

# Space-Time Network Coding

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**Abstract**—Traditional cooperative communications can improve communication reliability. However, transmissions from multiple relay nodes are challenging in practice. Single transmissions in time-division multiple-access (TDMA) manner cause large transmission delay, but simultaneous transmissions from two or more nodes using frequency-division multiple access (FDMA) and code-division multiple access (CDMA) are associated with the issue of imperfect frequency and timing synchronization. In this work, a novel framework for cooperative communications is proposed to achieve full spatial diversity with low transmission delay and eliminate the issue of imperfect synchronization. This is realized by the use of space-time network codes (STNCs) associated with a novel concept of wireless network cocast. For a network of  $N$  client nodes,  $R$  relay nodes and a base node, the STNCs provide a diversity order of  $(R + 1)$  for each symbol with  $(N + R)$  time slots, a reduction from  $2N$  time slots in traditional FDMA and CDMA cooperative communications for  $N$  being usually greater than  $R$  and from  $N(R + 1)$  time slots in traditional TDMA cooperative communications. The STNCs are also applied in networks, where the client nodes located in a cluster act as relays to help one another to improve their transmission performance. The performance in clustering setting is studied to show the improvement in power saving, range extension, and transmission rate.

**Index Terms**—Cooperative communications, frequency synchronization, linear network coding, space-time network codes, timing synchronization, wireless network cocast.

## I. INTRODUCTION

IT is well known that the performance of communication systems degrades greatly when operating in radio frequency environments characterized by multipath propagation such as urban environments. Diversity techniques like time, frequency and spatial diversity can be utilized to mitigate the multipath effect. The recent multiple-input multiple-output (MIMO) technology, in which communication devices are equipped with multiple transmit and/or multiple receive antennas, can significantly increase communication reliability through the use of spatial diversity [1]–[5]. However, the use of MIMO technology possesses a number of issues in practical applications whose size, weight and power are dominant factors in choosing a technology as in military hand-held and man-packed devices. First, the required separation among antennas, usually at least

a half of wavelength, makes MIMO unsuitable for low transmit frequencies, which associate with low free-space path loss and thus a longer battery life. Second, the use of multiple radio frequency chains at a device increases the size and weight of the device and thus limits certain MIMO applications such as those in wireless sensor networks and mobile networks.

To overcome the MIMO issues while maintaining MIMO benefits in improving reliability, cooperative communications [6] have recently received much attention. Cooperative communications make use of broadcast nature of wireless transmission and nodes in a network acting as relays can retransmit overheard information to a destination. The distributed antennas from the source and the relays form a virtual antenna array and spatial diversity is achieved without the need to use multiple antennas at the source node.

Various cooperative diversity protocols have been proposed and analyzed in [7]–[12]. Several strategies for single-relay cooperative communications such as decode-and-forward (DF) and amplify-and-forward (AF) were introduced in [7] and their performance in terms of outage behavior was analyzed. In DF protocol, each relay decodes the overheard symbol from the source, re-encodes it and then forwards it to the destination. AF protocol is a simpler technique, in which each relay simply amplifies the overheard signal and forwards it to the destination. In [8], user cooperation strategy and performance analysis of a code-division multiple access system for two cooperative users were presented. Symbol error rate (SER) performance for multi-node DF protocols was analyzed in [9]. Outage analysis for multi-node AF relay networks was studied in [10]. In [9] and [10], optimal power allocation was also derived for DF and AF protocols. Various relay selection schemes were proposed in [11] that achieve high bandwidth efficiency and full diversity order. Finally in [12], distributed space-time-coded DF and AF protocols, which are based on conventional MIMO space-time coding, were proposed and their outage performance was analyzed.

Cooperative communications often consist of two phases: source transmission and relay transmission [9], [10], [12]. In the first phase, a source broadcasts its information to relays, which then forward the overheard information to the destination in the second phase. Much research in cooperative communications has focused on simultaneous transmissions from two or more nodes by using frequency-division multiple access (FDMA), code-division multiple access (CDMA), or distributed space-time codes with an assumption of perfect frequency and timing synchronization [9], [10], [12], [13]. However, such an assumption is difficult to be met in practice, especially in mobile conditions where nodes move at different speeds and in different directions. For timing synchronization, the coordination to make signals received simultaneously at the destination is challenging due to differences in propagation

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time among nodes, processing time in each radio and timing estimation error. The frequency synchronization issue occurs when each node has an independent local oscillator generating a transmit frequency with certain variation to the nominal. The transmit frequencies from different nodes, therefore, are different. Moreover, signals from nodes in mobile conditions with different directions and speeds are under different Doppler effects. Together, various frequency mismatches occur at once at the destination and make it difficult to estimate and compensate all the frequency offsets. The imperfect synchronization causes the intersymbol interference, which is the source of system performance degradation [14], [15].

To overcome the imperfect synchronization issue that prohibits two or more nodes from transmitting at the same time, time-division multiple access (TDMA) [9], where each relay node take turn to forward the overheard symbols, would be the most commonly-used technique in many applications. A phased-locked loop device at a receiver can easily overcome the frequency mismatch and timing error from a single arriving signal. However, TDMA requires  $N(R + 1)$  time slots for  $N$  source nodes and  $R$  relay nodes, causing large transmission delays as  $N$  and  $R$  increase. Therefore, there is an essential need to overcome the issue of imperfect frequency and timing synchronization while maintaining the spatial diversity and reducing the total required time slots.

In this work, we leverage a novel concept of wireless network cocast (WNC) [16] and propose its associated space-time network codes (STNCs) to achieve the foretold objectives. Cocast, an abbreviation of *cooperative cast*, is a newly defined transmission method, in which information from different nodes are jointly combined within a relay and transmitted to the intended destinations. Many previous works consider cooperative communications as a direct extension of MIMO communications, leveraging the distributed antennas in a network to achieve the MIMO benefits. Hence, the issue of imperfect synchronization due to the simultaneous transmissions from multiple nodes arises and prevents cooperative communications from emerging in practice. We understand that a number of previous works such as delay-tolerant codes [17]–[19] or distributed carrier synchronization [20], [21] tried to mitigate this issue. The novelty of this work is that it lays out a new framework for the cooperation among nodes in a network that totally eliminates the said issue.

We consider two general cases of multipoint-to-point (M2P) and point-to-multipoint (P2M), where  $N$  client nodes transmit and receive their information to and from a common base node, respectively, with the assistance from  $R$  relay nodes. We denote them as M2P-WNCR and P2M-WNCR, where “R” implies the use of independent relay nodes. Both DF and AF protocols in cooperative communications are considered in the general WNCR schemes. We derive the exact and the asymptotic SER expressions<sup>1</sup> for general  $\mathcal{M}$ -PSK modulation for DF protocol. The extension to general  $\mathcal{M}$ -QAM can follow directly. For AF protocol, we offer a conditional SER expression given the channel knowledge.

The STNCs of WNCR schemes provide general codings that can be applied in many wireless network settings. The most

interesting setting is where a group of client nodes located in a close proximity to one another as in a cluster cooperate together to improve their performance. An example can be found in wireless sensor networks or cellular networks with transmission exchange between sensors and a fusion center or between a base station and clients. In this case, the client nodes can also be relays, helping one another in transmissions between themselves and the base node. We refer this network setting as clustering setting and the application of WNC schemes in this network are denoted as WNC schemes. We study the performance improvement using WNC schemes over a direct transmission scheme (DTX), where a client node transmits directly to the base node without assistance from any other nodes. Simulations show that given the same quality of service represented by a SER, the use of WNC schemes results in a great improvement in terms of power saving, range extension and transmission rate.

The rest of this paper is organized as follows. After this introduction section, the concept of STNCs is introduced in Section II. From this framework, the system models for WNCR schemes are presented in this section. Signal detection is followed in Section III. The performance analysis is presented in Section IV to provide the exact and the asymptotic SER expressions for DF protocol and the conditional SER expression for AF protocol. In addition, simulations are conducted to verify the performance analysis. The performance improvement by using WNC schemes in clustering settings is studied in Section V. Lastly, we draw some conclusions in Section VI.

*Notation:* Lower and upper case bold symbols denote column vectors and matrices, respectively.  $*$ ,  $\mathcal{T}$ , and  $\mathcal{H}$  denote complex conjugate, transpose and Hermitian transpose, respectively.  $\text{diag}\{\cdot\}$  represents a diagonal matrix.  $|\cdot|$  denotes the magnitude or the size of a set.  $\mathcal{CN}(0, \sigma^2)$  is the circular symmetric complex Gaussian random variable with zero mean and variance  $\sigma^2$ .  $r$  refers to the index of a relay node and  $n$  denotes the index of a transmitted symbol of interest and also the index of a client node.  $a_{uv}$  with generic indexes  $u$  and  $v$  denotes a signal coefficient.

## II. SPACE-TIME NETWORK CODING—A CONCEPT AND SYSTEM MODELS

### A. A Concept of Space-Time Network Coding

Let us consider a network consisting of  $N$  client nodes denoted as  $U_1, U_2, \dots, U_N$  having their own information that need to be delivered to a common base node  $U_0$ . Moreover, we assume  $R$  relay nodes, denoted as  $R_1, R_2, \dots, R_R$ , helping the client nodes in forwarding the transmitted information. Without loss of generality, the transmitted information can be represented by symbols, denoted as  $x_1, x_2, \dots, x_N$ . Nevertheless, the client nodes and relay nodes in practice will transmit the information in packets that contains a large number of symbols. The destination will collect all the transmitted packets and then jointly detect the transmitted information as in traditional network coding [22], [23]. Note that the relay nodes can be the client nodes themselves or they can be nodes dedicated to only relaying information for others such as relay towers.

We realize that after the source transmission phase, in which the  $N$  source nodes broadcast their information, the relay nodes

<sup>1</sup>Asymptotic SER performance is a performance at high signal-to-noise ratio.

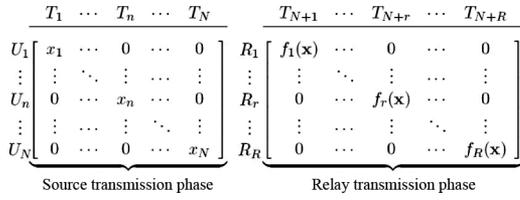


Fig. 1. A general framework of space-time network coding.

possess a set of overheard symbols, denoted as a vector  $\mathbf{x} = [x_1, x_2, \dots, x_N]^T$ , from the client nodes. Instead of allowing multiple relay nodes transmitting at the same time as in the traditional cooperative communications, each relay node forms a single signal by encoding the set of overheard symbols, denoted as  $f_r(\mathbf{x})$  for relay node  $R_r$  for  $r = 1, 2, \dots, R$  and transmits it to the intended destination in its own dedicated time slot. The transmissions from the  $N$  client nodes and  $R$  relay nodes in the source and relay transmission phases, respectively, are illustrated in Fig. 1. Each set of encoding functions  $f_r$ 's, denoted as  $\{f_r\}_{r=1}^R$ , will form a STNC that governs the cooperation and the transmission among the nodes in the network. The STNC will provide appropriate spatial diversity with only  $(N + R)$  time slots, a reduction from  $2N$  time slots in the traditional CDMA and FDMA cooperative communications for  $N$  being usually greater than  $R$  and from  $N(R + 1)$  time slots in the traditional TDMA cooperative communications. Moreover, the foretold issue of imperfect frequency and timing synchronization is overcome because a single transmission is granted at any given time slot in the network.

The concept of our proposed STNCs is very general. Fundamentally, it involves combining information from different sources at a relay node, which gives rise to the concept of network coding and transmitting the combined signal in a dedicated time slot, which makes the space-time concept. Since the cooperating nodes are distributed around the network, the space dimension (or node locations) can be an important aspect of designing a STNC. We expect that various traditional combining techniques can be used; however, a major distinction is that the combining of symbols from different sources, giving rise to the received signal at a destination, happens within a transmitter instead of through the air. These techniques could include CDMA-like, FDMA-like and TDMA-like techniques. As the names suggest, each source information in CDMA-like technique is assigned a spreading code [24] while FDMA-like technique uses a group of subcarriers as in the well-know orthogonal frequency-division multiple access (OFDMA) [25]. In TDMA-like technique, each relay is assigned a large time slots to be able to concatenate the  $N$  symbols along the time axis for transmission. As in the traditional TDMA scheme, the TDMA-like scheme can overcome the imperfect synchronization issue but also leads to the issue of long transmission delay due to the concatenation of symbols along the time axis. The combining techniques could also be transform-based techniques [26]–[28] with the use of Hadamard or Vandermonde matrices. In addition, they could probably be the traditional network coding [22], [23], which linearly combines symbols from different sources over Galois field. Clearly, each combining technique requires an

appropriate multi-user detection technique to separate the transmitted symbols from single coded-signals.

The STNCs that we propose in this work are the CDMA-like, FDMA-like and TDMA-like techniques. Although they are expected to provide comparable SER performance, we favor the first two techniques since they can provide lower transmission delay in comparison with the third one. CDMA and FDMA has been used in cooperative communications [8], [9], [13], where multiple relay nodes transmit at the same time with the assumption of perfect synchronization. This assumption is hard to be met in practice as we discussed in Section I. In our work, each relay node forms a linearly coded signal from the overheard symbol within the node itself and transmits the signal in its assigned time slot. Note that our proposed CDMA-like, FDMA-like and TDMA-like schemes do not provide more bandwidth efficiency, measured by the number of bits per second per Hertz (bit/s/Hz), than the traditional TDMA, FDMA and CDMA schemes in cooperative communications. In multinode transmissions, time, frequency and code are interchangeable resources. To reduce the number of required time slots, more frequency resource is needed. However, the use of these resources is governed by practical applications and constraints. For example in traditional non-cooperative networks, FDMA such as the OFDMA employed in WiMAX systems [25] is preferred over TDMA for applications that require low transmission delay. On the other hand, CDMA system with its spreading techniques is in favor due to its ability to overcome intentional interference such as jamming signals in military applications. Our proposed scheme is to solve a practical issues of imperfect synchronization and large transmission delay in the traditional TDMA, FDMA and CDMA schemes.

The general framework in Fig. 1 can be applied in M2P and P2M transmissions and we denote these schemes as M2P-WNCR and P2M-WNCR. In M2P-WNCR, the  $N$  client nodes  $U_1, U_2, \dots, U_N$  transmit their symbols  $x_1, x_2, \dots, x_N$ , respectively, to the base node  $U_0$  while the client nodes are the destinations for these symbols from the base node in P2M-WNCR. The channels are modeled as narrowband Rayleigh fading with additive white Gaussian noise (AWGN). Quasi-static channels are assumed, where they remain constant over each time slot and change independently between consecutive slots. The channel coefficient between an arbitrary receiver  $u$  and transmitter  $v$  is defined as  $h_{uv} \sim \mathcal{CN}(0, \sigma_{uv}^2)$ , where  $\sigma_{uv}^2 = d_{uv}^{-\alpha}$  is the channel variance with  $d_{uv}$  and  $\alpha$  being the distance between the two nodes and the path loss exponent, respectively. The transmit power  $P_n$  associated with transmitted symbol  $x_n$  is distributed among the source node ( $U_n$  or  $U_0$  in M2P-WNCR or P2M-WNCR, respectively) and the relay nodes. We have  $P_n = P_{nn} + \sum_{r=1}^R P_{rn}$ , where  $P_{nn}$  and  $P_{rn}$  are the power allocated at the source  $U_n$  or  $U_0$  and the relay  $R_r$ , respectively.

In the CDMA-like STNC, each symbol  $x_n$  is assigned a complex-valued signature waveform (also called a spreading code)  $s_n(t)$  to protect it against the interference from other symbols. The cross correlation between  $s_n(t)$  and  $s_m(t)$  is defined as  $\rho_{nm} = \langle s_n(t), s_m(t) \rangle$ , where  $\langle f(t), g(t) \rangle \triangleq \frac{1}{T} \int_0^T f(t)g^*(t)dt$  is the inner product between  $f(t)$  and  $g(t)$  with the symbol interval  $T$ . We assume  $\rho_{nn} = \|s_n(t)\|^2 = 1$ . In the FDMA-like

and TDMA-like,  $s_n(t)$  represents the dedicated carrier and the symbol duration associated with symbol  $x_n$ . In this work, orthogonal and nonorthogonal codes refer to the cross-correlation characteristics of the signature waveforms. When  $\rho_{nm} \neq 0$  for  $m \neq n$ , we have the nonorthogonal code. We assume that the relay nodes know the signature waveforms associated with the client nodes. In the sequel, we will describe in details the STNCs for WNCR schemes.

### B. Space-Time Network Code for M2P-WNCR Transmissions

Fig. 2(a) illustrates the transmissions in the source and relay transmission phases of the M2P-WNCR, in which the  $N$  client nodes  $U_1, U_2, \dots, U_N$  transmit their symbols to the common base node  $U_0$ . As shown in the figure, the STNC requires  $(N + R)$  time slots to complete the transmissions and guarantees a single transmission in the network at any given time slot to eliminate the issue of imperfect synchronization in traditional cooperative communications. In the source transmission phase, client node  $U_n$  for  $n = 1, 2, \dots, N$  is assigned a time slot  $T_n$  to broadcast its symbol  $x_n$  to the base node and all relay nodes. The signals received at  $U_0$  and  $R_r$  are

$$y_{0n}(t) = h_{0n}\sqrt{P_{nn}}x_n s_n(t) + w_{0n}(t) \quad (1)$$

and

$$y_{rn}(t) = h_{rn}\sqrt{P_{nn}}x_n s_n(t) + w_{rn}(t), \quad (2)$$

respectively, where  $w_{0n}(t)$  and  $w_{rn}(t)$  are zero-mean and  $N_0$ -variance AWGN. At the end of this phase, each relay node  $R_r$  for  $r = 1, 2, \dots, R$  possesses a set of  $N$  symbols  $x_1, x_2, \dots, x_N$  from the client nodes. In the relay transmission phase,  $R_r$  forms a single linearly coded signal, a linear combination of these symbols and transmits the signal to the base node in its dedicated time slot  $T_r$ .  $R_r$  can simply amplify the signal in (2) and combine with others to form the linearly coded signal, the so called AF protocol. It can also detect the symbol  $x_n$  based on (2), whose detection will be discussed later in Section III and re-encode it in the linearly coded signal if the decoding is successful, the so called DF protocol. A detection state, a success or a failure in detecting a symbol, can be determined based on the amplitude of the estimated channel coefficient [7] or the received signal-to-noise ratio (SNR) [9]. Notice that this DF scheme is also called the selective-relaying protocol in the literature [7]. In practice, information is transmitted in packets [29] that contain a large number of symbols. Each packet is detected as a whole and a cyclic redundancy check [30] is sufficient to determine the detection state of the packet.

The received signal at  $U_0$  from  $R_r$  in the relay transmission phase is

$$y_{0r}(t) = h_{0r} \sum_{k=1}^N \sqrt{\tilde{P}_{rk}} x_k s_k(t) + w_{0r}(t) \quad (3)$$

including symbol  $x_n$  when  $k = n$ . In (3),

$$\tilde{P}_{rk} = \begin{cases} P_{rk}, & \text{if } R_r \text{ decodes } x_k \text{ correctly} \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

$$\begin{array}{c} \begin{array}{cccc} T_1 & \cdots & T_n & \cdots & T_N \\ \hline U_1 & \begin{bmatrix} x_1 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ U_n & \begin{bmatrix} 0 & \cdots & x_n & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ U_N & \begin{bmatrix} 0 & \cdots & 0 & \cdots & x_N \end{bmatrix} \end{array} \end{array} \\ \hline \text{Source transmission phase} \end{array} & \begin{array}{cccc} T_{N+1} & \cdots & T_{N+r} & \cdots & T_{N+R} \\ \hline R_1 & \begin{bmatrix} \sum_{k=1}^N a_{1k} x_k & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ R_r & \begin{bmatrix} 0 & \cdots & \sum_{k=1}^N a_{rk} x_k & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ R_R & \begin{bmatrix} 0 & \cdots & 0 & \cdots & \sum_{k=1}^N a_{Rk} x_k \end{bmatrix} \end{array} \end{array} \\ \hline \text{Relay transmission phase} \end{array} \end{array} \quad (a)$$

$$\begin{array}{c} \begin{array}{cccc} T_1 & \cdots & T_n & \cdots & T_N \\ \hline U_0 & \begin{bmatrix} x_1 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ U_0 & \begin{bmatrix} 0 & \cdots & x_n & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ U_0 & \begin{bmatrix} 0 & \cdots & 0 & \cdots & x_N \end{bmatrix} \end{array} \end{array} \\ \hline \text{Source transmission phase} \end{array} & \begin{array}{cccc} T_{N+1} & \cdots & T_{N+r} & \cdots & T_{N+R} \\ \hline R_1 & \begin{bmatrix} \sum_{k=1}^N a_{1k} x_k & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ R_r & \begin{bmatrix} 0 & \cdots & \sum_{k=1}^N a_{rk} x_k & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ R_R & \begin{bmatrix} 0 & \cdots & 0 & \cdots & \sum_{k=1}^N a_{Rk} x_k \end{bmatrix} \end{array} \end{array} \\ \hline \text{Relay transmission phase} \end{array} \end{array} \quad (b)$$

Fig. 2. Space-time network codes for (a) M2P-WNCR and (b) P2M-WNCR schemes.

for the case of DF and

$$\tilde{P}_{rk} = \frac{P_{rk} P_{kk} |h_{rk}|^2}{P_{kk} |h_{rk}|^2 + N_0} \quad (5)$$

for the case of AF and  $w_{0r}(t)$  is zero-mean and  $N_0 f_{0r}$ -variance AWGN, where

$$f_{0r} = \begin{cases} 1, & \text{for DF} \\ 1 + \sum_{k=1}^N \frac{P_{rk} |h_{0r}|^2}{P_{kk} |h_{rk}|^2 + N_0}, & \text{for AF} \end{cases} \quad (6)$$

is a factor representing the impact on  $U_0$  performance due to the noise amplification at  $R_r$ . Notice that in (3),  $U_0$  receives a combined signal of multiple transmitted symbols that is formed within a relay node, not through the air as in traditional CDMA or FDMA schemes, where the symbols are from different relays and hence overcomes the prominent issue of imperfect frequency and timing synchronization in these technologies.

### C. Space-Time Network Code for P2M-WNCR Transmissions

P2M-WNCR also consists of a source transmission phase and a relay transmission phase, in which the base node  $U_0$  transmits symbols  $x_1, x_2, \dots, x_N$  to the  $N$  client nodes  $U_1, U_2, \dots, U_N$ . Fig. 2(b) illustrates the transmissions in the source and relay transmission phases of the P2M-WNCR. The STNC also requires  $(N + R)$  time slots to complete the transmissions and the foretold issue of imperfect synchronization is eliminated.

As shown in the figure, the signal model for this STNC can be derived in the same manner of that in M2P-WNCR scheme. The received signals at client node  $U_n$  and relay node  $R_r$  in the source transmission phase are

$$y_{n0}(t) = h_{n0}\sqrt{P_{nn}}x_n s_n(t) + w_{n0}(t) \quad (7)$$

and

$$y_{r0}(t) = h_{r0}\sqrt{P_{nn}}x_n s_n(t) + w_{r0}(t), \quad (8)$$



transmitted in packets [29] that contain a large number of symbols. Each packet is detected as a whole and a cyclic redundancy check [30] is sufficient to determine the detection state of the packet. Thus, one bit per packet results in a minimal overhead.

As shown in Section II, the transmission of symbol  $x_n$  in WNCR schemes shares a similar signal model, regardless where it is transmitted from. The symbol is first transmitted by the source node  $U_n$  or  $U_0$  in M2P or P2M transmissions, respectively and then forwarded by the relay nodes  $R_1, R_2, \dots, R_R$  to the destination node  $U_0$  or  $U_n$ . Thus, the same detection technique can be used in the two STNCs. In this section, we present a detailed signal detection in M2P-WNCR and the detection in P2M-WNCR can follow in a straight manner. To achieve a tractable performance analysis, we use a multi-user detection technique that includes a decorrelator and a maximal-ratio combining detector. Nevertheless, one can use minimum mean-square error (MMSE) detector, which is optimal among linear detectors. At high SNR, however, we expect that MMSE detector and our multiuser detector have comparable performance. The detection for an arbitrary symbol  $x_n$  is as follows.

After the completion of the two phases, the base node  $U_0$  in M2P-WNCR receives  $(R+1)$  signals that contain symbol  $x_n$ . From these signals, it extracts  $(R+1)$  soft symbols and uses a maximal-ratio combiner to detect the symbol. The first soft symbol of  $x_n$  comes directly from the source node  $U_n$  in the source transmission phase by applying matched-filtering to signal  $y_{0n}(t)$  in (1) with respect to signature waveform  $s_n(t)$  to obtain

$$y_{0nn} = \langle y_{0n}(t), s_n(t) \rangle = h_{0n} \sqrt{P_{nn}} x_n + w_{0nn}. \quad (12)$$

The remaining  $R$  soft symbols are collected from the  $R$  relaying signals  $y_{0r}(t)$  in (3) in the relay transmission phase as follows. For each signal  $y_{0r}(t)$ ,  $U_0$  applies a bank of matched-filtering to the signal with respect to signature waveforms  $s_m(t)$  for  $m = 1, 2, \dots, N$  to obtain

$$y_{0rm} = \langle y_{0r}(t), s_m(t) \rangle = h_{0r} \sum_{k=1}^N \sqrt{\tilde{P}_{rk}} x_k \rho_{km} + w_{0rm}. \quad (13)$$

Then it forms an  $N \times 1$  vector comprised of the  $y_{0rm}$ 's as

$$\mathbf{y}_{0r} = h_{0r} \mathbf{R} \mathbf{A}_r \mathbf{x} + \mathbf{w}_{0r} \quad (14)$$

where  $\mathbf{y}_{0r} = [y_{0r1}, y_{0r2}, \dots, y_{0rN}]^T$ ,  $\mathbf{A}_r = \text{diag} \left\{ \sqrt{\tilde{P}_{r1}}, \sqrt{\tilde{P}_{r2}}, \dots, \sqrt{\tilde{P}_{rN}} \right\}$ ,  $\mathbf{x} = [x_1, x_2, \dots, x_N]^T$ ,  $\mathbf{w}_{0r} = [w_{0r1}, w_{0r2}, \dots, w_{0rN}]^T \sim \mathcal{CN}(\mathbf{0}, N_0 f_{0r} \mathbf{R})$  with  $f_{0r}$  in (6) and

$$\mathbf{R} = \begin{bmatrix} 1 & \rho_{21} & \cdots & \rho_{N1} \\ \rho_{12} & 1 & \cdots & \rho_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1N} & \rho_{2N} & \cdots & 1 \end{bmatrix}. \quad (15)$$

Assume that  $\mathbf{R}$  is invertible with the inverse matrix  $\mathbf{R}^{-1}$ . This assumption is easy to achieve since the combining of symbols

is done within a relay node. The signal vector  $\mathbf{y}_{0r}$  is then decorrelated to obtain

$$\tilde{\mathbf{y}}_{0r} = \mathbf{R}^{-1} \mathbf{y}_{0r} = h_{0r} \mathbf{A}_r \mathbf{x} + \tilde{\mathbf{w}}_{0r}, \quad (16)$$

where  $\tilde{\mathbf{w}}_{0r} \sim \mathcal{CN}(\mathbf{0}, N_0 f_{0r} \mathbf{R}^{-1})$ . Since  $\mathbf{A}_r$  is a diagonal matrix, the soft symbol of  $x_n$  from  $R_r$  is

$$\tilde{y}_{0rn} = h_{0r} \sqrt{\tilde{P}_{rn}} x_n + \tilde{w}_{0rn} \quad (17)$$

where  $\tilde{w}_{0rn} \sim \mathcal{CN}(0, N_0 f_{0r} \varepsilon_n)$  with  $\varepsilon_n$  being the  $n$ th diagonal element of matrix  $\mathbf{R}^{-1}$  associated with symbol  $x_n$ . Since there are  $R$  relaying signals from  $R_r$ 's,  $r = 1, 2, \dots, R$ ,  $U_0$  obtains  $R$  soft symbols of  $x_n$  in the above manner.

From the soft symbols of  $x_n$  in (12) and (17),  $U_0$  forms an  $(R+1) \times 1$  signal vector

$$\mathbf{y}_{0n} = \mathbf{a}_{0n} x_n + \mathbf{w}_{0n} \quad (18)$$

where

$$\mathbf{a}_{0n} = \left[ h_{0n} \sqrt{P_{nn}}, h_{01} \sqrt{\tilde{P}_{1n}}, \dots, h_{0r} \sqrt{\tilde{P}_{rn}}, \dots, h_{0R} \sqrt{\tilde{P}_{Rn}} \right]^T$$

and  $\mathbf{w}_{0n} \sim \mathcal{CN}(0, \mathbf{K}_{0n})$ . We can show that  $\mathbf{K}_{0n} = \text{diag} \{ N_0, N_0 f_{01} \varepsilon_n, \dots, N_0 f_{0r} \varepsilon_n, \dots, N_0 f_{0R} \varepsilon_n \}$ . Let

$$\mathbf{b}_{0n} = \left[ h_{0n} \sqrt{P_{nn}} / N_0, h_{01} \sqrt{\tilde{P}_{1n}} / (N_0 f_{01} \varepsilon_n), \dots, h_{0r} \sqrt{\tilde{P}_{rn}} / (N_0 f_{0r} \varepsilon_n), \dots, h_{0R} \sqrt{\tilde{P}_{Rn}} / (N_0 f_{0R} \varepsilon_n) \right]^T.$$

Then the desired symbol  $x_n$  can be detected based on

$$\hat{x}_{0n} \triangleq \mathbf{b}_{0n}^H \mathbf{y}_{0n} = a_{0n} x_n + w_{0n} \quad (19)$$

where

$$a_{0n} \triangleq \mathbf{b}_{0n}^H \mathbf{a}_{0n} = \frac{P_{nn} |h_{0n}|^2}{N_0} + \sum_{r=1}^R \frac{\tilde{P}_{rn} |h_{0r}|^2}{N_0 f_{0r} \varepsilon_n} \quad (20)$$

and  $w_{0n} \triangleq \mathbf{b}_{0n}^H \mathbf{w}_{0n} \sim \mathcal{CN}(0, \sigma_{0n}^2)$  with  $\sigma_{0n}^2 = a_{0n}$ .

The detection of  $x_n$  at the relay node  $R_r$  can follow a matched-filtering applied to signal  $y_{rn}(t)$  in (2) with respect to the signature waveform  $s_n(t)$  as

$$y_{rn} = \langle y_{rn}(t), s_n(t) \rangle = h_{rn} \sqrt{P_{nn}} x_n + w_{rn} \quad (21)$$

where  $w_{rn} \sim \mathcal{CN}(0, N_0)$ .

#### IV. PERFORMANCE ANALYSIS

In this section, we derive the exact and the asymptotic SER expressions for the use of  $\mathcal{M}$ -PSK modulation in DF protocol in M2P-WNCR. The performance analysis for the case of P2M-WNCR, M2P-WNC and P2M-WNC can easily follow and thus we will offer only the final expressions for use in later sections. Notice that a similar approach can be used to obtain SER expressions for the case of  $\mathcal{M}$ -QAM modulation. For AF protocol, we

offer the conditional SER expression given the channel knowledge.

### A. Exact SER Expressions

For M2P-WNCR, the detection in (19) provides the maximal conditional signal-to-interference-plus-noise ratio  $\gamma_{0n}$  corresponding to the desired symbol  $x_n$  as

$$\gamma_{0n} = \frac{a_{0n}^2}{\sigma_{0n}^2} = \frac{P_{nn}|h_{0n}|^2}{N_0} + \sum_{r=1}^R \frac{\check{P}_{rn}|h_{0r}|^2}{N_0 f_{0r} \varepsilon_n}. \quad (22)$$

For DF protocol, let  $\beta_{rn} \in \{0, 1\}$  for  $r = 1, 2, \dots, R$  represent a detection state associated with  $x_n$  at  $R_r$ . Because  $R_r$  forwards  $x_n$  only if it has successfully detected the symbol,  $\check{P}_{nr} = P_{nr}\beta_{rn}$ . All  $\beta_{rn}$ 's form a decimal number  $S_n = [\beta_{1n} \dots \beta_{rn} \dots \beta_{Rn}]_2$ , where  $[\cdot]_2$  denotes a base-2 number, that represents one of  $2^R$  network detection states associated with  $x_n$  of the  $R$  relay nodes  $R_r$ 's. Because the detection is independent from one relay node to the others,  $\beta_{rn}$ 's are independent Bernoulli random variables with a distribution

$$G(\beta_{rn}) = \begin{cases} 1 - \text{SER}_{rn} & \text{if } \beta_{rn} = 1 \\ \text{SER}_{rn} & \text{if } \beta_{rn} = 0, \end{cases} \quad (23)$$

where  $\text{SER}_{rn}$  is the SER of detecting  $x_n$  at  $R_r$ . Hence, the probability of  $x_n$  detection in state  $S_n$  is

$$\Pr(S_n) = \prod_{r=1}^R G(\beta_{rn}). \quad (24)$$

Given a detection state  $S_n$ , we rewrite (22) as

$$\gamma_{0n|S_n} = \frac{P_{nn}|h_{0n}|^2}{N_0} + \sum_{r=1}^R \frac{P_{rn}\beta_{rn}|h_{0r}|^2}{N_0 \varepsilon_n} \quad (25)$$

where we have used  $f_{0r} = 1, \forall r$  for DF protocol.

In general, the conditional SER for  $\mathcal{M}$ -PSK modulation with conditional SNR  $\gamma$  for a generic set of channel coefficients  $\{h_{uv}\}$  is given by [33]

$$\text{SER}_{|\{h_{uv}\}} = \Psi(\gamma) \triangleq \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(-\frac{b\gamma}{\sin^2 \theta}\right) d\theta \quad (26)$$

where  $b = \sin^2(\pi/M)$ . Based on (21), the SNR of detecting  $x_n$  at  $R_r$  given the channel gain is  $\gamma_{rn} = P_{nn}|h_{rn}|^2/N_0$ . By averaging (26) with respect to the exponential random variable  $|h_{nr}|^2$ , the SER in detecting  $x_n$  at  $R_r$  can be shown as [33]

$$\text{SER}_{rn} = F\left(1 + \frac{bP_{nn}\sigma_{rn}^2}{N_0 \sin^2 \theta}\right) \quad (27)$$

where

$$F(x(\theta)) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \frac{1}{x(\theta)} d\theta. \quad (28)$$

Given a detection state  $S_n$ , which can take  $2^R$  values, the conditional SER in detecting  $x_n$  at  $U_0$  can be calculated using the law of total probability [34] as

$$\text{SER}_{n|\{h_{0n}, h_{0r}, r=1, 2, \dots, R\}} = \sum_{S_n=0}^{2^R-1} \Pr(\hat{x}_n \neq x_n | S_n) \cdot \Pr(S_n) \quad (29)$$

where  $\Pr(S_n)$  follows (24) and

$$\Pr(\hat{x}_n \neq x_n | S_n) = \Psi(\gamma_{0n|S_n}) \quad (30)$$

with  $\gamma_{0n|S_n}$  following (25). By averaging (29) with respect to the exponential random variables  $\{|h_{0n}|^2, |h_{0r}|^2, r=1, 2, \dots, R\}$ , the exact SER in detecting  $x_n$  at  $U_0$  can be given by [33]

$$\text{SER}_n = \sum_{S_n=0}^{2^R-1} F\left(\left(1 + \frac{bP_{nn}\sigma_{0n}^2}{N_0 \sin^2 \theta}\right) \times \prod_{r=1}^R \left(1 + \frac{bP_{rn}\beta_{rn}\sigma_{0r}^2}{N_0 \varepsilon_n \sin^2 \theta}\right)\right) \prod_{r=1}^R G(\beta_{rn}) \quad (31)$$

where  $G(\cdot)$  and  $F(\cdot)$  follows (23) and (28), respectively.

For AF protocol, the conditional SER is

$$\text{SER}_{n|\{h_{uv}\}} = \Psi(\gamma_n) \quad (32)$$

where  $\Psi(\cdot)$  is defined in (26) and  $\gamma_n$  follows (22).

For P2M-WNCR, the information flows from the base node  $U_0$  to the client nodes  $U_1, U_2, \dots, U_N$  through the relay nodes  $R_1, R_2, \dots, R_R$ . The exact SER expression for detecting  $x_n$  at  $U_n$  for the DF protocol in P2M-WNCR can be shown as

$$\text{SER}_n = \sum_{S_n=0}^{2^R-1} F\left(\left(1 + \frac{bP_{nn}\sigma_{n0}^2}{N_0 \sin^2 \theta}\right) \times \prod_{r=1}^R \left(1 + \frac{bP_{rn}\beta_{rn}\sigma_{nr}^2}{N_0 \varepsilon_n \sin^2 \theta}\right)\right) \prod_{r=1}^R G(\beta_{rn}) \quad (33)$$

where  $G(\beta_{rn})$  follows (23) with the SER in (27), where  $\sigma_{rn}^2$  is replaced by  $\sigma_{r0}^2$ .

For M2P-WNC and P2M-WNC as described in Section II-D, the  $(N-1)$  client nodes act as relays to help the other client node. The exact SER expressions are

$$\text{SER}_n = \sum_{S_n=0}^{2^{(N-1)}-1} F\left(\left(1 + \frac{bP_{nn}\sigma_{0n}^2}{N_0 \sin^2 \theta}\right) \times \prod_{\substack{r=1 \\ r \neq n}}^N \left(1 + \frac{bP_{rn}\beta_{rn}\sigma_{0r}^2}{N_0 \varepsilon_n \sin^2 \theta}\right)\right) \prod_{\substack{r=1 \\ r \neq n}}^N G(\beta_{rn}) \quad (34)$$

and

$$\text{SER}_n = \sum_{S_n=0}^{2^{(N-1)}-1} F \left( \left( 1 + \frac{bP_n \sigma_{n0}^2}{N_0 \sin^2 \theta} \right) \times \prod_{\substack{r=1 \\ r \neq n}}^N \left( 1 + \frac{bP_{rn} \beta_{rn} \sigma_{nr}^2}{N_0 \varepsilon_n \sin^2 \theta} \right) \right) \prod_{\substack{n=1 \\ r \neq n}}^N G(\beta_{rn}), \quad (35)$$

respectively.

### B. Asymptotic SER Expressions

To obtain the asymptotic SER performance, i.e., performance at sufficiently high SNR, in detecting  $x_n$  at  $U_0$  in M2P-WNCR, a number of approximations are needed. First, we expect that  $\text{SER}_{rn}$  at high SNR is sufficiently small compared to 1 and thus we can assume  $(1 - \text{SER}_{rn}) \simeq 1$ . Second, because of high SNR, we can ignore the 1's in the argument of  $F(\cdot)$ . Hence, we rewrite (31) as shown in (36) at the bottom of the page, where  $\alpha_{nn} = \frac{P_{nn}}{P_n}$  and  $\alpha_{rn} = \frac{P_{rn}}{P_n}$  denote the fraction of power  $P_n$  allocated at the source node  $U_n$  and a relay node  $R_r$ .

Let  $\Omega_{n0}$  and  $\Omega_{n1}$  denote subsets of the indexes of nodes that decode  $x_n$  erroneously and correctly, respectively. Then  $\Omega_{n0} = \{r : \beta_{rn} = 0\}$  and  $\Omega_{n1} = \{r : \beta_{rn} = 1\}$ . Furthermore,  $|\Omega_{n0}|$  and  $|\Omega_{n1}| \in \{0, 1, \dots, R\}$  and  $|\Omega_{n0}| + |\Omega_{n1}| = R$  for any network detection state  $S_n$ . Hence, in (36), we can show that

$$\mathcal{A} = \left( \frac{N_0}{bP_n} \right)^{1+|\Omega_{n1}|} \frac{g(1 + |\Omega_{n1}|)}{\alpha_{nn} \sigma_{0n}^2 \prod_{r \in \Omega_{n1}} \alpha_{rn} \left( \frac{\sigma_{0r}^2}{\varepsilon_n} \right)} \quad (37)$$

$$\mathcal{B} = \left( \frac{N_0}{bP_n} \right)^{|\Omega_{n0}|} \frac{[g(1)]^{|\Omega_{n0}|}}{\alpha_{nn}^{|\Omega_{n0}|} \prod_{r \in \Omega_{n0}} \sigma_{rn}^2} \quad (38)$$

where

$$g(x) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} [\sin(\theta)]^{2x} d\theta. \quad (39)$$

Consequently, (36) can be rewritten as

$$\text{SER}_n \simeq \left( \frac{bP_n}{N_0} \right)^{-(R+1)} \frac{1}{\sigma_{0n}^2} \times \sum_{S_n=0}^{2^R-1} \frac{g(1 + |\Omega_{n1}|) [g(1)]^{|\Omega_{n0}|}}{\alpha_{nn}^{1+|\Omega_{n0}|} \prod_{r \in \Omega_{n1}} \alpha_{rn} \left( \frac{\sigma_{0r}^2}{\varepsilon_n} \right) \prod_{r \in \Omega_{n0}} \sigma_{rn}^2}. \quad (40)$$

A similar derivation can be applied for the case of P2M-WNCR, M2P-WNC and P2M-WNC and the asymptotic SER

expressions for these cases are

$$\text{SER}_n \simeq \left( \frac{bP_n}{N_0} \right)^{-(R+1)} \frac{1}{\sigma_{n0}^2} \times \sum_{S_n=0}^{2^R-1} \frac{g(1 + |\Omega_{n1}|) [g(1)]^{|\Omega_{n0}|}}{\alpha_{nn}^{1+|\Omega_{n0}|} \prod_{r \in \Omega_{n1}} \alpha_{rn} \left( \frac{\sigma_{nr}^2}{\varepsilon_n} \right) \prod_{r \in \Omega_{n0}} \sigma_{r0}^2} \quad (41)$$

$$\text{SER}_n \simeq \left( \frac{bP_n}{N_0} \right)^{-N} \frac{1}{\sigma_{0n}^2} \times \sum_{S_n=0}^{2^{(N-1)}-1} \frac{g(1 + |\Omega_{n1}|) [g(1)]^{|\Omega_{n0}|}}{\alpha_{nn}^{1+|\Omega_{n0}|} \prod_{r \in \Omega_{n1}} \alpha_{rn} \left( \frac{\sigma_{0r}^2}{\varepsilon_n} \right) \prod_{r \in \Omega_{n0}} \sigma_{rn}^2} \quad (42)$$

and

$$\text{SER}_n \simeq \left( \frac{bP_n}{N_0} \right)^{-N} \frac{1}{\sigma_{n0}^2} \times \sum_{S_n=0}^{2^{(N-1)}-1} \frac{g(1 + |\Omega_{n1}|) [g(1)]^{|\Omega_{n0}|}}{\alpha_{nn}^{1+|\Omega_{n0}|} \prod_{r \in \Omega_{n1}} \alpha_{nr} \left( \frac{\sigma_{nr}^2}{\varepsilon_n} \right) \prod_{r \in \Omega_{n0}} \sigma_{r0}^2}, \quad (43)$$

respectively.

### C. Diversity Order and Interference Impact on Signal-to-Noise Ratio

Diversity order of a communication scheme is defined as

$$\text{div} = - \lim_{\gamma \rightarrow \infty} \frac{\log \text{SER}(\gamma)}{\log \gamma} \quad (44)$$

where  $\text{SER}(\gamma)$  is the SER with the SNR  $\gamma \triangleq P/N_0$ . From (40), the interference impact  $\varepsilon_n$  does not affect the diversity gain and  $x_n$  for all  $n$  is clearly received with a diversity order of  $(R+1)$  at the base node  $U_0$  in M2P-WNCR, as expected. In other words, the use of nonorthogonal codes with cross-correlations  $\rho_{nm} \neq 0$ ,  $n \neq m$ , in WNCR schemes still guarantees full diversity.

To see the interference impact on the SNR due to the use of nonorthogonal code, we consider unique cross-correlations  $\rho_{nm} = \rho$  for all  $n \neq m$ . In this case, we can write (15) as

$$\mathbf{R} = \begin{bmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \cdots & 1 \end{bmatrix} = (1 - \rho) (\mathbf{I} + \mathbf{c}\mathbf{d}^T) \quad (45)$$

where  $\mathbf{I}$  is an  $N \times N$  identity matrix and  $\mathbf{c} = \frac{\rho}{1-\rho} \mathbf{d}$  and  $\mathbf{d} = [1, 1, \dots, 1]^T$  are  $N \times 1$  vectors. Because  $1 + \mathbf{d}^T \mathbf{c} = 1 + \frac{N\rho}{1-\rho} \neq 0$ , it can be shown that [35]

$$\mathbf{R}^{-1} = \frac{1}{1 - \rho} \left( \mathbf{I} - \frac{\mathbf{c}\mathbf{d}^T}{1 + \mathbf{d}^T \mathbf{c}} \right) \quad (46)$$

$$\text{SER}_n \simeq \sum_{S_n=0}^{2^R-1} F \left( \underbrace{\left( \frac{bP_n \alpha_{nn} \sigma_{0n}^2}{N_0 \sin^2 \theta} \right) \prod_{r=1: \beta_{rn}=1}^R \left( \frac{bP_n \alpha_{rn} \sigma_{0r}^2}{N_0 \varepsilon_n \sin^2 \theta} \right)}_{\mathcal{A}} \right) \underbrace{\prod_{r=1: \beta_{rn}=0}^R F \left( \frac{bP_n \alpha_{nn} \sigma_{rn}^2}{N_0 \sin^2 \theta} \right)}_{\mathcal{B}} \quad (36)$$

and thus the diagonal elements of the inverse matrix are

$$\varepsilon_n = \frac{1 + (N - 2)\rho}{(1 - \rho)(1 + (N - 1)\rho)} \triangleq \varepsilon \quad (47)$$

the same for all  $n$ . Because  $\varepsilon \geq 1$  and  $|\Omega_{n1}| \leq R$ ,  $\varepsilon^{|\Omega_{n1}|} \leq \varepsilon^R$ . Hence, we rewrite (40) as

$$\text{SER}_n \lesssim \left(\frac{bP_n}{N_0}\right)^{-(R+1)} \frac{\varepsilon^R}{\sigma_{0n}^2} \times \sum_{S_n=0}^{2^R-1} \frac{g(1 + |\Omega_{n1}|) [g(1)]^{|\Omega_{n0}|}}{\alpha_{nn}^{1+|\Omega_{n0}|} \prod_{r \in \Omega_{n1}} \alpha_{rn} \sigma_{0r}^2 \prod_{r \in \Omega_{n0}} \sigma_{rn}^2} \quad (48)$$

for M2P-WNCR. A similar result can be shown for P2M-WNCR. Based on (48), given the same required SER, the additional SNR in using unique nonorthogonal codes can be shown to be at most  $\Delta\gamma = \frac{R}{R+1} 10 \log \varepsilon$  (dB), where  $\varepsilon$  follows (47). Furthermore,

$$\lim_{N \rightarrow \infty} \Delta\gamma = -\frac{R}{R+1} 10 \log(1 - \rho) \text{ (dB)}. \quad (49)$$

Equation (49) reveals the significance of WNCR schemes when using nonorthogonal codes. The use of such codes, which permit broader applications than orthogonal codes, still guarantees full diversity with the most SNR penalty in (49), regardless of the number of client nodes  $N$ . For instance, when using  $\rho = 0.5$  and  $R = 1$ , the additional SNR is just at most 1.5 dB, relatively small compared to the gain provided by the achieved diversity, as seen in subsequent subsections.

#### D. SER Performance

We perform computer simulations to validate the performance analysis of WNCR schemes. For simulation setup, a network of ten client nodes ( $N = 10$ ) and various numbers of relay nodes with BPSK modulation are considered. In this setup, the relay nodes are placed with equal distance to the base node and the client nodes and thus we assume  $\sigma_{n0}^2 = \sigma_{0n}^2 = 1$ ,  $\sigma_{nr}^2 = \sigma_{rn}^2 = \sigma_{r0}^2 = \sigma_{0r}^2 = 6$  for all  $n$  and  $r$ . The transmit power  $P_n$  corresponding to  $x_n$  is assumed the same for all  $n$  and thus denoted as  $P$  and the equal power allocation [9] is used, where  $P_{nn} = P/2$  and  $P_{nr} = P/(2R)$ . We also assume unit noise variance, i.e.,  $N_0 = 1$  and unique cross-correlations  $\rho_{nm} = \rho$  for  $n \neq m$  and we take  $\rho = 0$  and  $\rho = 0.5$  for orthogonal and nonorthogonal codes, respectively. With this setup, the performance is expected to be the same for all  $x_n$  and hence we will present the performance associated with  $x_1$ .

Fig. 5 presents the SER performance for DF and AF protocols of WNCR schemes; the SER performance of DTX is also displayed in the figure for a comparison. In DTX, each client node transmits its symbol directly to the base node. Without any help from the relay nodes, the transmit power  $P$  is allocated entirely to the source node. In Fig. 5, curves marked with Exact, Asymptotic, Numerical and Simulation correspond to the exact, the asymptotic, the numerical and the simulation performances. The Exact and Asymptotic curves are generated based on (31) and (40), respectively, for DF protocol while (32) is used in AF protocol to obtain the numerical curves.

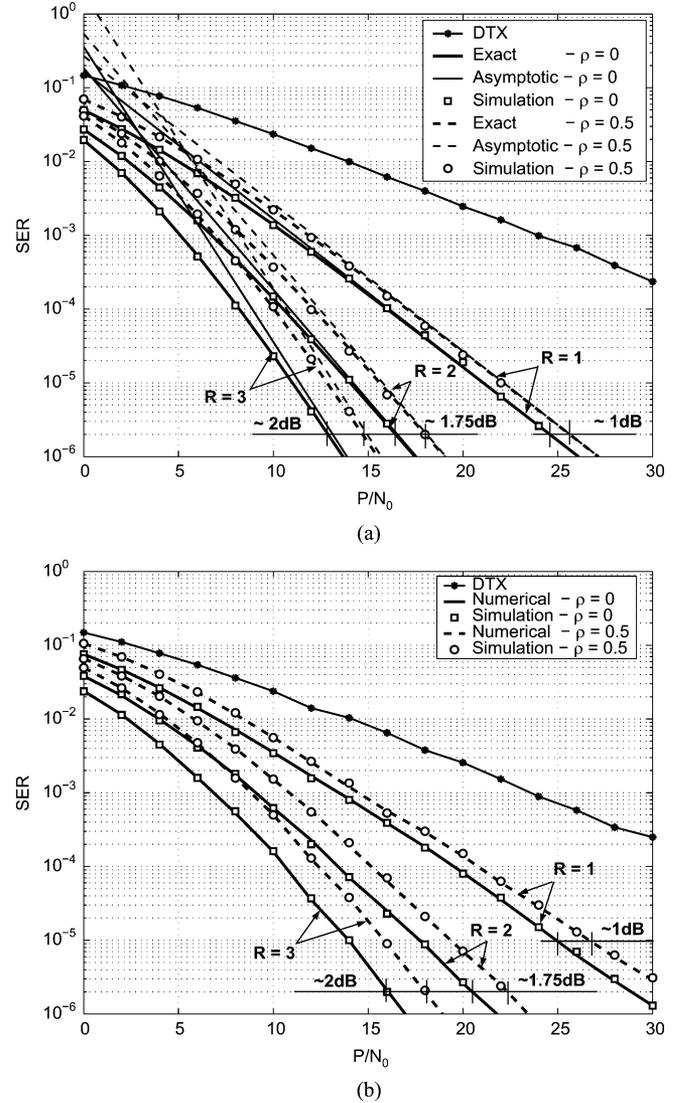


Fig. 5. SER versus SNR performance for BPSK modulation in WNCR schemes with  $N = 10$  and various numbers of relay nodes (a) DF protocol and (b) AF protocol.

From the figure, the simulation curves in DF protocol match the corresponding Exact curves well. Likewise, the Simulation curves and the Numerical curves also match together in AF protocol. In addition, the Asymptotic curves are tight to the Exact curves at high SNR. These validate our performance analysis. The figure also shows that WNCR schemes clearly provide the expected diversity order of  $(R + 1)$  in both DF and AF. Moreover, the performance difference between the orthogonal and nonorthogonal codes is well confined even for  $N = 10$  used in the figure. The gaps between the two performance curves are about 1, 1.75, and 2 dB for  $R = 1, 2,$  and  $3$ , respectively. Although additional SNR is required for transmitting a symbol when using nonorthogonal codes, the SNR gain over direct transmission by the spatial diversity greatly exceeds the loss in SNR, as revealed in the figure.

In Fig. 6, the SER performance of the proposed WNCR schemes is compared with that of a traditional TDMA scheme. A similar simulation setup in Fig. 5 with  $\rho = 0$ ,  $R = 1$  and

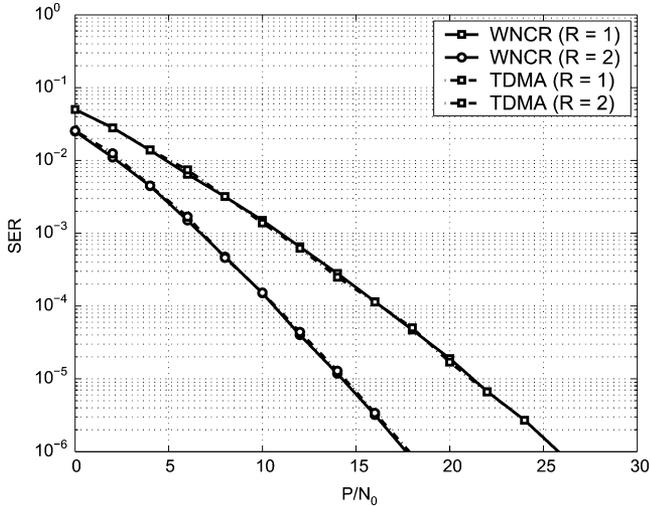


Fig. 6. SER versus SNR performance of WNCR (solid curves) and traditional TDMA (dotted–dashed curves) schemes in DF protocol.

2 and DF protocol is used. For a fair comparison, a relay in the TDMA scheme detects a source symbol based solely on the source signal as in the proposed WNCR schemes. From the figure, the performance of the WNCR schemes is very comparable with that of the traditional TDMA scheme. Note that unlike the WNCR schemes, the traditional TDMA scheme suffers the long transmission delay as discussed in Section I.

## V. PERFORMANCE IMPROVEMENT BY WNC IN CLUSTERING SETTING

The STNCs of WNCR schemes provide general codings that can be applied in many wireless network settings. The most interesting setting is where a group of client nodes located in a close proximity to one another as in a cluster cooperate together to improve their performance. In this case, the client nodes also act as relays, helping one another in transmission between themselves and the distant base node. We refer this network setting as clustering setting and its STNCs were discussed in Section II-D. In this section, we study the performance of this network in terms of SER performance, transmit power saving, range extension and transmission rate improvement in comparison with DTX.

To see the benefits of using WNC schemes, the channel variances between any two client nodes are assumed the same and denoted as  $\sigma_{cc}^2$ . Likewise is for the channel variances between any client nodes and the base node, denoted as  $\sigma_{bc}^2$ . Since client nodes are in a cluster that is distant from the base node, it is assumed that  $\sigma_{bc}^2 = 1$  and  $\sigma_{cc}^2 = 30$ . Equal power allocation is used in this study. In addition, BPSK modulation and the same cross-correlation  $\rho = 0.5$  are used.

### A. SER Performance of WNC Schemes

In clustering setting, the channel links among the client nodes are much stronger than the links between a client node and the base node. This could impact the performance of M2P-WNC and P2M-WNC differently although they share similar SER expressions.

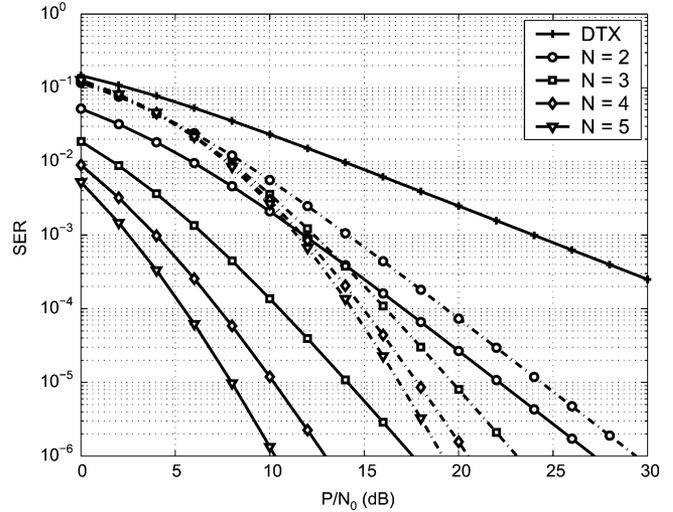


Fig. 7. SER versus SNR performance of DF P2M-WNC (solid curves) and M2P-WNC (dotted–dashed curves):  $\sigma_{bc}^2 = 1$ ,  $\sigma_{cc}^2 = 30$  and  $\rho = 0.5$ .

Fig. 7 reveals the performance of P2M-WNC (with the solid curves) and M2P-WNC (with the dotted–dashed curves) in a clustering setting for various  $N$  values. P2M-WNC clearly outperforms M2P-WNC greatly. The larger the number of nodes in the cluster, the larger the SNR gain given the same SER. For instance, a gain of 2 dB for  $N = 2$  increases to 9 dB for  $N = 5$  for the same SER of  $10^{-6}$ . The reason relates to the strength of the source signal and the relaying signals. Both schemes have the same source signal strength since they share the same source power and source-destination channel variance. However, P2M-WNC has stronger relaying signals, due to the higher relay-destination channel variances. A strong relaying signal ensures a correctly-detected symbol to be forwarded with high quality. This behavior suggests that in applications to M2M transmissions as discussed in Section II-D, P2M-WNC should be used. In other words, the cooperation in M2M transmissions should be imposed at the receiving cluster.

### B. Power Saving

Given the same SER and transmission range as in DTX, we examine the power saving using WNC schemes over DTX in this subsection. The power saving of scheme 1 over scheme 2 is defined as the difference in transmit power between scheme 2 and scheme 1 to achieve the same SER. Fig. 8 reveals the power saving for various SER with a fixed number of client nodes  $N = 4$ . From the figure, the lower the SER is the higher the power saving over DTX. For instance, at  $SER = 10^{-4}$ , the saving associated with M2P-WNC and P2M-WNC are 19 and 27 dB, respectively. The saving increases to 26 and 34 dB, respectively, as SER of  $10^{-5}$ . P2M-WNC achieves a better saving as expected due to its better performance in clustering setting over M2P-WNC, as discussed in Section V-A.

In Fig. 9, we study the power saving as the number of client nodes  $N$  varies. The SER is kept fixed at  $SER = 10^{-6}$ . From the figure, higher power saving is achieved by WNC schemes as  $N$  increases. This is due to the increment in the diversity order that the two schemes offer. However, the increment in power saving becomes saturate at high  $N$  values. The reason relates to the

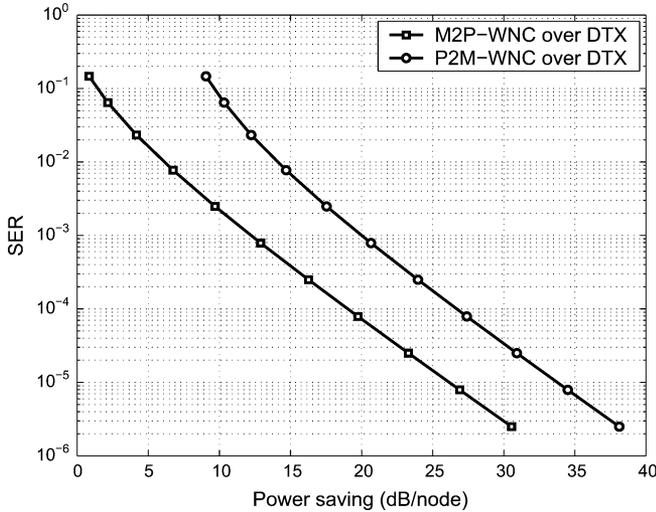


Fig. 8. Power saving per transmitted symbol of WNC over DTX for various SER:  $\sigma_{bc}^2 = 1$ ,  $\sigma_{cc}^2 = 30$ ,  $N = 4$ ,  $\rho = 0.5$  and  $\alpha = 2.5$ .

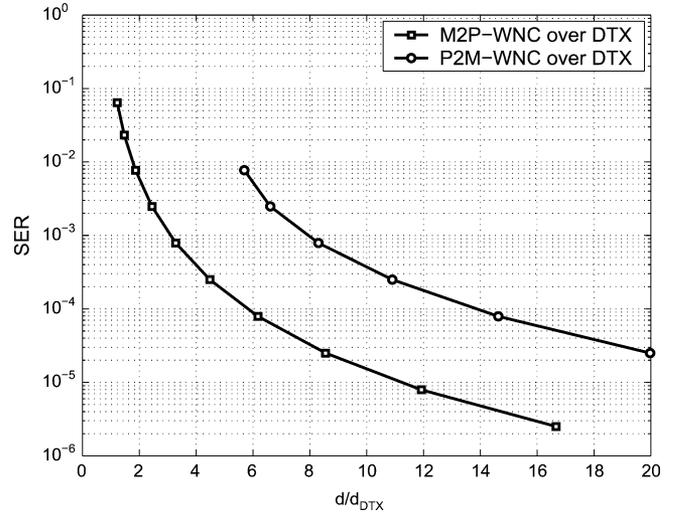


Fig. 10. Range extension of WNC over DTX for various SER:  $\sigma_{cc}^2 = 30$ ,  $N = 4$ ,  $\rho = 0.5$ , and  $\alpha = 2.5$ .

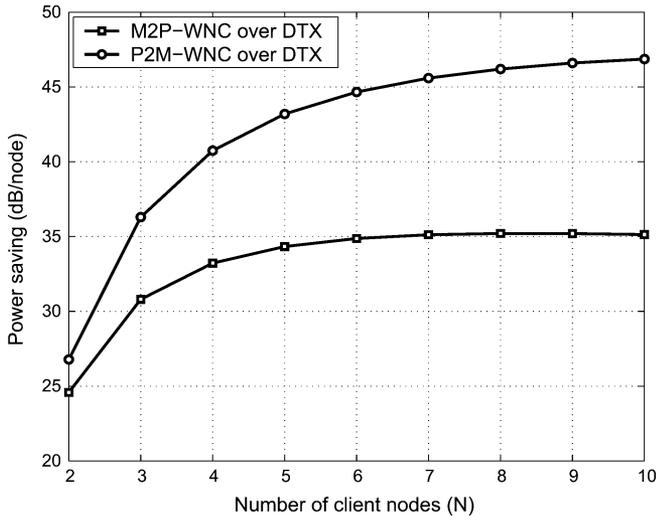


Fig. 9. Power saving per transmitted symbol of WNC over DTX for various number of client nodes  $N$ :  $\sigma_{bc}^2 = 1$ ,  $\sigma_{cc}^2 = 30$ ,  $\rho = 0.5$ , and SER =  $10^{-6}$ .

reduction of the marginal gain in power saving, defined as the difference in power saving between two consecutive numbers of client nodes. This suggests that we should not use WNC for large numbers of client nodes because the use does not provide much gain in power saving but would lead to a high system complexity. When the number of client nodes in a cluster is large, we can form sub-clusters with an appropriate value of  $N$ . WNC can be applied within each group to achieve the desired diversity order.

### C. Range Extension

The diversity achieved by WNC schemes can be used to extend the transmission range between the client nodes and the base node in comparison with DTX. Given the same quality of service, represented by a required SER and the same transmit power, the range extension of scheme 1 over scheme 2 is defined as the ratio of the distance between the client nodes and the base node in scheme 1 over that in scheme 2. Fig. 10 shows the range

extension for various SER with BPSK modulation and a fixed number of client nodes  $N = 4$ . In the figure, we keep  $\sigma_{cc}^2 = 30$  and vary  $\sigma_{bc}^2$  to achieve the range extension with the assumption of the path loss exponent  $\alpha = 2.5$ . This scenario replicates a group of client nodes that move away from the base node. Similar trend as in Fig. 8 can be seen here. From the figure, the lower the required SER is the larger the range extension for WNC over DTX. Moreover, the range extension is quite tremendous at low required SERs. For instance, at SER =  $10^{-5}$ , the range extension is 11 and 30 times for M2P-WNC and P2M-WNC, respectively. This is due to the higher power saving at lower SER as revealed in Fig. 8 that can turn into extended transmission ranges. Again, P2M-WNC results in a better performance with the higher range extension.

### D. Transmission Rate Improvement

Given the same quality of service and transmission range as in DTX, the power saving in WNC schemes can be used to transmit the signal with a larger constellation size and hence to increase the transmission rate. In this subsection, we study the transmission rate improvement over DTX by using WNC schemes. We assume transmission in DTX uses fixed modulation of BPSK and thus the transmission rate of DTX is always 1 bit per time slot (bpts) in this study. For WNC, we start with BPSK modulation and search for the maximum constellation size  $\mathcal{M}$  such that the resulting SER does not exceed the SER in DTX, given that they all have the same transmit power. In this way, the SER performance of WNC scheme should be the same or better than that of DTX. For  $\mathcal{M} = 2^m$ , where  $m$  is the number of bits associated with the constellation size  $\mathcal{M}$ , the transmission rate in WNC is  $m/2$  bpts since it requires  $2N$  time slots to transmit  $N$  symbols.

Fig. 11 shows the transmission rates that can be achieved by DTX and WNC for various SNR. From the figure, several points are worth noted. First, WNC schemes interestingly can provide higher transmission rates than DTX although they take more time to deliver a symbol. As seen in the figure, only M2P-WNC

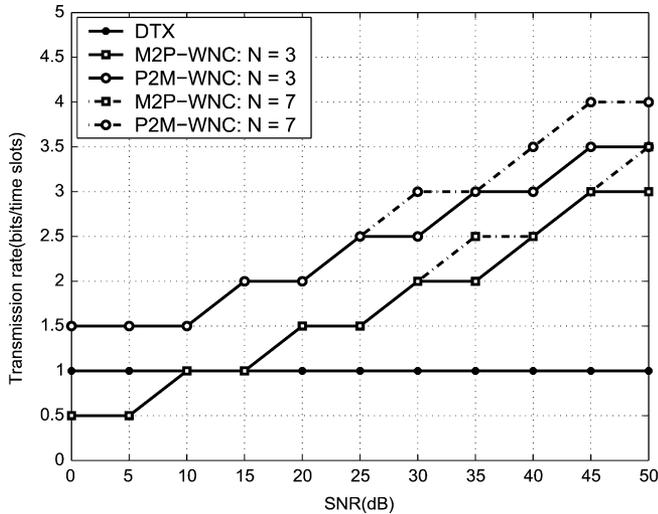


Fig. 11. Transmission rate of WNC and DTX for various SNR:  $\sigma_{bc}^2 = 1$ ,  $\sigma_{cc}^2 = 30$ , and  $\rho = 0.5$ .

with  $\text{SNR} < 10$  dB results in a smaller transmission rate, compared to DTX. The reason relates to the power gaps in SER performance between these schemes and DTX that allows them to increase the constellation size of the  $\mathcal{M}$ -PSK modulation and thus the transmission rate. Secondly, the increase in the number of the client nodes does not lead to substantial increase in transmission rate at low and moderate SNR, as shown in the figure. This suggests that we should not use WNC for large numbers of client nodes. When the number of client nodes in a cluster is large, we can form sub-clusters and WNC is applied on each group.

## VI. CONCLUSION

In this paper, we proposed a novel framework for cooperative communications that help to achieve spatial diversity with low transmission delay and eliminate the issue of imperfect frequency and timing synchronization. The objective was realized by the use of WNC schemes and their associated STNCs that were applied in a network consisting of  $N$  client nodes and a base node with the assistance from  $R$  relay nodes. Signal model for the proposed STNCs was presented, signal detection was introduced and SER performance was analyzed to confirm that a full diversity order of  $(R+1)$  was achieved for each transmitted symbol. The STNCs require only  $(N+R)$  time slots, a reduction from  $2N$  time slots in traditional FDMA and CDMA cooperative communications given that  $N$  is usually greater than  $R$  and  $N(R+1)$  time slots in traditional TDMA cooperative communications. We also applied WNC schemes to M2P and P2M transmissions, where the client nodes acted as relays to help one another to improve their transmission performance. The performance in clustering setting was studied to reveal the improvement in power saving, range extension and transmission rate.

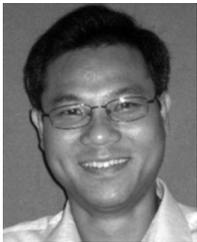
## VII. ACKNOWLEDGEMENTS

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