Performance Analysis of Space-Time Codes over Correlated Rayleigh Fading Channels

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***Abstract*— The potential for capacity increase in multi-antenna wireless communication systems has drawn considerable attention to space-time codes. However, most of the existing space-time code construction methods have assumed ideal channel models: either quasi-static fading or fast fading. In this work, we derive the performance criteria for space-time coded wireless communication systems taking into account both spatial and temporal channel correlation. We show that if the space-time correlation matrix is of full rank, the space-time code design problem for correlated channels can be reduced to the code design problem for fast fading channels. Some simulation results are also presented to support the theory.**

I. Introduction

In wireless communications, diversity techniques have been used extensively to improve the quality of transmission at high data rates. Space-time (ST) codes represent a combination of transmit diversity, modulation, and possibly forward error correction. The performance criteria for both quasi-static channels (the channel stays constant over one frame period) and fast fading channels (the channel changes independently from channel symbol period to channel symbol period) were derived in [1], characterizing the ST codes with two quantities: the diversity advantage, which describes the asymptotic error rate decrease as a function of the signal to noise ratio (SNR), and the coding advantage, which determines the vertical shift of the error performance curve.

Most of the existing ST code construction methods have assumed ideal channel models: either quasi-static fading or fast fading. For the quasi-static channel model, the authors of [1] proposed design rules for two transmit antennas to achieve the maximum diversity advantage. Later works [2], [11] described systematic trellis code design methods for an arbitrary number of transmit antennas.

The first ST trellis code construction method for the fast fading channel model was described in [3]. ST codes for 2 transmit antennas and QPSK modulation were designed using the idea of signal set partitioning. In [4], the design of ST codes for fast fading channels was also considered. The authors found ST codes for 2 transmit antennas and QPSK and 8PSK modulations through computer search. A systematic trellis code design method for fast fading channels was proposed in [12].

The problem of code design for correlated fading channels was addressed in [5]. In that work, it was assumed that for a communication system having K transmit antennas, the channel stays constant for K channel symbol periods. The performance criteria for this channel model were derived, and hand crafted ST codes were proposed for a small number of transmit antennas. In [6], the quasi-static channel model was adopted, and the achievable diversity level was analyzed as a function of the spatial correlation.

In this paper, we consider the problem of characterizing the performance of ST codes taking into account both spatial and temporal channel correlation. We derive the performance criteria for a channel model in which the channel changes from channel symbol period to channel symbol period in a correlated manner, assuming that the space-time correlation matrix is of full rank. We show that for this transmission scenario, the space-time code design problem for correlated channels can be reduced to the code design problem for fast fading channels. Moreover, we present some computer experiments to illustrate the theoretical results using a physical propagation model.

II. System Model and Notation

Consider a wireless communication system with K transmit and L receive antennas (the transmit antennas are indexed by $k,\ k \in \{0, 1, \ldots, K-1\}$, and the receive antennas are indexed by $l,\ l \in \{0, 1, \ldots, L-1\}$). The input bit stream is divided into b_s bit long blocks, forming B-ary ($B = 2^{b_s}$) source symbols. At discrete time t ($t = 0, 1, \ldots, T-1$), the ST encoder takes the current source symbol, b_t ($b_t \in \{0, 1, \ldots, B-1\}$), and outputs K B-ary channel symbol indices. We denote the channel symbol index for antenna k at time t by i_t^k. The channel symbol indices are mapped onto channel symbols (or constellation points) by the modulators and transmitted through the transmit antennas. In the sequel, $c(i)$ will represent the constellation point corresponding to channel symbol index i (For example, in case of B-ary PSK, $c(i) = \exp(j2\pi i/B)$, where $j = \sqrt{-1}$.). All the constellations are assumed to be normalized so that the average energy of the constellation is unity (if the channel symbols are equally likely). $c_t^k = c(i_t^k)$ will denote the constellation point output by antenna k at time t. We will also use the channel symbol vector, defined as: $\mathbf{c}_t = [\, c_t^0, \ldots, c_t^{K-1} \,]^T$.

The transmission medium is assumed to be a flat (frequency non-selective), correlated Rayleigh fading channel. $\alpha_{k,l}(t)$ will represent the path gain between transmit antenna k and receive antenna l at time t. These path gains are

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modeled as complex, zero mean, Gaussian random variables with unit variance, and are assumed to be known by the receiver. Based on these assumptions, after down-conversion, matched filtering and sampling, r_t^l, the received signal at receive antenna l at discrete time t, can be expressed as

$$r_t^l = \sum_{k=0}^{K-1} \sqrt{\frac{E_0}{K}} \alpha_{k,l}(t) c_t^k + z_t^l, \quad (1)$$

where E_0 is the average transmission energy per source symbol (each transmit antenna transmits with E_0/K average transmit energy). The receiver noise, denoted by z_t^l, is taken from samples of independent, complex, zero mean, Gaussian random variables with variance N_0. The average SNR per source symbol at receive antenna l will be defined as $SNR_l = E_0/N_0$.

Due to decoding errors, the receiver may decode a different sequence of channel symbols. The erroneously decoded channel symbol for transmit antenna k at time t will be denoted by $\hat{c}_t^k$, and the vector of decoded channel symbols at time t will be given by $\hat{\mathbf{c}}_t = [\,\hat{c}_t^0, \ldots, \hat{c}_t^{K-1}\,]^T$.

In the sequel, the notation $\mathrm{diag}(a_1, a_2, \ldots, a_N)$ will be used to represent a diagonal matrix with scalar elements $a_1, a_2, \ldots, a_N$ along the main diagonal. The entries in the vectors, and the rows and columns of the matrices will be indexed from 0. All vectors are assumed to be column vectors, unless mentioned otherwise.

III. Performance Criteria

In this section, we derive the performance criteria for space-time correlated Rayleigh fading channels. The criteria are based on an upper bound on the pairwise error probability [7], derived for a general transmission scenario in which the received signal vector can be expressed as

$$\mathbf{r} = \sqrt{\frac{E_0}{K}} \mathbf{\Gamma} \alpha + \mathbf{z}. \quad (2)$$

In (2), $\mathbf{\Gamma}$ denotes the matrix of sent channel symbols, α stands for the complex, zero mean, Gaussian path gain vector with correlation matrix $\mathbf{R} = E(\alpha\alpha^H)$, and $\mathbf{z}$ denotes the receiver noise vector consisting of complex, zero mean, independent, Gaussian random variables with variance N_0. It can be shown [7] that the probability that the maximum likelihood decoder erroneously decodes the channel symbol matrix $\hat{\mathbf{\Gamma}}$ if $\mathbf{\Gamma}$ was sent can be upper bounded as

$$P(\hat{\mathbf{\Gamma}}|\mathbf{\Gamma}) \leq \frac{\binom{2r-1}{r-1} \left(\frac{E_0}{KN_0}\right)^{-r}}{\Pi_{i=1}^r \gamma_i}, \quad (3)$$

where r and γ_i's are the rank and the nonzero eigenvalues of the matrix $\mathbf{\Delta R \Delta}^H$, respectively, and $\mathbf{\Delta}$ is the channel symbol difference matrix, defined as $\mathbf{\Delta} = \mathbf{\Gamma} - \hat{\mathbf{\Gamma}}$.

The performance criteria are obtained by evaluating (3) when the received signal is described by (1). Toward this end, we can define the matrices

$$\mathbf{\Gamma}^k = \mathrm{diag}(c_0^k, c_1^k, \ldots, c_{T-1}^k),$$
$$\mathbf{\Gamma_x} = [\,\mathbf{\Gamma}^0, \mathbf{\Gamma}^1, \ldots, \mathbf{\Gamma}^{K-1}\,],$$

and the row vectors

$$\mathbf{r}^l = [r_0^l, r_1^l, \ldots, r_{T-1}^l],$$
$$\alpha_{k,l} = [\alpha_{k,l}(0), \alpha_{k,l}(1), \ldots, \alpha_{k,l}(T-1)],$$
$$\mathbf{z}^l = [z_0^l, z_1^l, \ldots, z_{T-1}^l].$$

Using these quantities, the LT by 1 received signal vector

$$\mathbf{r} = [\mathbf{r}^0, \mathbf{r}^1, \ldots, \mathbf{r}^{L-1}]^T$$

is given by (2), with the LT by KLT channel symbol matrix $\mathbf{\Gamma} = \mathrm{diag}(\mathbf{\Gamma_x}, \mathbf{\Gamma_x}, ..., \mathbf{\Gamma_x})$, the LT by 1 noise vector $\mathbf{z} = [\,\mathbf{z}^0, \mathbf{z}^1, \ldots, \mathbf{z}^{L-1}\,]^T$, and the KLT by 1 path gain vector

$$\alpha = [\,\alpha_{0,0}, \alpha_{1,0}, \ldots, \alpha_{K-1,0}, \alpha_{0,1}, \alpha_{1,1}, \ldots$$
$$\ldots, \alpha_{0,L-1}, \alpha_{1,L-1}, \ldots, \alpha_{K-1,L-1}\,]^T.$$

The correlation matrix $\mathbf{R}$ has KLT rows and KLT columns, and it is assumed to be of full rank (i.e. its eigenvalues are real and positive). Defining the matrix

$$\hat{\mathbf{\Gamma}}^k = \mathrm{diag}(\hat{c}_0^k, \hat{c}_1^k, \ldots, \hat{c}_{T-1}^k),$$

the erroneously decoded channel symbol matrix, $\hat{\mathbf{\Gamma}}$, can be expressed similarly to $\mathbf{\Gamma}$, resulting in the LT by KLT channel symbol difference matrix $\mathbf{\Delta}$.

Assume that for τ time instants $t_0, t_1, \ldots, t_{\tau-1}$, the sent and the erroneously decoded channel symbol vectors are different, i.e. $\mathbf{c}_t - \hat{\mathbf{c}}_t \neq \mathbf{0}$ for $t \in \{t_0, t_1, \ldots, t_{\tau-1}\}$, and for the rest of the time instants, they are the same. Therefore, the sent and decoded channel symbol vectors corresponding to the times $t \notin \{t_0, t_1, \ldots, t_{\tau-1}\}$ will produce all zero rows and columns in the $\mathbf{\Delta}$ channel symbol difference matrix. These rows and columns can be eliminated from the analysis in the following way. For each $t \notin \{t_0, t_1, \ldots, t_{\tau-1}\}$, rows $t, t+T, t+2T, \ldots, t+(L-1)T$ and columns $t, t+T, t+2T, \ldots, t+(KL-1)T$ are removed from the matrix $\mathbf{\Delta}$, producing a new $L\tau$ by $KL\tau$ channel symbol difference matrix, $\mathbf{\Delta}'$. $\mathbf{\Delta}'$ has a structure similar to $\mathbf{\Gamma}$, but the matrices $\mathbf{\Gamma}^k$ are replaced with

$$\mathbf{\Delta}'^k = \mathrm{diag}(c_{t_0}^k - \hat{c}_{t_0}^k, c_{t_1}^k - \hat{c}_{t_1}^k, \ldots, c_{t_{\tau-1}}^k - \hat{c}_{t_{\tau-1}}^k).$$

Note that $\mathbf{\Delta}'$ has full row rank.

In addition, rows and columns $t, t+T, t+2T, \ldots, t+(KL-1)T$ must also be removed from $\mathbf{R}$, resulting in the $KL\tau$ by $KL\tau$ matrix $\mathbf{R}'$. Since only all zero rows and columns have been deleted from $\mathbf{\Delta}$, the nonzero eigenvalues of $\mathbf{\Delta R \Delta}^H$ and $\mathbf{\Delta' R' \Delta'}^H$ are the same. It is shown in the Appendix that the relation

$$\prod_{i=1}^{r} \gamma_i = \det(\mathbf{\Delta' R' \Delta'}^H) \geq \det(\mathbf{\Lambda}_{min}(L\tau)) \det(\mathbf{\Delta' \Delta'}^H) \quad (4)$$

holds, where $\mathbf{\Lambda}_{min}(L\tau)$ is a $L\tau$ by $L\tau$ diagonal matrix with the $L\tau$ smallest eigenvalues of $\mathbf{R}$ along the diagonal. Since $\mathbf{R}$ is positive definite, $\det(\mathbf{\Lambda}_{min}(L\tau))$ is strictly positive. Moreover, $\mathbf{\Delta'}$ has full row rank, so $\det(\mathbf{\Delta' \Delta'}^H)$ is also strictly positive. Consequently, the matrices $\mathbf{\Delta R \Delta}^H$ and $\mathbf{\Delta' R' \Delta'}^H$ are both of rank $L\tau$. Combining (3) with (4), and recognizing that

$$\det(\mathbf{\Delta' \Delta'}^H) = \prod_{i=0}^{\tau-1} ||\mathbf{c}_{t_i} - \hat{\mathbf{c}}_{t_i}||^{2L},$$

where $||\mathbf{x}|| = \sqrt{\mathbf{x}^H \mathbf{x}}$, we arrive at the upper bound

$$P(\hat{\mathbf{\Gamma}}|\mathbf{\Gamma}) \leq \left(\frac{E_0}{KN_0}\right)^{-L\tau} \binom{2L\tau-1}{L\tau-1} \cdot \frac{1}{\det(\mathbf{\Lambda}_{min}(L\tau))} \prod_{i=0}^{\tau-1} ||\mathbf{c}_{t_i} - \hat{\mathbf{c}}_{t_i}||^{-2L}. \quad (5)$$

The performance criteria now can be formulated to minimize the maximum value of $P(\hat{\mathbf{\Gamma}}|\mathbf{\Gamma})$:

1. Design for diversity advantage (distance criterion): The minimum number of time instants when the correct and the decoded channel symbol vectors are different (the minimum value of τ) taken over all possible correct and erroneously decoded channel symbol vector sequences must be maximized.

2. Design for coding advantage (product criterion): The minimum of the norm products

$$\delta = \prod_{i=0}^{\tau-1} ||\mathbf{c}_{t_i} - \hat{\mathbf{c}}_{t_i}||^2$$

taken over all possible correct and erroneously decoded channel symbol vector sequences must be maximized.

Note that the these performance criteria are the same as the performance criteria proposed for fast (independently) fading channels [1]. This is not surprising since in case of independent fading, the matrices $\mathbf{R}$, $\mathbf{R}'$ and $\mathbf{\Lambda}_{min}(L\tau)$ become identity matrices, and (5) simplifies to a form similar to the upper bound derived in [1].

In the above derivation, the matrix $\mathbf{R}$ was assumed to have full rank. If the magnitudes of the correlation values $E\left[\alpha_{k_1 l_1}(t_1)\alpha^*_{k_2 l_2}(t_2)\right]$ diminish fast enough as $|k_1 - k_2|, |l_1 - l_2|$, and $|t_1 - t_2|$ increase, this assumption will be true. This corresponds to the condition that the magnitude of the correlation decays fast enough as the transmit and receive antenna separation and the time separation increase. If this condition holds, the space-time code design problem for correlated channels can be reduced to the code design problem for fast fading channels. Moreover, the correlation only causes coding advantage loss, and it is possible to achieve full diversity advantage. Here we define full diversity as the level of diversity achievable by a communication system having K transmit and L receive antennas operating in fast (independent) fading environment. However, if this condition is not satisfied, the correlation matrix may become rank deficient, causing loss of diversity advantage. In this case, the analysis can be performed by deleting more rows and columns from $\mathbf{R}$ and $\mathbf{\Delta}$.

IV. Simulation Results

To illustrate the above analytical results, we performed some computer simulations. In this section, we present the bit error rate curves of the ST trellis codes of [3], [4] and [12] designed for fast fading channels, and the ST trellis codes of [8] designed for quasi-static fading channels. All of these codes have very good performance in fast fading environment [12]. The symbol N will denote the number of encoder states.

The source symbols were transmitted in frames of length 130, and the Viterbi algorithm with decoding depth of 20 state transitions was used to decode the received signals. For the fast fading channel model, the path gains between the transmit and the receive antennas were independent, complex, zero mean, Gaussian random variables with unit variance at each discrete time instant.

In the correlated fading case, the path gains were generated according to the statistical model described in [13]. The base station (BS) was the transmitter and the mobile terminal (MT) was the receiver. Both the BS and the MT were assumed to have a uniform, linear array of isotropic antennas, and the MT was surrounded by a ring of scatterers. The model parameters were: d_B - BS antenna separation, d_M - MT antenna separation, D - distance between the BS and the MT, R - radius of the scatterer ring, N_s - number of scatterers, β - direction of the BS antenna array, γ - direction of the MT antenna array, σ - direction of the MT movement, v - the magnitude of the MS speed, f_c - the carrier frequency (or λ_c - the carrier wavelength), and T_s - the channel symbol period.

The ith ($i = 0, 1, \ldots, N_s - 1$) scatterer was at an angle θ_i from the middle point of the MT antenna array. For each frame, the scatterer angles were randomly generated in the range $[-\pi, \pi]$ with uniform distribution. The effect of scatterer i was modeled as multiplication of the incident signal by a scattering coefficient S_i. The scattering coefficients were modeled as independent, complex, zero mean, Gaussian random variables with variance $1/N_s$.

During the simulations, we used the following parameter values: $D = 1\text{km}$, $R = 20\text{m}$, $N_s = 20$, $\beta = \frac{3\pi}{4}$ rad,

$\gamma = \frac{\pi}{4}$ rad, $\sigma = \frac{3\pi}{4}$ rad, and $v = 70\text{km/h}$. Three cases were considered: (a) high correlation ($T_s = 50\mu$s, $f_c = 900$MHz, $d_B = 5\lambda_c$, $d_M = 0.6\lambda_c$), (b) low correlation ($T_s = 500\mu$s, $f_c = 2$GHz, $d_B = 25\lambda_c$, $d_M = 5\lambda_c$), and (c) fast fading (no correlation). Note that the value of d_M is significant only if the MT has multiple receive antennas.

Figure 1 depicts the performance of the ST code designed by the method described in [12] for 2 transmit antennas and QPSK constellation ($K = 2$, $B = 4$, $N = 4$) with 1 receive antenna. The bit error rate curves for the same code with 2 receive antennas are shown in Figure 2. Both curves demonstrate that the spatio-temporal correlation has a significant impact on the performance. Moreover, it can be observed that in the low correlation case, the bit error probability curve becomes approximately parallel to the fast fading bit error probability curve at high SNR. Therefore, they achieve the same diversity level, validating our analysis.

Figure 3 compares the performance of the ST trellis codes from [3], [4], [12] and[8] constructed for a 2 transmit antenna system and QPSK modulation ($K = 2$, $B = 4$, $N = 4$) with 1 receive antenna. It is observed that all codes have essentially the same performance, with the ST code from [8] being a little better in the high correlation case and being a little worse in the fast fading case. Note that the ST code of [12], the result of a systematic code design method, achieves the same performance as the ST code of [4], which was found by exhaustive search.

The bit error rate curves for 3 transmit antennas and QPSK modulation ($K = 3$, $B = 4$, $N = 16$) with 1 receive antenna are depicted in Figure 4. The ST code given in [12] is compared to the ST code described in [8]. The performance of the two codes is almost identical, and the bit error curves for the fast fading channel model and for the low correlation channel model are approximately parallel at high SNR. The ST code of [8], also found by computer search, performs slightly better in the high correlation case, which is expected since this code was designed for quasi-static channels.

V. Conclusion

We derived the performance criteria for correlated Rayleigh fading channels, assuming that the space-time correlation matrix has full rank. The obtained performance criteria imply that in this case, the space-time code design problem for correlated channels can be reduced to the code design problem for fast fading channels. Moreover, the maximum achievable diversity is the same as the achievable diversity over fast (independently) fading channels, and the channel correlation only causes coding advantage loss.

Appendix

For simplicity, we assume that $\mathbf{R}$ is p by p, $\mathbf{R}'$ is m by m, and $\mathbf{\Delta}'$ is n by m, with $p \geq m \geq n$. In our case, $p = KLT$, $m = KL\tau$ and $n = L\tau$. Let us denote the positive and real eigenvalues of $\mathbf{R}$ by $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_p$. Using the singular value decomposition, $\mathbf{\Delta}'$ can be expressed as $\mathbf{\Delta}' = \mathbf{X}[\mathbf{\Sigma}\ \mathbf{0}]\mathbf{Y}^H$, where $\mathbf{X}$ is an n by n unitary matrix, $\mathbf{Y}$ is an m by m unitary matrix, $\mathbf{\Sigma}$ is an n by n diagonal matrix with the singular values along the diagonal, and $\mathbf{0}$ is an n by $(m-n)$ zero matrix. The matrix $\mathbf{R}'$ admits the spectral decomposition $\mathbf{R}' = \mathbf{U}\mathbf{\Lambda}'\mathbf{U}^H$, with an m by m unitary matrix $\mathbf{U}$, and a diagonal matrix $\mathbf{\Lambda}' = \text{diag}(\lambda'_1, \lambda'_2, \ldots \lambda'_m)$. The quantities $\lambda'_1 \geq \lambda'_2 \geq \ldots \geq \lambda'_m$ are the real eigenvalues of $\mathbf{R}'$. We can define $\mathbf{Z}$, the m by m unitary matrix, as $\mathbf{Z} = \mathbf{Y}^H\mathbf{U}$, and partition $\mathbf{Z}$ into an n by m matrix $\mathbf{Z_1}$, and an $(m-n)$ by m matrix $\mathbf{Z_2}$ as $\mathbf{Z} = \begin{bmatrix}\mathbf{Z_1}\\ \mathbf{Z_2}\end{bmatrix}$. The matrix $\mathbf{Q} = \mathbf{Z}\mathbf{\Lambda}'\mathbf{Z}^H$ will have the same eigenvalues as $\mathbf{R}'$. If $\mathbf{Q}$ is partitioned as $\mathbf{Q} = \begin{bmatrix}\mathbf{Q_{11}} & \mathbf{Q_{12}}\\ \mathbf{Q_{21}} & \mathbf{Q_{22}}\end{bmatrix}$, where $\mathbf{Q_{11}} = \mathbf{Z_1}\mathbf{\Lambda}'\mathbf{Z_1}^H$ is an n by n principal submatrix of $\mathbf{Q}$, $\mathbf{\Delta}'\mathbf{R}'\mathbf{\Delta}'^H$ can be expressed as

$$\mathbf{\Delta}'\mathbf{R}'\mathbf{\Delta}'^H = \mathbf{X}\mathbf{\Sigma}\mathbf{Q_{11}}\mathbf{\Sigma}^H\mathbf{X}^H. \tag{6}$$

Since $\mathbf{\Delta}'$ has full row rank, the matrix $\mathbf{\Sigma}$ has full rank. Using Fisher's inequality [9], it can be easily verified that $\mathbf{Q_{11}}$ also has full rank. Moreover, all matrices on the right hand side of (6) are n by n. As a consequence, we have the relationship

$$\begin{aligned}\det(\mathbf{\Delta}'\mathbf{R}'\mathbf{\Delta}'^H) &= \det(\mathbf{Q_{11}})\det(\mathbf{\Sigma}\mathbf{\Sigma}^H)\det(\mathbf{X}\mathbf{X}^H)\\ &= \det(\mathbf{Q_{11}})\det(\mathbf{\Delta}'\mathbf{\Delta}'^H). \end{aligned} \tag{7}$$

To obtain a lower bound on $\det(\mathbf{Q_{11}})$, we use Cauchy's interlacing theorem [10] (also known as the inclusion principle [9]), stated as follows: Let $\mathbf{Q}$ be an m by m Hermitian matrix with real eigenvalues $\lambda'_1 \geq \lambda'_2 \geq \ldots \geq \lambda'_m$. Furthermore, let $\mathbf{Q_{11}}$ be an n by n $(m \geq n)$ principal submatrix of $\mathbf{Q}$, with real eigenvalues $\mu_1 \geq \mu_2 \geq \ldots \geq \mu_n$. Then we have

$$\lambda'_i \geq \mu_i \geq \lambda'_{m-n+i}, \quad \text{for } i = 1, 2, \ldots, n.$$

Moreover, since $\mathbf{R}'$ is a principal submatrix of $\mathbf{R}$, we can apply Cauchy's interlacing theorem to obtain

$$\lambda_i \geq \lambda'_i \geq \lambda_{p-m+i}, \quad \text{for } i = 1, 2, \ldots, m.$$

Therefore, if we form the diagonal matrix $\mathbf{\Lambda}_{min}(n)$ from the n smallest eigenvalues of $\mathbf{R}$ (i.e. $\mathbf{\Lambda}_{min}(n) = \text{diag}(\lambda_{p-n+1}, \lambda_{p-n+2}, \ldots, \lambda_p)$), we obtain the bound

$$\det(\mathbf{Q_{11}}) \geq \det(\mathbf{\Lambda}_{min}(n)). \tag{8}$$

Note that (8) also shows that $\mathbf{Q_{11}}$ has full rank. Finally, combining (7) with (8) yields (4).

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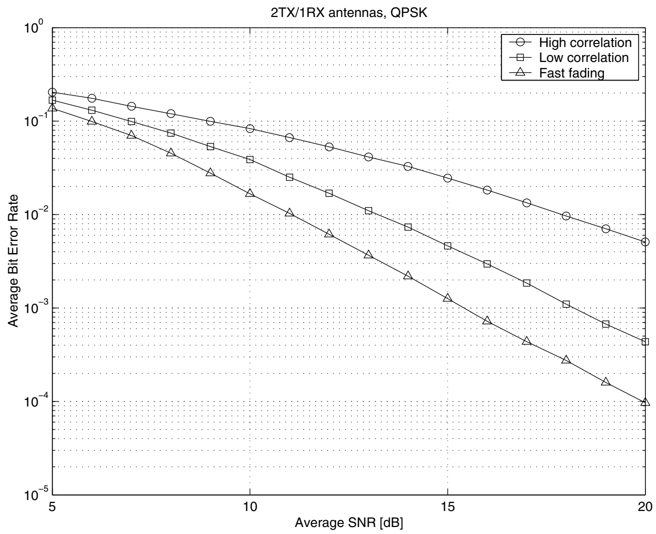

Fig. 1. ST code for 2 antennas, QPSK

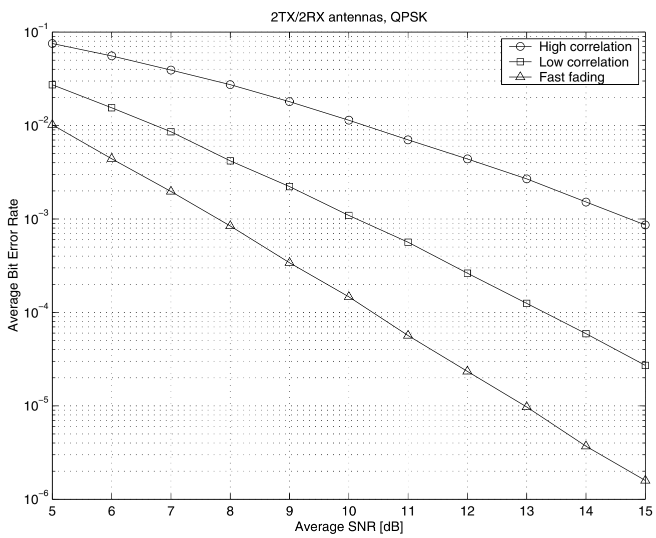

Fig. 2. ST code for 2 antennas, QPSK

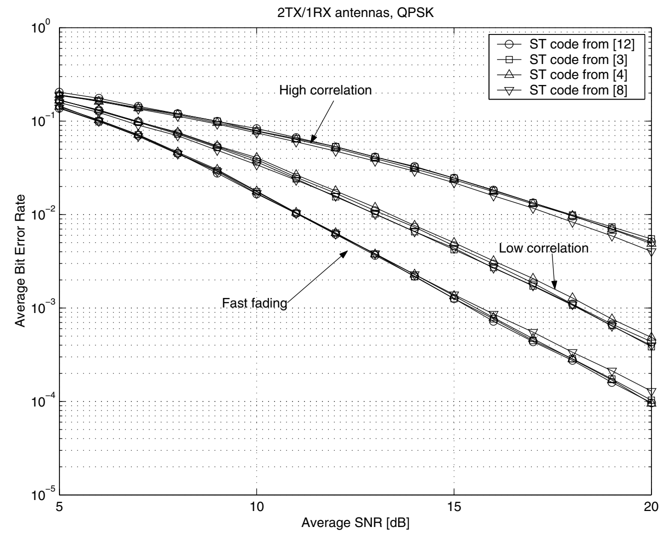

Fig. 3. ST codes for 2 antennas, QPSK

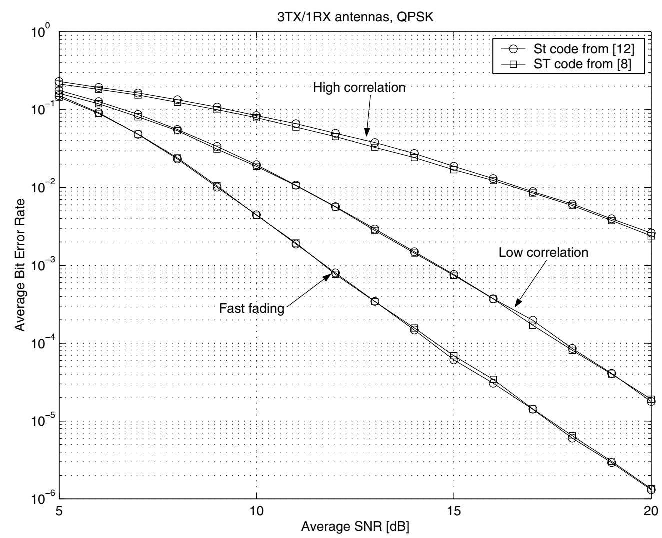

Fig. 4. ST codes for 3 antennas, QPSK

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