Rate Control for DS-CDMA Wireless Systems Using Power Control and Orthogonally Coded Substreams

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Abstract— *There is an increase of multimedia applications in wireless networks. The increase in data transmission requirements for some multimedia applications poses a problem for the service providers who must overcome the high interference environment to provide this higher transmission rate. One method to service multimedia users is by splitting their data streams into separately coded substreams. We propose an algorithm to choose the optimal number of substreams for each multimedia user, and to use power control techniques to control the rate of each substream.*

I. Introduction

Consider a wireless service network where users with multimedia services are flagged and targeted for rate increase. We study a DS-CDMA system with automatic power control, which is used to control the signal to interference and noise level (SINR) for each user [1], [2]. In addition, we take each multimedia user's data stream and divide it into smaller data streams, which are independently coded with short orthogonal pseudo random sequences [3].

Each user may have a different number of substreams and each substream is treated as an independent user, with its own power assignment and channel capacity. We use power control to adjust the capacity for each substream. The goal of our method is to maximize the capacity for each user, by choosing how many substreams each must is assigned, and what is the SINR level for each substream of each user.

Given that the wireless channel has a certain level of interference and noise, the set of allowable SINR levels is limited to be below some certain *feasible* levels. The algorithms presented in this paper try to find the highest feasible SINR levels, given that each user may have several substreams. The system we work with uses power control and space-time diversity [4].

It can be shown that finding the globally optimum SINR levels is an intractable problem [5], however there are algorithms that find suboptimal solutions to the quality allocation problem [6]. With this allocation, our algorithm find the number of substreams per user and the channel rate for each substream that maximizes the overall capacity.

A multimedia user will normally have a variety of data types that can be sent in separate substreams with different rates (video, audio, or data). Each substream should, therefore, be constrained to perform within certain SINR constraints. We place special emphasis on the choice of SINR levels allowed to each substream, since this has a marked effect on the overall solution.

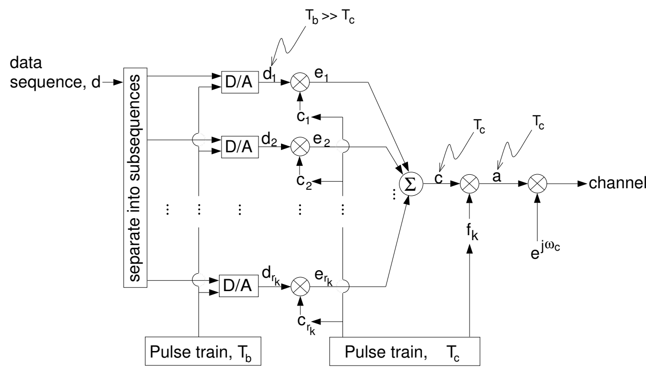

Fig. 1. Proposed transmitter for user k. There are r_k substreams. Substream d_j is spread with a SOPRaS code, c_j. All substreams are then added and then spread with the user spreading code, f_k.

II. System Model

The proposed transmitter for our system would look like Fig. 1. Each user, k, has r_k substreams. Each data stream, i, of user k is spread with a SOPRaS code, c_i, for $i = 1, \cdots, r_k$. The SOPRaS spread substreams are then added together and spread with user k's pseudo-random spreading sequence, f_k. The output of that is then modulated and transmitted. Thus, the signal before modulation for user k is:

$$s_k(t) = \sqrt{P_k} \sum_{i=1}^{r_k} d_{k,i}(t) c_i(t) f_k(t) \ . \qquad (1)$$

The proposed receiver can be seen in Fig. 2. The base station has an antenna array, and RAKE receivers for space-time diversity. The received signal, $\mathbf{x}(t)$, is the addition of all multipaths from all users and noise which is assumed to be white and Gaussian:

$$\mathbf{x}(t) = \sum_{k=1}^{M} \sum_{l=1}^{L_k} s_k(t - \tau_{k,l}) G_{k,l} \mathbf{a}_{k,l} e^{-j\omega_c \tau_{k,l}} + \mathbf{n}(t) \ , \qquad (2)$$

where $G_{k,l}$, $\mathbf{a}_{k,l}$, and $\tau_{k,l}$ are the pathloss, array response vec-

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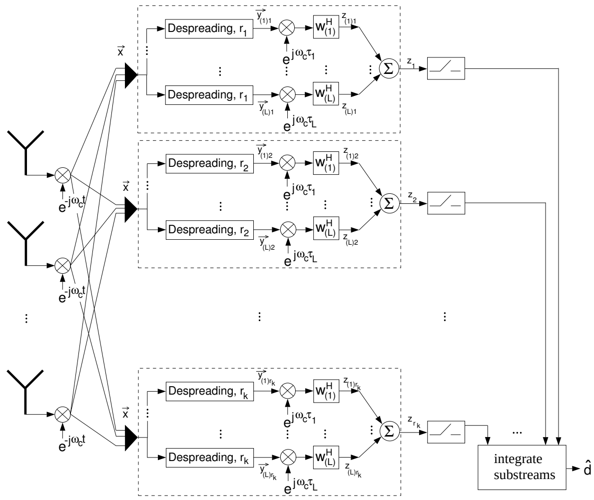

Fig. 2. Proposed receiver. The antenna array has D elements. Each vector portrayed before and after the spreading boxes, $\mathbf{x}_{(j)i}$ and $\mathbf{y}_{(j)i}$, are D dimensional. Each dotted box represents one of users k's r_k substreams. Each substream box has L multipath fingers.

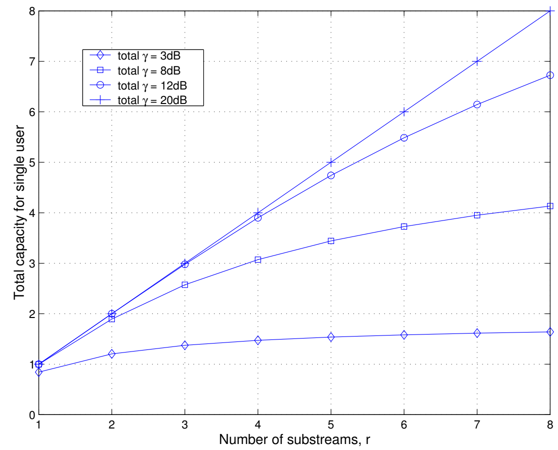

Fig. 3. Total capacity for a user without restrictions on the SINR of each substream.

tor, and the delay of the l^{th} multipath of the k^{th} user, respectively.

Vector $\mathbf{x}$ passes through the despreading block corresponding to the i^{th} substream for the k^{th} user with delay for multipath l, resulting in $\mathbf{y}^k_{(l)i}$. The space-time weights are applied to $\mathbf{y}^k_{(l)i}$ producing a scalar, $z^k_{(l)i} = (\mathbf{w}^k_{(l)})^H \mathbf{y}^k_{(l)i}$. Obtaining the space-time weights and estimating the multipath delays has been addressed in [7]. The space-time processing outputs for all substreams are then assembled to reconstruct the original sequence.

III. Rate Control for All Users

Our objective is to maximize the channel rate for all multimedia users. Let I denote the set of indexes of all multimedia users. We control the channel rate for any given user k by controlling the number of substreams and the channel rate of each of its r_k substreams.

Assuming that we use the minimum noise variance criteria to calculate the beamforming/RAKE vector, and that the number of antenna elements, D, in the antenna arrays is large enough so that we can assume that the self-interference can be neglected, then the signal to noise ratio for the i^{th} substream of user k, is [8]

$$\gamma^k_i = \frac{P_{k,i}\Psi^k}{\sum\limits_{j\neq k}\sum\limits_{i=1}^{r_j} P_{ji}\Phi^{k,j} + N_k}\,, \tag{3}$$

where $\Psi^k = L\sum_{l,\breve{m}=1}^{L_k} G_{k,l}G^*_{k,\breve{m}}((\mathbf{w}^k_l)^H\mathbf{a}_{k,l})((\mathbf{w}^k_{\breve{m}})^H\mathbf{a}_{k,\breve{m}})^*$ and $\Phi^{k,j} = \sum_{l=1}^{L_j} G^2_{kj_l}|((\mathbf{w}^k_l)^H\mathbf{a}_{kj_l})|^2$, where L is the processing gain.

For simplicity, assume we use PSK, then the probability of bit error conditioned on the SINR is $Pr(error|\gamma_k) = \mathcal{Q}(\sqrt{2\gamma_k})$, which we shall denote $p_{e,k}$. Here, $\mathcal{Q}(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{\frac{-t^2}{2}}dt$. Then, the channel capacity for substream i of user k, given γ^k_i is $C_k = 1 - H(p_{e,k,i})$, where $H(\cdot)$ denotes the entropy due to the probability of error $p_{e,k,i}$. Note that we may use any modulation technique and any probability of error model resulting in a capacity function which is a concave function of the SINR for the SINR levels of interest.

Requirement	Index Set Name
$\mu^*_{2i-1} = 0$ and $\mu^*_{2i} = 0$	A
$\mu^*_{2i-1} = 0$ and $\gamma^*_i = m_i$	B
$\mu^*_{2i} = 0$ and $\gamma^*_i = M_i$	C
$\gamma^*_i = m_i$ and $\gamma^*_i = M_i$	$\emptyset$

TABLE I

Possible requirements for optimal solution.

Now, if the transmission rate for all users is $\hat{R}_t$, for $k = 1,\cdots,M$, then the reception rate for each substream of user k is $R_{k,i} = (1 - H(p_{e,k,i}))\hat{R}_t$, for $i = 1,\cdots,r_k$. Therefore, the total capacity for all multimedia users is $R_{tot} = \hat{R}_t\sum_{k\in I}\sum_{i=1}^{r_k}(1 - H(p_{e,k,i})$.

For each user, k, we wish to choose the best number of substreams, r_k, and for each of the substreams, i, we wish to choose the optimum target SINR assignment, γ^k_i, in order to maximize the total capacity for that k. We control the SINR for each substream through automatic power control. To write an optimization algorithm for the rate over $\{r_k, \gamma^k_i,\ i = 1,\cdots,r_k\}_{k\in I}$, we identify a cost function, J:

$$J \triangleq \sum_{k\in I}\sum_{i=1}^{r_k}(1 - H(p_{e,k}(\gamma^k_i)))\,. \tag{4}$$

The SINR levels that we assign cannot be arbitrarily large. If the target γ^k_i are too high,then the power control algorithm does not converge to a positive power vector. We have shown that we can guarantee the feasibility of a selection of γ^k_i if $|\rho(\Gamma F)| < 1$ [8], where Γ is an $M\times M$ diagonal matrix with diagonal entries $\sum_{i=1}^{r_k}\gamma^k_i$, and:

$$F = \begin{pmatrix} 0 & \frac{\Phi^{1,2}}{\Psi^1} & \cdots & \frac{\Phi^{1,M}}{\Psi^1} \\ \frac{\Phi^{1,2}}{\Psi^2} & 0 & \cdots & \frac{\Phi^{2,M}}{\Psi^2} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\Phi^{M,1}}{\Psi^M} & \frac{\Phi^{M,2}}{\Psi^M} & \cdots & 0 \end{pmatrix}. \tag{5}$$

1.	For each index assignment in D, do:
2.	If $A \neq \emptyset$, then
2.a.	$\bar{\gamma}_{new} = \bar{\gamma} - \sum_{i \in B} \gamma_i^* - \sum_{i \in C} \gamma_i^*$
2.b.	Set $\gamma_i^* = \gamma_A^* = \frac{\bar{\gamma}_{new}}{r_A}$ for all γ_i with $i \in A$.
2.c.	If γ_A^* is not in the constraining interval for some $i \in A$, then go to Step 4.
2.d.	Find $\psi^* = \frac{\partial c(\gamma_A^*)}{\partial \gamma_i}$.
2.e.	If $\mu_{2i-1}^* = \frac{\partial c(M_i)}{\partial \gamma_i} - \psi^*,\ i \in C$, is negative, then go to Step 4.
2.f.	If $\mu_{2i}^* = \psi^* - \frac{\partial c(m_i)}{\partial \gamma_i},\ i \in B$, is negative, then go to Step 4.
2.g.	Set all $\gamma_i^* = m_i$ for $i \in B$, and all $\gamma_i^* = M_i$ for $i \in C$ and return $\{\gamma_i^*\}_{i=1,\cdots,r}$. Exit.
3.	If $A = \emptyset$, then
3.a.	If $\max_{i \in B} \frac{\partial c(m_i)}{\partial \gamma_i} > \min_{i \in C} \frac{\partial c(M_i)}{\partial \gamma_i}$, then go to Step 4.
3.b.	If $\sum_{i \in B} m_i + \sum_{i \in C} M_i \neq \bar{\gamma}$, then go to Step 4.
3.c.	Set all $\gamma_i^* = m_i$ for $i \in B$, and all $\gamma_i^* = M_i$ for $i \in C$ and return $\{\gamma_i^*\}_{i=1,\cdots,r}$. Exit.
4.	There is no solution for this D, go to the next D and go to Step 2.
5.	If there were no solutions for any combinations in D, then there is no solution. Exit.

TABLE II

ALGORITHM FOR OBTAINING THE OPTIMAL SET OF γ_i^*, GIVEN r AND $\bar{\gamma}$.

This is notable in that the feasibility test only places constraints on the cumulative SINR for each user, $\sum_{i=1}^{r_k} \gamma_i^k$, and not on each individual substream of each user, γ_i^k.

Let the N-dimensional vector $\vec{\gamma}$ be defined by $\vec{\gamma} = (\gamma_1^1, \cdots, \gamma_{r_1}^1 \cdots, \gamma_1^M \cdots, \gamma_{r_M}^M)$, and say that we constrain the SINR levels for each substream to be inside some prespecified intervals. Due to the nature of multimedia data, we can assign higher SINR intervals to substreams that require higher rates and lower SINR intervals to those which require lower rates. We have shown that the optimization problem:

$$\begin{array}{ll} \max_{\vec{\gamma}, r_1, \cdots, r_M} \ J & \text{such that} \\ |\rho(\Gamma F)| < 1 \ , & \\ \gamma_i^k \in (\gamma_{k,i}^{min}, \gamma_{k,i}^{max}) \subset \mathcal{R}^+, & i = 1, \cdots r_k \ , \\ r_k \in \{1, 2, \cdots, r_{max}\}, & k = 1, \cdots, M. \end{array} \tag{6}$$

is NP-hard [8].

Finding the optimal selection of SINR levels, and choosing the optimal number of substreams per user is an optimization problem with two types of variables, real, such as the γ_i^k, and integer, such as the r_k.

A. Optimal Number of Substreams and Substream SINR for Each User

We can use the algorithms introduced in [6] to find a suboptimal total SINR allocation for each user k, $\bar{\gamma} = \sum_{i=1}^{r_k} \gamma_i^k$. Now, for each user, we need to choose r_k and each γ_i^k. For this latter problem, we developed an algorithm that solves this constrained optimization problem.

1.	Choose $r = 1$ and $I_1 = [m_1^1, M_1^1]$
2.	For a sampling of $\bar{\gamma} \in [\bar{\gamma}_{min}, \bar{\gamma}_{max}]$, get the optimum $\{\gamma_{r_i}^*\}_{i=1,\cdots,r}$, if they exist.
3.	Calculate the total capacity, given $\{\gamma_{r_i}^*\}_{i=1,\cdots,r}$: $\sum_{i=1}^{r} c(\gamma_{r_i}^*)$
4.	Set $r = r + 1$, and define $I_1 = [m_1^r, M_1^r], \cdots, I_r = [m_r^r, M_r^r]$.
5.	Repeat from Step 2 until $r = r_{max}$.
6.	For each $\bar{\gamma}$, compare the resulting $\sum_{i=1}^{r} c(\gamma_{r_i}^*)$ among those r which had an optimal solution.
7.	For each $\bar{\gamma}$, choose $r^* = \arg \max_{r=1,\cdots,r_{max}} \sum_{i=1}^{r} c(\gamma_{r_i}^*)$
8.	Assign $\{\gamma_{r_i^*}^*\}_{i=1,\cdots,r^*}$ to each substream.

TABLE III

ALGORITHM FOR OBTAINING THE OPTIMAL r AND γ_i^* FOR VARIOUS $\bar{\gamma}$.

We proved that when there are no constraints on each individual γ_i^k, then the optimal SINR allocation is that all substreams have the same SINR, $\gamma_i^k = \bar{\gamma}/r_k$, for $i = 1, \cdots, r_k$,[8]. And we also showed that the higher the number of substreams, r_k, the higher the rate. This can be seen in Fig. 3, where we see the capacity for a given user, normalized by the transmission rate, for different r_k. As can be seen, for each $\bar{\gamma}$, the capacity grows with r_k.

This suggests that the optimal solution for unconstrained γ_i^k is when one has an unlimited number of substreams, and each substream would have a very low SINR level. There are at least two reasons to avoid this. First, the transmitter and receiver would have to handle a very large number of substream blocks, which must each have decorrelators, space-time diversity processors, etc. Another reason is that an arbitrarily low γ_i^k contribute to high bit error rates, which require sophisticated coding methods. These methods require a great deal of computing power, which translate into depleting the battery resources.

It is more reasonable to consider a constraint on r_k, say we define the maximum number of substreams any user can have, r_{max}, and we limit the range of SINR levels for each substream.

We rewrite Problem (6) for a single user with r substreams, in a format with we can use constrained optimization methods. Let $\gamma = (\gamma_1, \cdots, \gamma_r)$, the problem is

$$\begin{array}{c} \min_{\gamma} f^0(\gamma) = -\sum_{i=1}^{r} c(\gamma_i) \quad \text{such that} \\ g(\gamma) = \bar{\gamma} - \sum_{i=1}^{r} \gamma_i = 0, \\ f^1(\gamma) = \gamma_1 - \gamma_1^{max} \leq 0, \\ f^2(\gamma) = \gamma_1^{min} - \gamma_1 \leq 0, \\ \vdots \\ f^{2r-1}(\gamma) = \gamma_r - \gamma_r^{max} \leq 0, \\ f^{2r}(\gamma) = \gamma_r^{min} - \gamma_r \leq 0 \ . \end{array} \tag{7}$$

It can be shown that if γ^* is a local minimizer for (7), and we constrain each γ_i^* to be in its interval with $\sum_{i=1}^{r} \gamma_i^* = \bar{\gamma}$, then there exists a $\mu^* > 0$, $\mu^* \in \mathcal{R}^{2r}$, and $\psi^* \in \mathcal{R}$ such that [8]

r	$I_i = [m_i^r, M_i^r]$, [dB]
1	$I_1 = [2, 30]$
2	$I_1 = [5, 30]$, $I_2 = [2, 15]$
3	$I_1 = [7, 30]$, $I_2 = [5, 20]$, $I_3 = [2, 15]$
4	$I_1 = [8, 30]$, $I_2 = [7, 20]$, $I_3 = [6, 18]$, $I_4 = [2, 15]$
5	$I_1 = [10, 30]$, $I_2 = [9, 25]$, $I_3 = [8, 20]$, $I_4 = [7, 18]$ $I_5 = [2, 15]$
6	$I_1 = [11, 30]$, $I_2 = [10, 25]$, $I_3 = [9, 20]$, $I_4 = [8, 18]$ $I_5 = [7, 16]$, $I_6 = [2, 15]$
7	$I_1 = [12, 30]$, $I_2 = [11, 26]$, $I_3 = [10, 25]$, $I_4 = [9, 20]$ $I_5 = [8, 18]$, $I_6 = [7, 16]$, $I_7 = [2, 15]$

TABLE IV
SINR CONSTRAINTS FOR DIFFERENT NUMBERS OF SUBSTREAMS FOR EXAMPLE 1.

$$\begin{pmatrix} \frac{-\partial c(\gamma_1^*)}{\partial \gamma_1} \\ \frac{-\partial c(\gamma_2^*)}{\partial \gamma_2} \\ \vdots \\ \frac{-\partial c(\gamma_r^*)}{\partial \gamma_r} \end{pmatrix} + \begin{pmatrix} \mu_1^* - \mu_2^* \\ \mu_3^* - \mu_4^* \\ \vdots \\ \mu_{2r-1}^* - \mu_{2r}^* \end{pmatrix} + \begin{pmatrix} \psi^* \\ \psi^* \\ \vdots \\ \psi^* \end{pmatrix} = 0, \tag{8}$$

$$\begin{cases} \mu_i^* \left(\gamma_{\frac{i+1}{2}}^* - M_{\frac{i+1}{2}}\right) = 0 & \text{for } i = 2, \cdots, 2r, \, i \text{ even} \\ \mu_i^* \left(m_{\frac{i}{2}} - \gamma_{\frac{i}{2}}^*\right) = 0 & \text{for } i = 1, \cdots, 2r, \, i \text{ odd.} \end{cases}$$

To find the local minimizer, the matrix equation of (8) requires that

$$\begin{aligned} \frac{-\partial c(\gamma_i^*)}{\partial \gamma_i} &= \frac{e^{-\gamma_i^*}}{2\ln(2)\sqrt{\pi\gamma_i^*}} \ln\left(\frac{Q}{1-Q}\right) \\ &= \mu_{2i}^* - \mu_{2i-1}^* - \psi^* \, , \end{aligned} \tag{9}$$

for $i = 1, \cdots, r$, where $Q \stackrel{\triangle}{=} Q(\sqrt{2\gamma_i^*})$. The last set of equations of (8) requires that for each $i = 1, \cdots, r$, we have that ($\mu_{2i-1}^* = 0$ or $\gamma_i^* = M_i$) and ($\mu_{2i}^* = 0$ or $m_i = \gamma_i^*$). These conditions allow us four possibilities for each substream i, which are detailed in Table I.

Table II shows an algorithm that uses the above equations to obtain γ_i^* for a given r. Recall that $\bar{\gamma}$ is given by the method outlined in [6]. This algorithm does an exhaustive search of all possible assignments of the indexes, $i = 1, \cdots, r$ to the sets A, B, and C, as defined in Table I. So, for example, if $r = 2$, then the algorithm considers all the following combinations for indexes $i = 1, 2$: $1, 2 \in A$, $1, 2 \in B$, $1, 2 \in C$, $1 \in A$ and $2 \in B$, $1 \in A$ and $2 \in C$, etc. We denote all of these possible combinations as D. Finally, to choose r, we follow the algorithm outlined in Table III.

From [9], we know that since $f^0(\cdot)$ and the constraints in (7) are convex, and $g(\cdot)$ is affine, then the solution we find with our algorithm, if it exists, is globally optimum.

One important factor that must be noted is that the choice of constraining intervals in (7) has an important impact on the solution. In the simulations we will see the difference which may result from this.

IV. SIMULATIONS

We compare the optimum r and γ_i for a user, given different constraining intervals, as specified in (7). We limit $r \leq 7$.

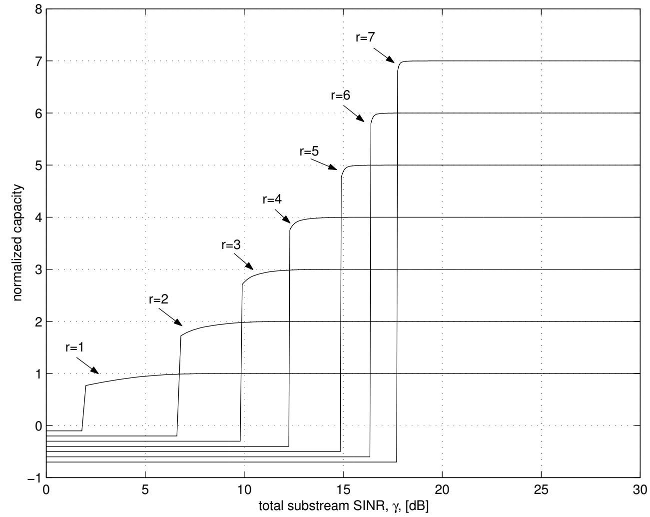

Fig. 4. The optimal number of substreams, r, can be seen for each total γ.

In the first example, the constraining intervals are seen in Table IV. In the second example, the constraining intervals are seen in Table V. For both examples, we allow $\bar{\gamma}$ to take on values between $0\ dB$ and $30\ dB$. For both cases, we plotted the capacity achieved by the optimum solution of γ_i, if a solution existed. If a solution did not exist, we plotted negative values. For example 1, the resulting graph is Figure 4. For example 2, the graph is Figure 5.

We can see in Figure 4 that for $\bar{\gamma} < 17.6\ dB$, there are no solutions, $\{\gamma_i^*\}_{i=1,\cdots,7}$ for $r = 7$. But for $\bar{\gamma} \geq 17.6\ dB$, the optimal capacity is achievable with seven substreams. Furthermore, seven substreams yields a higher overall user capacity than any other choice of r.

By defining our intervals, I_i, differently in example 2, we obtain different graphs, which in turn, show different thresholds for us to change from one number of substreams to another number of substreams. For example, in Figure 5, we can see that for $\bar{\gamma} \leq 14\ dB$, the only possible solution is with $r = 1$. When $\bar{\gamma} = 14.05\ dB$, the optimum capacity is achieved when $r = 3$.

Note that in example 2, the optimum number of substreams can only belong to a limited set, $r \in \{1, 3, 4, 7\}$. Optimum capacity will not be achieved with $r = 2,\ 5$ or 6. This does not pose a problem, however, this example illustrates another issue that must be addressed when choosing the I_i. Note that in Figure 5 the normalized capacity for the user increases negligibly between $\bar{\gamma} \in [12, 14]\ dB$. This choice reflects a waste of resources, since the system can handle higher cumulative SINR levels for the user, yet the user benefits negligibly from this. The choice made in example 1 has the advantage that, up to about $18\ dB$, an increase in $\bar{\gamma}$ results in an increase in capacity for the user.

These considerations and algorithms are all done off-line, for the purpose of generating a look-up table that will be used online. Once the intervals are defined, we implement the algorithm described in Table II and Table III, and we generate a

r	$I_i = [m_i^r, M_i^r]$, [dB]
1	$I_1 = [2, 30]$
2	$I_1 = [15, 30]$, $I_2 = [2, 15]$
3	$I_1 = [12, 30]$, $I_2 = [9, 20]$, $I_3 = [2, 15]$
4	$I_1 = [15, 30]$, $I_2 = [12, 20]$, $I_3 = [7, 18]$, $I_4 = [2, 10]$
5	$I_1 = [18, 30]$, $I_2 = [15, 25]$, $I_3 = [12, 20]$, $I_4 = [7, 18]$ $I_5 = [3, 15]$
6	$I_1 = [18, 30]$, $I_2 = [15, 25]$, $I_3 = [12, 20]$, $I_4 = [8, 18]$ $I_5 = [6, 16]$, $I_6 = [2, 15]$
7	$I_1 = [12, 30]$, $I_2 = [11, 26]$, $I_3 = [10, 25]$, $I_4 = [9, 20]$ $I_5 = [8, 18]$, $I_6 = [7, 16]$, $I_7 = [2, 15]$

TABLE V

SINR CONSTRAINTS FOR DIFFERENT NUMBERS OF SUBSTREAMS FOR EXAMPLE 2.

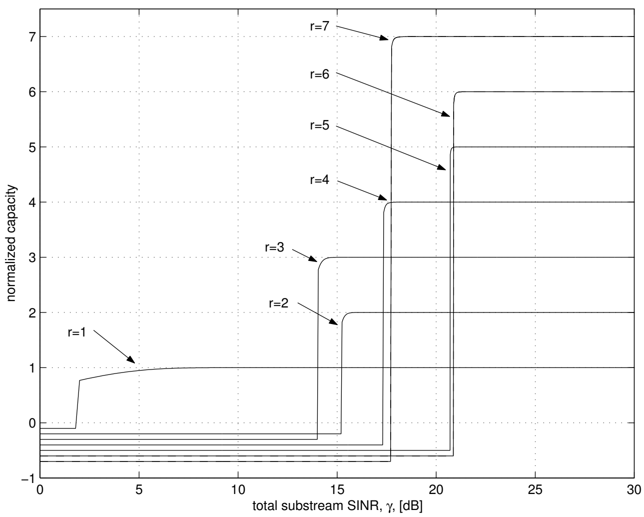

Fig. 5. The optimal number of substreams, r, can be seen for each total γ.

table that can be referenced on-line by the system. Each time a user is assigned a $\bar{\gamma}$, the system checks the table to check what is the best number of substreams and what $\{\gamma_i^*\}$ should be assigned to each substream. Using the intervals from Table IV, we have created a sample look-up table, Table VI.

Finally, the method explained in [6] is used to find the optimum power vector and antenna array weights.

V. Conclusions

We use the algorithms presented in this paper to generate a look-up table for wireless systems which indicates how many substreams each user may have and what is the optimum power of each substream in order to maximize the overall capacity.

The method is intended to provide higher data rates for users which have multimedia wireless services, and require higher rates.

The method first assigns a cumulative SINR level to each user, which can be solved by implementing the fine-tuning algorithm introduced in [6]. Then, the system checks the look-up table to find the optimum r and γ_i, $i = 1, \cdots, r$, for each user.

$\bar{\gamma}$ $[dB]$	r	$\gamma_1^*, \cdots \gamma_r^*$ $[dB]$
< 2	-	no solution
$2-6$	1	$\gamma_1^* = \bar{\gamma}$
7	2	$\gamma_1^* = 2.6708,\ \gamma_2^* = 5$
8	2	$\gamma_1^* = 4.9794,\ \gamma_2^* = 5$
9	2	$\gamma_1^* = \gamma_2^* = 5.9897$
10	3	$\gamma_1^* = 2.6147,\ \gamma_2^* = 5,\ \gamma_3^* = 7$
11	3	$\gamma_1^* = \gamma_2^* = 5.7849,\ \gamma_3^* = 7$
12	3	$\gamma_1^* = \gamma_2^* = \gamma_3^* = 7.2288$
13	4	$\gamma_1^* = \gamma_2^* = 6.3504,\ \gamma_3^* = 7,\ \gamma_4^* = 8$
14	4	$\gamma_1^* = \gamma_2^* = \gamma_3^* = 7.9725,\ \gamma_4^* = 8$
15	5	$\gamma_1^* = 3.7255,\ \gamma_2^* = 7,$ $\gamma_3^* = 8,\ \gamma_4^* = 9,\ \gamma_5^* = 10$
16	5	$\gamma_1^* = \gamma_2^* = \gamma_3^* = 8.6268,\ \gamma_4^* = 9,\ \gamma_5^* = 10$
17	6	$\gamma_1^* = \gamma_2^* = \gamma_3^* = 8.1483,$ $\gamma_4^* = 9,\ \gamma_5^* = 10,\ \gamma_6^* = 11$
18	7	$\gamma_1^* = \gamma_2^* = 7.1620,\ \gamma_3^* = 8,\ \gamma_4^* = 9,$ $\gamma_5^* = 10,\ \gamma_6^* = 11,\ \gamma_7^* = 12$
19	7	$\gamma_1^* = \cdots = \gamma_5^* = 10.0855,$ $\gamma_6^* = 11,\ \gamma_7^* = 12$
20	7	$\gamma_1^* = \cdots = \gamma_6^* = 11.4691,\ \gamma_7^* = 12$
21	7	$\gamma_1^* = \cdots = \gamma_7^* = 12.5490$
22	7	$\gamma_1^* = \cdots = \gamma_7^* = 13.5490$
23	7	$\gamma_1^* = \cdots = \gamma_7^* = 14.5490$

TABLE VI

SAMPLE LOOK-UP TABLE.

The solutions reflected in the look-up table are globally optimum in the sense of maximizing capacity, given $\bar{\gamma}$.

Finally, we showed how the choice of interval constraints for the substream SINR levels must be done carefully in order to ensure that an increase in $\bar{\gamma}$, as permitted by the interference and noise of the system, will result in an increase in capacity for each user.

References

[1] F. Rashid-Farrokhi, L. Tassiulas, and K.J.R. Liu, "Joint Optimal Power Control and Beamforming in Wireless Networks Using Antenna Arrays," *IEEE Transactions on Communications*, Vol. 46, number 10, pp. 1313–1324, October 1998.

[2] F. Rashid-Farrokhi, and L. Tassiulas, "Power Control and Space-Time Diversity," *IEEE GLOBECOM*, Vol. 4, pp. 1313–1324, November 1998.

[3] Y. Liang, F. Chin, and K.J.R. Liu, "Downlink Beamforming for DS-CDMA Mobile Radio with Multimedia Services," *Proc. of IEEE 50th Vehicular Technology Conference*, Vol. 1, pp. 17–21, 1999.

[4] A. Paulraj, and C. Papadias,"Space-Time Processing for Wireless Communications," *IEEE Signal Processing Magazine*, Vol. 14, pp. 49–83, November 1997.

[5] A. Mercado, and K.J.R. Liu,"NP-hardness of the stable matrix in unit interval family problem in discrete time," *Systems and Control Letters*, Vol. 42, pp. 261–265, April 2001.

[6] A. Mercado, and K.J.R. Liu,"Adaptive QoS for Mobile Multimedia Applications Using Power Control and Smart Antennas," *Proc. of IEEE International Conference on Communications*, Vol. 1, pp. 60–64, 2000.

[7] A. Naguib, *Adaptive Antennas for CDMA Wireless Networks.* Ph.D. Thesis, Stanford University, 1996.

[8] A. Mercado, *Adaptive Service Rate for Integrated Multimedia over Wireless Networks.* Ph. D. Dissertation, University of Maryland, 2001.

[9] A. Tits, *Notes for ENEE 664: Optimal Control (draft).* August, 1998.