Blocking Probability of Handoff Calls and Carried Traffic in Wireless Networks with Antenna Arrays

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Abstract

In this paper we consider a wireless network with beamforming capabilities at the receiver. We derive the blocking probabilities of the calls in the system under different traffic policies for such a network. For a set of co-channel transmitters, their success probabilities for being captured by separate antenna beams are computed. These success probabilities are taken into account in the queueing model of the system. Our analytical and also numerical results show that adaptive beamforming at the receiver indeed reduces the blocking probability of the calls and increases the total carried traffic *in the system.*

1. Introduction

In a cellular wireless communication network, as a mobile user crosses the boundries of cells (coverage area of each base station) the call must be handed over to the new base station (**BS**) that is the closest. This operation is called a *handoff* and ideally goes unnoticed by the customer [1]. An important problem arising from the mobility of the users in a cellular system is the possible blocking of a handoff call [1, 2, 3]. Namely, we can have a call that was initiated in one cell in the system and moves into a new cell where all the channels (servers) happen to be already occupied by other calls. This entering call will then have to be cleared from the system involuntarily, which is obviously an unpleasant situation for the customer, and it is called *forced termination of a call.* Such an event results in a disconnection in the middle of a call, which is highly undesirable. The increasing popularity of wireless communication services together with the limited amount of the available radio spectrum calls for highly effecient usage of resources (traffic channels) in the system. The interference reduction capability of antenna arrays have been considered as a means to increase the capacity of wireless systems [4]-[10]. Specifically, using beamforming techniques at the receiver, two or more transmitters can share the same traffic channel to communicate with the base station at the same time.

In this paper, We consider a wireless network with beamforming capabilities at the receiver. We derive the blocking probabilities of the calls in the system under different traffic policies for such a network. We model each cell by a multiuser/multiserver service facility, with servers being the traffic channels assigned to the cell. Two types of arrivals are distinguished, corresponding to handoff calls (calls already in progress that enter the cell) and originating calls (calls initiated inside the cell). The queueing system used, assumes *Poisson* distributed arrivals with different rates for the two types of customers. For a set of co-channel transmitters, their success probabilities for being captured by separate antenna beams are computed. These success probabilities are taken into account in the queueing model of the system and from this generalized model we derive closed form solutions for blocking probabilities of the calls and total carried traffic in the system under different traffic policies. We show that adaptive beamforming at the receiver reduces the *blocking probability* of the calls and increases the *total carried traffic* in the system.

2. Approach

2.1. Review of Spatial Multiplexing Using Adaptive Beamforming

A set of J co-channel transmitters is considered. Let E_i denote the received power from the desired transmitter, and E_j for $j = 1, 2, \cdots, J$ and $j \neq i$ denote the interference power due to $J-1$ cochannel transmitters. The carrier to interference ratio (CIR) is defined as

$$\Gamma_i = \frac{E_i}{\sum_{j=1, j\neq i}^{J} E_j + \frac{N_0}{2}} \tag{1}$$

where $\frac{N_0}{2}$ is the noise power. The desired signal will be captured (with acceptable quality) if $\Gamma_i \geq \gamma$. An antenna

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array consisted of M elements is considered at the receiver. We shall use adaptive beamforming capabilities of antenna arrays to maintain a unity gain for the signal along the direction of interest and adjusting the nulls so as to reject the cochannel interference. The beamformer tries to minimize the output power subject to maintaining a distortionless response in the direction of interest such that

$$\mathbf{w}_i^H \mathbf{a}_i(\theta_i) = 1$$

. It can be shown that maximum CIR for signal of interest is

$$\Gamma_{i,\,max} = \frac{P_i G_i}{\sum_{j=1,\, j\neq i}^{J} P_j G_j \hat{\mathbf{w}}_i^H \mathbf{a}_j(\theta_j) \mathbf{a}_j^H(\theta_j) \hat{\mathbf{w}}_i + \frac{N_0}{2} \hat{\mathbf{w}}_i^H \hat{\mathbf{w}}_i} \qquad (2)$$

which is achieved by optimum weight vector given by $\hat{\mathbf{w}}_i = \frac{\Phi^{-1}\mathbf{a}_i(\theta_i)}{\mathbf{a}_i^H(\theta_i)\Phi^{-1}\mathbf{a}_i(\theta_i)}$, where P_i is the power of the i^{th} transmitter, $\mathbf{\Phi} = \sum_{j=1,\, j\neq i}^{J} P_j G_j \mathbf{a}_j(\theta_j)\mathbf{a}_j^H(\theta_j) + \frac{N_0}{2} I$, G_i is the link gain between the i^{th} transmitter and the base station, and $\mathbf{a}(\theta_i)$ is the array response to the signal arriving from direction θ_i. The *spatial information* of the transmitters is used to discriminate them. This provides a means of *spatial multiplexing* of the signals on the top of the other multiplexing schemes such as *FDMA/TDMA/CDMA*.

2.2. Network With Adaptive Array under Guard Channels Traffic Policy

In this section we analyze the network with adaptive array under traffic policies proposed in [3]. In our network model, we assume that when a new call (hand off or originating call) arrives, the adaptive array points one beam toward that user and assigns one channel out of those L channels to that user. Each channel c_i *for* $i = 1, 2, \cdots, L$ can be assigned to K users by K separate beams, using K beamfomer (section 3.1) in parallel for each channel, as it is shown in Figure ??. If the first beam of all channels are already occupied by one user (L users are in the system) the new call can be assigned by another separate beam to any of the L channels. If we assume that $(i-1)$ cochannel transmitters successfully share the same channel, for acceptable link quality, the newly arrived i^{th} transmitter is allowed to share that channel with them if $\Gamma_i \geq \gamma$, where Γ_i is given by (2). To avoid any degradations in system performance, using a *call admission control (CAC)* mechanism, if $\Gamma_i < \gamma$ or $i > K$ we prevent that user from being accepted into the system. Now we wish to compute the probability of the event $\Gamma_i \geq \gamma$ in the system. This would give us the success probability $p_{i+1|i,M}$ that $(i+1)^{th}$ transmitter can share the same channel, given that i transmitters are already using that channel provided M antenna elements at the receiver. The Monte Carlo simulation results for computing the success probabilities $p_{3|2,M}$, success probabilities for 3-beam adaptive arrays, (with M antenna elements) is shown in Figure 1.

So far we have computed the success probabilities $p_{i+1|i,M}$ (where M denotes the number of antenna elements). Since there are L distinct channels in the system and each channel may be reused up to K times we shall define *probability of successful reception of* $(n+1)^{th}$ *user into the system given that there are already n user in the system*

$$\begin{aligned} & q_{N_{t+1}|N_t}(N_{t+1} = n+1 \mid N_t = n) \\ = \; & Pr\{(n+1)^{th}\ user\ is\ successfully\ accepted\ into\ the \\ & system \mid n\ users\ were\ already\ being\ served\} \end{aligned}$$

where N_t is the number of users in the system at time t before a new call (user) arrives into the system (t is the time index which increases by one at each epoch corresponding to a new handoff or originating call). For a K-beam adaptive array system, we have shown that success probabilities $q_{n+1|n}$ are

$$\begin{aligned} q_{n+1|n} \;=\; & \sum_{\{x_2 \cdots x_K | x_K \leq \cdots \leq x_2\}} \hat{q}(n+1, x_2, \cdots, x_K) \\ & p[(x_2, \cdots, x_K) \mid x_2 + \cdots + x_K = n - L] \qquad (3) \end{aligned}$$

where $\hat{q}(n+1, x_2, \cdots, x_K) = 1 - (1 - p_{2|1,M})^{(L-x_2)} \times (1 - p_{3|2,M})^{(x_2 - x_3)} \times \cdots \times (1 - p_{K|K-1,M})^{(x_{K-1} - x_K)}$, and $(x_2, x_3, \cdots, x_K)$ denotes the vector representing the number of calls assigned to the channels in the second beam up to the K^{th} beam. Let $\alpha = \lambda + \gamma$, $\mu = \eta + \nu$, $a = \frac{\alpha}{\mu}$, $b = \frac{\gamma}{\mu}$, $c = \frac{\lambda}{\mu}$, where γ and η are arrival rate and service rate for originating calls, λ and ν are arrival rate and service rate for hand off calls respectively. In the network with adaptive array the effective arrival rate into the cell at state n, $\lambda_{eff}(n)$, is

$$\lambda_{eff}(n) = \begin{cases} \alpha q_{n+1|n} & n = 0, 1, \cdots, KL - g - 1 \\ \lambda q_{n+1|n} & n = KL - g, \cdots, KL - 1. \end{cases} \qquad (4)$$

Similarly, the effective service rate in the system at a given state n, $\mu(n) = n\mu \quad n = 1, 2, \cdots, KL$. Figure 2 illustrates the queuing model of the system with adaptive arrays under guard channel traffic policy. Using this model, the state probabilities, $P(n)$ $n = 0, 1, \cdots, KL$, (where n is the number of ongoing calls in the cell) are derived [11, 12] and from there the blocking probability B_O of originating calls and handoff calls blocking probability B_H are given by:

$$\begin{aligned} B_O \;=\; & \Bigg[\sum_{n=L}^{KL-g-1} (1 - q_{n+1|n}) \frac{a^n}{n!} \prod_{j=0}^{n-1} q_{j+1|j} \\ & + a^{KL-g} \sum_{n=KL-g}^{KL} \frac{c^{n-(KL-g)}}{n!} \prod_{j=0}^{n-1} q_{j+1|j} \Bigg] P(0) \\ B_H \;=\; & \Bigg[\sum_{n=L}^{KL-g-1} (1 - q_{n+1|n}) \frac{a^n}{n!} \prod_{j=0}^{n-1} q_{j+1|j} \end{aligned}$$

$$+ \sum_{n=KL-g}^{KL-1} (1 - q_{n+1|n}) \frac{a^{KL-g} c^{n-(KL-g)}}{n!} \prod_{j=0}^{n-1} q_{j+1|j} + \frac{a^{KL-g} c^{g}}{(KL)!} \prod_{j=0}^{KL-1} q_{j+1|j} \Bigg] P(0) \quad (5)$$

2.3. Network with Adaptive Array under Queueing of Handoff Calls Policy

In this section, we investigate other traffic policies that further decrease the blocking probability of handoff calls at the expense of slightly increasing the blocking probability of originating calls [3]. We first assume that infinite queues are allowed for handoff calls. In a network that infinite queues are allowed for handoff calls, call blocking probabilities B_O and B_H are again computed and it is shown that the *probability that a handoff call is being delayed* (due to queuing) is given by

$$P_H(> 0) = \frac{a^{KL-g} c^{g}}{(KL-1)!(KL-c)} \prod_{j=0}^{KL-1} q_{j+1|j} P(0) \quad (6)$$

. Also the probability that a hand off call waits more than a certain time t is given by

$$\begin{aligned} P_H(> t) &= \frac{a^{KL-g} c^{g}}{(KL-1)!(KL-c)} e^{-\mu t(KL-c)} \prod_{j=0}^{KL-1} q_{j+1|j} P(0) \\ &= e^{-\mu t(KL-c)} \times P_H(> 0) \end{aligned} \quad (7)$$

From (7) the *average delay* W_H of handoff calls is $W_H = \frac{1}{\mu(KL-c)} \times P_H(> 0)$ and the *average delay of handoff calls that actually do experience a delay* is given by $D_H = \frac{1}{\mu(KL-c)}$. The same quantities are derived for a network with adaptive array under finite queuing of handoff calls traffic policy. Due to the space limitations, reporting of these results in this summary is omitted.

3. Numerical Results

In this section we present the numerical results to show the effectiveness of the network with adaptive arrays from communication traffic point of view. We evaluate the performance of the system with parameters drawn from [3]. We choose a cell with $L = 44$ channels with a total offered traffic $a = 40$ Erlangs (heavy traffic), and a handoff traffic $c = 8$ Erlangs. The blocking probabilities for handoff calls B_H, for different number of antenna elements in a 2-beam (3-beam) adaptive array system are plotted in Figure 3. This figure illustrates that for a given SNR and threshold γ, as the number of antenna elements M increases, the B_H decrease. Of course because of array limitations the blocking probability B_H in a 2-beam system ($K = 2,\ L = 44$) is slightly higher than the B_H for the system with $K = 1,\ L = 88$. These numerical results confirm that by using a 2-beam adaptive array in the limit by increasing the number of antenna elements we almost get the same effect as doubling the total available channels in the system. Figure 4 illustrates the blocking probabilities of handoff calls versus number of antenna elements M. From this figure we can see as M increases B_H decreases rapidly. An important consequence of deploying adaptive arrays at the base station is the significant improvement in the total carried traffic (actual traffic that goes through) in the system. It means that network with adaptive array can effectively handle higher offered traffic intensities a, compared to the network with single omnidirectional antenna. Higher carried traffic means smaller number of blocked calls. It is therefore highly desirable to maintain high total carried traffic in the network. The total carried traffic in the system can be easily derived as follows

$$C \;=\; (a - c)(1 - B_O) + c(1 - B_H) \quad (8)$$

where C is the total carried traffic in the system, a is the total offered traffic and c is the total hand off traffic in the system. Figure 4, illustrates the effect of adaptive array on the total carried traffic in the system. The total carried traffic for different number of antenna elements M in a 3-beam system are plotted in Figure 4.

4. Conclusion

A wireless communication network with beamforming capabilities at the base station was considered and the blocking probabilities of the calls in the system under different traffic policies for such a network was derived. The usefulness of adaptive arrays for improving the traffic characteristics of the network was proved through analytical as well as numerical results. Our analytical and also numerical results show that adaptive beamforming at the receiver reduces the *blocking probability* of the calls and increases the *total carried traffic* in the system. The result of our analysis confirms that wireless networks with adaptive arrays are very promising in terms of traffic improvements in the network.

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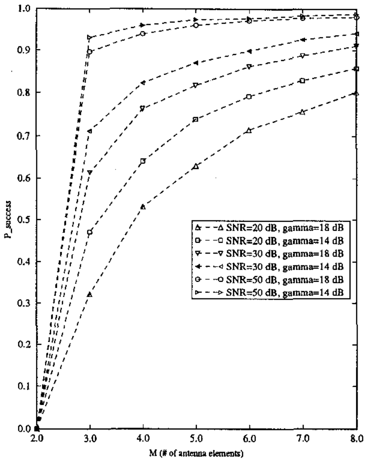

Figure 1. Success probability $p_{3|2,M}$ for a 3-beam adaptive array for different values of M (# of antenna elements), SNR and threshold γ.

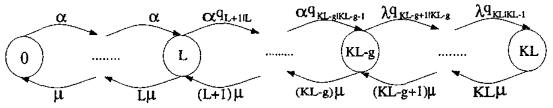

Figure 2. State transition diagram of discrete Markov chain, the network model of the system with an adaptive array at the base station and guard channels.

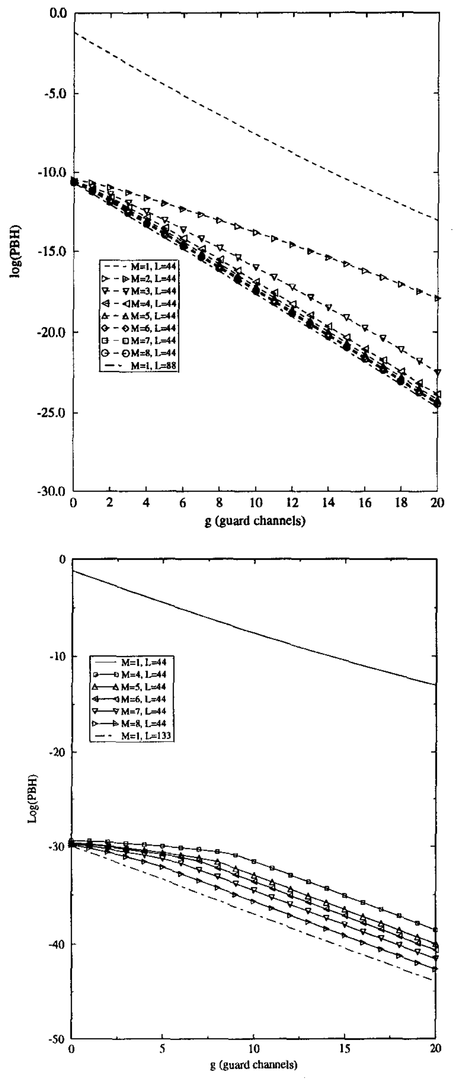

Figure 3. Blocking probabilities of hand off calls (B_H) for different number of antenna elements M with $SNR = 30$, $\gamma = 14\ dB$, $a = 40$, $c = 8$ $\beta = 4$ in a 2-beam (top) 3-beam adaptive array system.

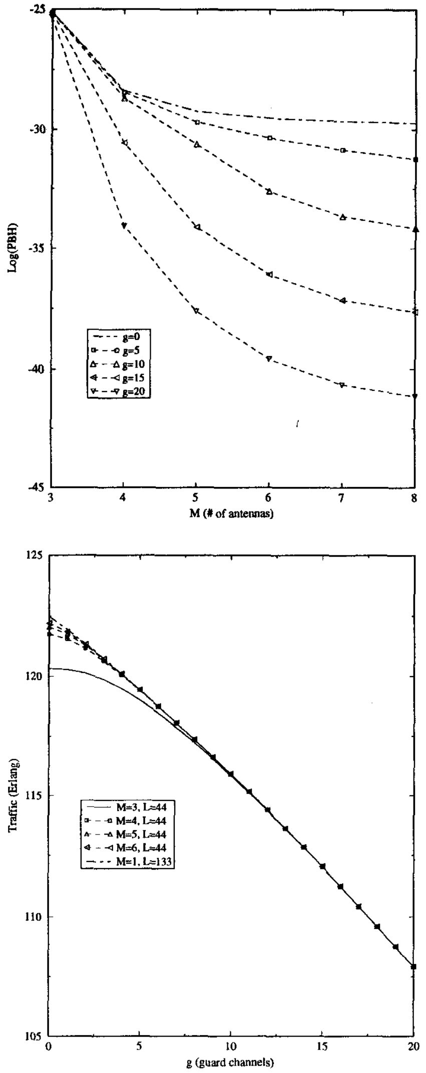

Figure 4. (Top) Blocking probabilities B_H versus number of antenna elements M, (bottom) Total carried traffic C for different number of antenna elements M with $SNR = 20$, $\gamma = 18\ dB$, $a = 130$, $c = 8$ (Erlangs) in a 3-beam adaptive array system.