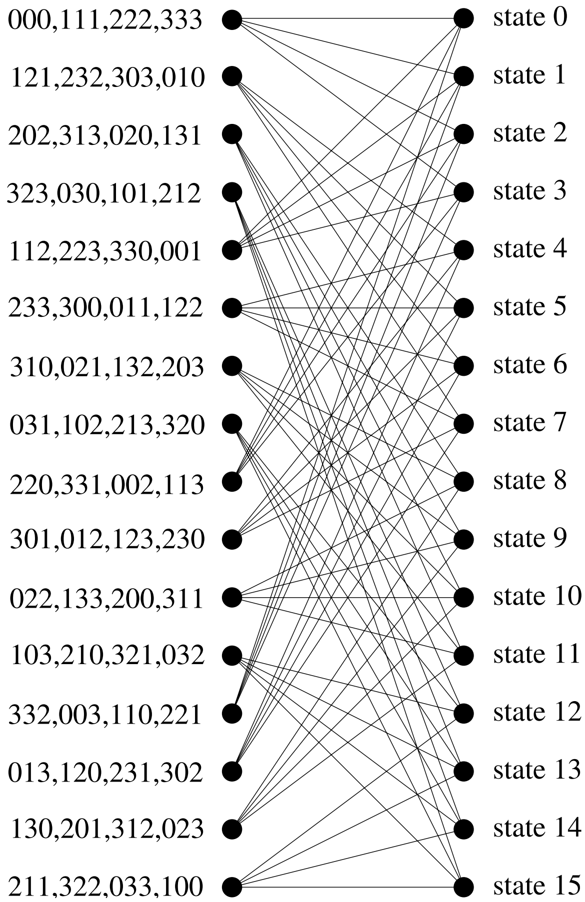

Fig. 1. Example ST code for 3 antennas, 4-ary modulation

the channel is assumed to be constant within a frame, so the channel variations cannot be exploited to increase the diversity order. From a mathematical prospective, the diversity advantage is governed by the rank of the channel symbol difference matrix. Increasing the multiplicity of the ST code will add new columns to the channel symbol difference matrix, but the number of rows, and therefore, the rank of the matrix will stay the same.

Increasing the multiplicity of the ST code also has some desirable properties from the viewpoint of decoding complexity. The computational effort to decode the traditional ST trellis codes using the Viterbi algorithm is exponential in the desired transmit diversity level. Furthermore, in some applications (for example: mobile, hand-held receiver), the number of receive antennas, and therefore, the achievable receive diversity, is limited. Since the decoding complexity is only linear in the multiplicity, increasing the number of channel symbols per state transition can be a reasonable alternative to increase the diversity advantage.

V. Variable Rate Space-Time Codes

In the previous section, it was shown how multiplicity can improve the performance of communication systems. However, this improvement does not come for free; we have to pay a price for it. If it is desirable to maintain the same SNR per source symbol at the receive antennas without bandwidth expansion and transmit power increase, increasing the number of channel symbols per state transition will decrease the effective source data rate. Moreover, practical considerations may also lead to the requirement that the channel symbol period be kept constant, and only the digital processing method should be changed. Therefore, if the channel symbol period is fixed, by using a ST code with a B-ary modulation and multiplicity M, the effective code rate becomes b_s/M source bits per channel symbol period. Equivalently, the spectral efficiency is reduced to b_s/M b/s/Hz. As a result, by changing the multiplicity, different tradeoffs between source rate and diversity order can be achieved.

This phenomenon is the driving force behind our variable rate ST code construction. By changing the multiplicity (M) of the ST code, we can provide a set of codes, each of them having different error correcting capabilities. The higher rate codes are embedded in the lower rate codes, so the same encoder and decoder (with minor modifications) can be used for communication at different data rates. This property is similar to that of the rate compatible punctured convolutional (RCPC) codes described in [9], but in our case, the puncturing is performed at the channel symbol level, as opposed to the bit level.

VI. Simulation Results

To illustrate the performance of the codes designed using the above described method, we present some simulation results. The curves show probability of bit error values as functions of the average signal to noise ratio (SNR) per source symbol. The simulated communication system had one receive antenna. The source symbols were transmitted in frames of length 130, and the Viterbi algorithm with decoding depth of 20 state transitions was used to decode the received signals. For each discrete time instant, the path gains between the transmit antennas and the receive antenna were modeled as independent, complex, zero mean, circularly symmetric Gaussian random variables with unit variance.

Figure 2 depicts the performance of the variable rate ST code designed for 2 transmit antennas and QPSK modulation. The high rate code ($M = 1$) can carry 2 bits per channel symbol period, while the low rate code has a throughput of 1 bit per channel symbol period. The figure clearly shows that the low rate code offers higher diversity advantage, even though both codes have 4 states.

The bit error curve of the variable rate ST code constructed for 3 transmit antennas and QPSK constellation is shown in Figure 3. These codes are based on the example code of Figure 1. The high rate ($M = 1$) and the low rate ($M = 2$) ST codes have spectral efficiencies of 2 b/s/Hz and 1 b/s/Hz, respectively. The tendencies observed here are similar to that of the previous figure.

The performance of the variable rate ST code designed for 3 antennas and 8PSK modulation is depicted in Figure 4. In this case, we simulated three multiplicities ($M = 1, M = 2$ and $M = 3$), resulting in 3 b/s/Hz, 1.5 b/s/Hz and 1 b/s/Hz spectral efficiencies. Finally, Figure 5 shows the simulation results in the case of 4 transmit antennas and 4ASK constellation. Both

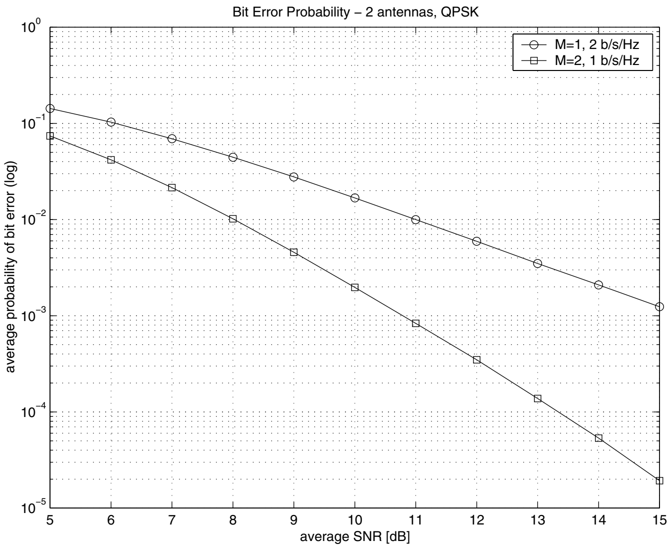

Fig. 2. Variable rate ST code for 2 antennas, QPSK

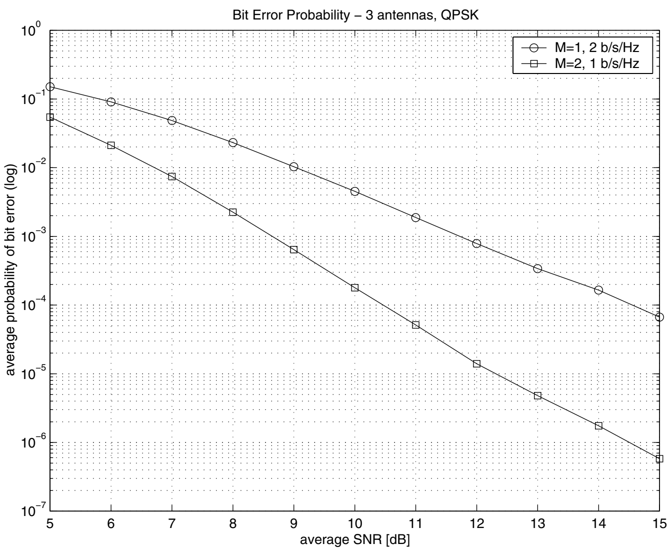

Fig. 3. Variable rate ST code for 3 antennas, QPSK

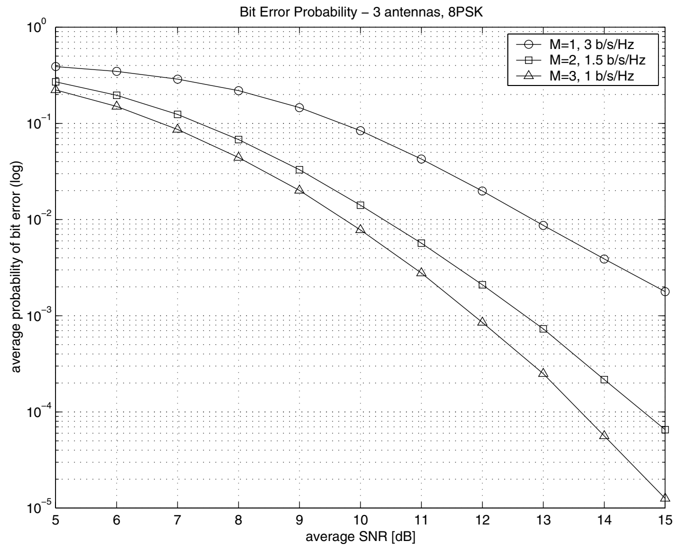

Fig. 4. Variable rate ST code for 3 antennas, 8PSK

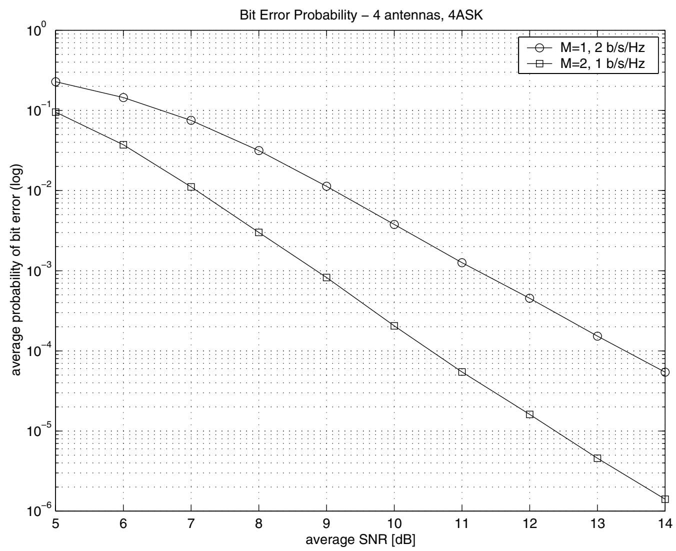

Fig. 5. Variable rate ST code for 4 antennas, 4ASK

figures demonstrate that ST codes with different rates (different multiplicities) achieve different diversity advantages.

VII. Conclusion

We described a systematic method to construct variable rate ST codes for an arbitrary number of transmit antennas and any memoryless constellation. By changing the number of output channel symbols per state transition, the ST codes can offer variable protection against the adverse effects of fading channels. The properties of the MTCM code construction can be exploited to achieve new tradeoffs between spectral efficiency, diversity and decoding complexity.

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