Performance Analysis of Space-Time Modulation over Time-Correlated Rayleigh Fading Channels

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Abstract — **In this paper, we address the problem of characterizing the performance of space-time modulation in time-correlated fading environment. We show that the space-time signals of square size achieving full diversity in quasi-static fading channels can also achieve full diversity in time-correlated fading channels, irrespectively of the time correlation matrix. We also generalize the notion of the diversity product to include the effect of temporal correlation.**

As most existing space-time coding and modulation schemes [1] were designed for ideal channel models, such as quasi-static and rapid fading, they may suffer serious performance degradation under more realistic channel conditions, such as correlated fading. The problem of code design for correlated fading channels was addressed in [2], and general performance criteria were derived for space-time-correlated Rayleigh fading channels. These general performance criteria were further simplified in [3], assuming that the space-time correlation matrix is of full rank. In this case, the design criteria simplified to the design criteria for rapid fading channels. In [4], characterizing the performance of space-time codes over correlated Rayleigh fading channels was also considered and the relationship between the achieved diversity order and the rank of the space-time correlation matrix was investigated.

In this work, we analyze the performance of space-time modulation in time-correlated fading environment. We consider a wireless communication system with M transmit antennas and N receive antennas. The space-time modulator divides the input bit stream into b bit long blocks, and for each block, it selects one space-time signal from the signal set of size $L = 2^b$. The selected signal is then transmitted through the channel over the M transmit antennas and T time slots. If c_t^i denotes the channel symbol transmitted by transmit antenna i ($i = 1, 2, \cdots, M$) at discrete time t ($t = 1, 2, \cdots, T$), each space-time signal can be expressed as a $T \times M$ matrix C, whose entry in the t-th row and the i-th column is given by c_t^i. The space-time signals are assumed to satisfy the energy constraint $E||C||_F^2 = MT$. We adopt a flat, Rayleigh fading channel model, where the channel coefficients are modeled as zero-mean, complex Gaussian random variables with unit variance. These coefficients are assumed to be known at the receiver, but unknown at the transmitter. We also assume that the channel fading has only temporal correlation, i.e., the channel coefficients are independent for different transmit or receive antenna pairs. In the sequel, we will use the notation $\Delta \triangleq (C - \tilde{C})(C - \tilde{C})^{\mathcal{H}}$ for two different space-time signals C and $\tilde{C}$, and R will stand for the $T \times T$ time correlation matrix of the channel coefficients, assuming that the time correlation is the same for all transmit and receive antenna pairs.

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Our first result establishes the maximum achievable diversity order given the earlier described channel model.

Theorem 1: For space-time signals of size $T \times M$ operating in time-correlated fading environment, the maximum achievable diversity is upper bounded by $\min(M\Gamma, T)N$, where Γ is the rank of the time-correlation matrix R.

The second result states that all of the space-time signals of square size designed for quasi-static channels (including orthogonal designs, cyclic codes, parametric codes and Cayley codes) can also be used for full-diversity transmission in arbitrary time-correlated fading environment.

Theorem 2: If a set of space-time signals of size $M \times M$ achieves full diversity (MN) for quasi-static fading channels, then it also achieves full diversity (MN) for any time-correlated fading channel, independent of the time correlation matrix R.

Assume that $\Delta \circ R$ is of full rank for any pair of distinct signals C and $\tilde{C}$. We define the generalized diversity product as

$$\zeta_R = \frac{1}{2\sqrt{M}} \min_{C \neq \tilde{C}} |\det(\Delta \circ R)|^{\frac{1}{2T}}$$

to incorporate the effects of the temporal correlation on the performance. The last result relates the generalized diversity product of the signal set to the diversity product for quasi-static channels, ζ_{static}, and the diversity product for rapid fading channels, ζ_{rapid}.

Theorem 3: If a set of space-time signals of size $T \times M$ has L elements, then ζ_R satisfies

$$\max\left\{\zeta_{static},\ |\det(R)|^{\frac{1}{2T}}\zeta_{rapid}\right\} \leq \zeta_R \leq \zeta_{rapid} \leq \sqrt{\frac{L}{2(L-1)}}.$$

The interested reader can find a more detailed discussion of these results in [5].

References

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