Space-Time Network Codes Utilizing Transform-Based Coding

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Abstract— **Cooperative communications can be used to improve communication reliability. However, the transmissions from different relaying nodes result in a great challenge in practice. The use of TDMA for relaying transmissions causes large transmission delay. In contrast, FDMA and CDMA associate with the issue of imperfect frequency and timing synchronization due to the simultaneous transmissions from the relaying nodes. In this work, we propose a space-time network coding (STNC) scheme utilizing transform-based coding to improve communication reliability while maintaining a stable network throughput and overcoming the issue of imperfect frequency and timing synchronization. Based on TDMA, a node in the network forms a linearly-coded signal from the overheard symbols and transmits it to the destination in its dedicated time slot. The pairwise error probability (PEP) is analyzed and design criteria of the STNC are derived to ensure achieving full diversity. Simulations are conducted to verify the performance of the proposed scheme and to reveal its advantage over a distributed space-time block coding scheme under timing synchronization errors.**

I. INTRODUCTION

It is well-known that performance of communication systems degrades greatly when operating in radio frequency environments characterized by multipath propagation such as urban environments. Diversity techniques such as time diversity, frequency diversity, and spatial diversity can improve transmission reliability. Among these techniques, spatial diversity achieved by cooperative communications [1] has become recently attractive.

In cooperative communications, nodes acting as relays retransmit the overheard information to a destination. The distributed antennas among nodes are used to provide spatial diversity without the need to use multiple antennas at the source. Various cooperative diversity protocols have been proposed and analyzed in [2], [3], [4]. They often consist of two phases: source transmission and relay transmission. In the first phase, a source broadcasts its information to a destination and the relays, which then forward the overheard information to the destination in the second phase.

Much of research in cooperative communications has focused on simultaneous transmission from two or more nodes with an assumption of perfect frequency and timing synchronization [4], [5]. However, such an assumption is difficult to be met in practice, especially in mobile conditions where nodes move at different speeds and in different directions. The imperfect synchronization causes the intersymbol interference (ISI), which is the source of system performance degradation [6], [7]. For timing synchronization, the coordination to make signals received simultaneously at the destination is challenging due to differences in propagation time among nodes, processing time in each radio, and timing estimation error. The frequency synchronization issue occurs when each node has an independent local oscillator generating a transmit frequency with certain variation to the nominal. Various frequency mismatches occur at once at the destination and make it difficult to estimate and compensate all the frequency offsets.

To overcome the imperfect synchronization issue that prohibits two or more nodes from transmitting at the same time, time-division multiple access (TDMA) would be the most commonly-used technique in many applications. However, TDMA is extremely inefficient in cooperative communications, especially for large numbers of relay nodes to achieve high diversity orders. For a network of N client nodes helping one another in transmitting their information to a common base node, TDMA requires N^2 time slots [4], causing large transmission delays as N increases. Therefore, there is an essential need to reduce the total required time slots in TDMA for cooperative communications while maintaining the spatial diversity and overcoming the issue of imperfect frequency and timing synchronization.

The recent work on space-time network coding (STNC) [8] reveals the mechanism to achieve the foretold objectives. A client node acting as a relay linearly combines the set of symbols as a unique signal and transmits it to the base node in its dedicated time slot. As a result, the foretold issue of imperfect synchronization disappears and only $2N$ time slots are required, a significant reduction from that in traditional cooperative communications using TDMA.

The combining technique in this work is different from that in [8], where symbols at a client node are formed in CDMA or OFDMA manners. In this work, transform-based coding [9], [10], whose coding matrices compose a set of parameters that are optimized for conventional signal constellations, is used to generate the code symbols. The combining of symbols at a client node here is also different from that in traditional network coding [11]. Such a combining technique could be possible and subject to a future investigation. We analyze the pairwise error probability (PEP) performance of the proposed STNC scheme and derive the design criteria of the network coding matrix. Simulations are provided to verify the proposed STNC achieves full diversity. In addition, a performance comparison between the proposed STNC and a distributed space-time block coding (DSTBC) scheme under timing synchronization errors is investigated.

The rest of this paper is organized as follows. After this introduction section, STNC utilizing transform-based coding is introduced in Section II. The system model and signal detection are also presented in this section. The performance analysis and the code design is derived in Section III. In Section IV, the impact of timing synchronization errors on DSTBC performance is presented. The simulations and conclusion are provided in Section V and Section VI, respectively.

Notation: Lower and upper case bold symbols denote column vectors and matrices, respectively. *, $^\mathcal{T}$, and $^\mathcal{H}$ de-
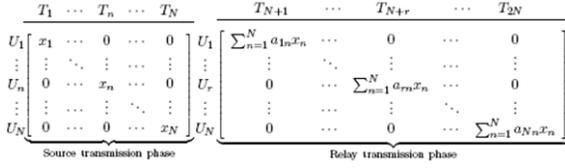
Fig. 1. Space-time network codes utilizing transform-based coding.

note complex conjugate, transpose, and Hermitian transpose, respectively. $\mathsf{E}\{.\}$, $\mathrm{diag}\{.\}$, and $|.|$ represent the expectation, a diagonal matrix, the size of a set, respectively.

II. SYSTEM MODEL AND SIGNAL DETECTION

We consider a wireless network that consists of $(N + 1)$ nodes denoted as $U_0, U_1, U_2, ..., U_N$, where $U_1, U_2, ..., U_N$ are source nodes transmitting their information to the common destination node U_0. For convenience, we refer U_0 as a base node and $U_1, U_2, ..., U_N$ as client nodes as in cellular networks. In this work, we consider the base node has M antennas while the client nodes are single-antenna devices. The channels are modelled as narrow-band Rayleigh fading with additive white Gaussian noise (AWGN). We assume that the antenna separations are at least a half of wavelengths apart and thus the channels are spatially uncorrelated. The channel coefficient between an arbitrary receiver u and transmitter v is defined as $h_{uv} \sim \mathcal{CN}\left(0, \sigma_{uv}^2\right)$, where σ_{uv}^2 is the channel variance.

A. System Model

The system model comprises a source transmission phase and a relay transmission phase. Each transmission phase consists of N time slots, and the network requires $2N$ time slots for transmissions of the N symbols to U_0. In the source transmission phase, each client node U_n for $n = 1, 2, ..., N$ is assigned a time slot, denoted T_n, to broadcast its symbol x_n to other client nodes U_r, $r \neq n$, where x_n is from an $\mathcal{M}$-QAM constellation $\mathbb{X}$. At the end of this phase, each client node U_r for $r = 1, 2, ..., N$ possesses a set of N symbols $x_1, x_2, ..., x_N$, comprising its own symbol x_r. In the relay transmission phase, U_r forms a single linearly-coded signal, a linear combination of the overheard symbols and its own one, and transmits the signal to the base node in its dedicated time slot T_{N+r}. U_r detects the symbol x_n based on the source signal and re-encode the symbol in its linearly-coded signal if the decoding is successful, the so called DF protocol. A detection state, a success or a failure in detecting a symbol, can be determined based on the amplitude of the estimated channel coefficient [2] or the received signal-to-noise ratio (SNR) [4]. Notice that this DF scheme is also called the selective-relaying protocol in literature [2]. The transmissions from the N client nodes in the source and relay transmission phases are illustrated in Figure 1, which guarantee a single transmission in the network at any given time slot to eliminate the issue of imperfect synchronization in traditional cooperative communications.

The received signal at U_r from U_n in the source transmission phase is

$$y_{rn} = h_{rn}\sqrt{P_n}x_n + w_{rn} \qquad (1)$$

where P_n is the transmit power at U_n in the source transmission phase and $w_{rn} \sim \mathcal{CN}(0, N_0)$ is the AWGN. In the relay

transmission phase, the received signal at the mth antenna, $m = 1, 2, ..., M$, of the base node U_0 from U_r is

$$y_{mr} = h_{mr}\sqrt{P_r}s_r + w_{mr} \qquad (2)$$

where P_r is the transmit power in the relay transmission phase, $w_{mr} \sim \mathcal{CN}(0, N_0)$ is the AWGN, and

$$s_r = \sum_{n=1}^{N} a_{rn}\beta_{rn}x_n = \mathbf{a}_r^{\mathcal{T}}\mathbf{B}_r\mathbf{x} \qquad (3)$$

is the linearly-coded symbol at U_r. In (3), $\mathbf{a}_r = [a_{r1}, ..., a_{rn}, ..., a_{rN}]^{\mathcal{T}}$, $\mathbf{a}_r^{\mathcal{T}}\mathbf{a}_r = 1$ to normalize the transmitted signal, is the code vector at U_r with a_{rn} being the code coefficient associated with x_n, $\mathbf{x} = [x_1, x_2, ..., x_N]^{\mathcal{T}}$ is the transmitted symbol vector, and $\mathbf{B}_r = \mathrm{diag}\{\beta_{r1}, ..., \beta_{rn}, ..., \beta_{rN}\}$ is a matrix representing the detection state at U_r with β_{rn} being detection state associated with x_n. We have $\beta_{rr} = 1$ always since U_r has its own symbol x_r and for $r \neq n$,

$$\beta_{rn} = \begin{cases} 1 & \text{if } U_r \text{ decodes } x_n \text{ correctly} \\ 0 & \text{otherwise} \end{cases}. \qquad (4)$$

The received signals at U_0 can be expressed in a matrix form as

$$\mathbf{Y} = \mathbf{HS} + \mathbf{W}, \qquad (5)$$

where $\mathbf{Y}$, $\mathbf{H}$, and $\mathbf{W}$ are $M \times N$ matrices comprising the received signals, the channel coefficients, and the AWGN's, and $\mathbf{S} = \mathrm{diag}\{\sqrt{P_1}s_1, ..., \sqrt{P_r}s_r, ..., \sqrt{P_N}s_N\}$ is the $N \times N$ code matrix.

B. Signal Detection

To detect the transmitted symbols, we assume that receivers have a full knowledge of the channel state information (CSI), which can be acquired using a preamble in the transmitted signal as usually done in systems such as 802.11 [12]. We also assume that the base node knows the detection states at the relay nodes. This can be done by using an N-bit indicator in the relaying signal. Notice that in practice, information is transmitted in packets [12] that contain a large number of symbols. Each packet is detected as a whole, and a CRC [13] is sufficient to determine the detection state of the packet. Thus one bit per packet results in a minimal overhead.

To derive code criteria in the next section, maximum-likelihood (ML) detectors, which is a minimum distance rule, are used. The detected symbol vector at the base node U_0 is

$$\widehat{\mathbf{x}} = \underset{\mathbf{x} \in \mathbb{X}^N}{\mathrm{argmin}} \left\{\| \mathbf{Y} - \mathbf{HS} \|_{\mathcal{F}}^2\right\}, \qquad (6)$$

where $\| . \|_{\mathcal{F}}$ denotes the Frobenius norm. As shown in (6), detecting $\mathbf{x}$ requires testing $|\mathbb{X}|^N$ possible vectors $\mathbf{x}$, which is computationally prohibitive as N and/or $|\mathbb{X}|$ are large. In that case, sphere decoder [14], which results in suboptimal decoding, can be used. The detection of x_n at client node U_r in the source transmission phase follows

$$\widehat{x}_n = \underset{x \in \mathbb{X}}{\mathrm{argmin}} \left\{|y_{rn} - h_{rn}\sqrt{P_{nn}}x|^2\right\}. \qquad (7)$$

III. PERFORMANCE ANALYSIS AND CODE DESIGN

Let us define $\mathbf{y} = \mathrm{vec}(\mathbf{Y})$, $\mathbf{h} = \mathrm{vec}(\mathbf{H})$, and $\mathbf{w} = \mathrm{vec}(\mathbf{W})$, where $\mathrm{vec}(.)$ denotes the vectorization of a matrix by stacking the columns of the matrix on the top of one another. Then the signal matrix in (5) can be rewritten in vector form as

$$\mathbf{y} = \mathbf{Dh} + \mathbf{w}, \qquad (8)$$

where $\mathbf{D} = \mathbf{S} \otimes \mathbf{I}_M$ with $\otimes$ denoting the Kronecker product [15] and $\mathbf{I}_M$ being the identity matrix of size M. Suppose that $\mathbf{D}$ and $\widehat{\mathbf{D}}$ are two different matrices related to two different code matrices $\mathbf{S}$ and $\widetilde{\mathbf{S}}$. The base node applies the ML detector in (6), and thus the conditional PEP is given by

$$Prob\left(\mathbf{S} \to \widetilde{\mathbf{S}} \mid \{\mathbf{B}_r\}, \mathbf{h}\right) = Prob\left(\| \mathbf{y} - \mathbf{Dh} \|^2 > \right.$$
$$\left. \| \mathbf{y} - \widetilde{\mathbf{D}}\mathbf{h} \|^2 \mid \{\mathbf{B}_r\}, \mathbf{h}, \mathbf{S} \text{ transmitted}\right). \quad (9)$$

The conditional PEP can be expressed in a quadratic form of a complex Gaussian random variable as

$$Prob\left(\mathbf{S} \to \widetilde{\mathbf{S}} \mid \{\mathbf{B}_r\}, \mathbf{h}\right) = Prob(Q < 0 \mid \{\mathbf{B}_r\}, \mathbf{h}), \quad (10)$$

where

$$Q = [\mathbf{z}_1^{\mathcal{H}} \ \mathbf{z}_2^{\mathcal{H}}] \begin{bmatrix} \mathbf{I}_{MN} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I}_{MN} \end{bmatrix} \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix}, \quad (11)$$

in which $\mathbf{0}$ represents a zero matrix, $\mathbf{z}_1 = (\mathbf{D} - \widetilde{\mathbf{D}})\mathbf{h} + \mathbf{w}$, and $\mathbf{z}_2 = \mathbf{w}$. Because $\mathbf{h}$ and $\mathbf{w}$ are mutually independent random vectors, the conditional PEP can be averaged over the channel realization $\mathbf{h}$. At high SNR, the conditional PEP between $\mathbf{S}$ and $\widetilde{\mathbf{S}}$ can be upper-bounded as [16]

$$Prob\left(\mathbf{S} \to \widetilde{\mathbf{S}} \mid \{\mathbf{B}_r\}\right) \le \left(\begin{array}{c} 2\nu - 1 \\ \nu - 1 \end{array} \right) N_0^{\nu} \prod_{i=1}^{\nu} \gamma_i^{-1}, \quad (12)$$

where ν and γ_i is the rank and the ith non-zero eigenvalue of the matrix $\left(\mathbf{D} - \widetilde{\mathbf{D}}\right) \mathbf{R}_{\mathbf{h}} \left(\mathbf{D} - \widetilde{\mathbf{D}}\right)^{\mathcal{H}}$ with $\mathbf{R}_{\mathbf{h}} \triangleq \mathsf{E}\{\mathbf{hh}^{\mathcal{H}}\} =$ diag$\{\sigma_{11}^2, ..., \sigma_{M1}^2, \sigma_{12}^2, ..., \sigma_{M2}^2, ..., \sigma_{1N}^2, ..., \sigma_{MN}^2\}$ being the correlation matrix of the channel vector $\mathbf{h}$. Based on the relationship between $\mathbf{D}$ and $\mathbf{S}$ in (8) and the fact that $\mathbf{D}, \widetilde{\mathbf{D}}$, and $\mathbf{R}_{\mathbf{h}}$ are all diagonal matrices, [15]

$$\left(\mathbf{D} - \widetilde{\mathbf{D}}\right) \mathbf{R}_{\mathbf{h}} \left(\mathbf{D} - \widetilde{\mathbf{D}}\right)^{\mathcal{H}} = (\boldsymbol{\Delta}_{\mathbf{S}} \otimes \mathbf{I}_M) \mathbf{R}_{\mathbf{h}}, \quad (13)$$

where we define

$$\boldsymbol{\Delta}_{\mathbf{S}} \triangleq \left(\mathbf{S} - \widetilde{\mathbf{S}}\right)\left(\mathbf{S} - \widetilde{\mathbf{S}}\right)^{\mathcal{H}}$$
$$= \text{diag}\left\{P_1|\mathbf{a}_1^{\mathcal{T}}\mathbf{B}_1\Delta\mathbf{x}|^2, ..., P_r|\mathbf{a}_r^{\mathcal{T}}\mathbf{B}_r\Delta\mathbf{x}|^2, \right.$$
$$\left. ..., P_N|\mathbf{a}_N^{\mathcal{T}}\mathbf{B}_N\Delta\mathbf{x}|^2\right\}, \quad (14)$$

in which $\Delta\mathbf{x} = \mathbf{x} - \widetilde{\mathbf{x}}$ with two distinct transmitted symbol vectors $\mathbf{x}$ and $\widetilde{\mathbf{x}}$. Given that rank$(\mathbf{R}_{\mathbf{h}}) = MN$, $\nu = M\nu_{\mathbf{S}}$, where $\nu_{\mathbf{S}} \triangleq$ rank$(\boldsymbol{\Delta}_{\mathbf{S}})$. In (14), the maximum rank of $\boldsymbol{\Delta}_{\mathbf{S}}$ is N. We should design the coding vectors $\mathbf{a}_r$'s to achieve this rank and the design criteria will be derived later in the section. In that case, the N eigenvalues of $\boldsymbol{\Delta}_{\mathbf{S}}$ are $\lambda_r = P_r|\mathbf{a}_r^{\mathcal{T}}\mathbf{B}_r\Delta\mathbf{x}|^2$ for $r = 1, 2, ..., N$, and we can express (12) as

$$Prob\left(\mathbf{S} \to \widetilde{\mathbf{S}} \mid \{\mathbf{B}_r\}\right) \le \left(\begin{array}{c} 2MN - 1 \\ MN - 1 \end{array} \right) N_0^{MN} \times$$
$$\left(\prod_{m=1}^{M} \prod_{r=1}^{N} \frac{1}{\sigma_{mr}^2}\right) \left(\prod_{r=1}^{N} \left(P_r|\mathbf{a}_r^{\mathcal{T}}\mathbf{B}_r\Delta\mathbf{x}|^2\right)^{-M}\right). \quad (15)$$

Averaging with respect to the detection state matrices $\{\mathbf{B}_r\}$ with a notice that the detection state matrices are mutually independent because the detection at each client node is

independent from one another, the PEP can be rewritten as

$$Prob\left(\mathbf{S} \to \widetilde{\mathbf{S}}\right) \le \left(\begin{array}{c} 2MN - 1 \\ MN - 1 \end{array} \right) N_0^{MN} \left(\prod_{m=1}^{M} \prod_{r=1}^{N} \frac{1}{\sigma_{mr}^2}\right)$$
$$\times \underbrace{\prod_{r=1}^{N} \mathsf{E}\left\{\left(P_r \left|\sum_{n=1}^{N} a_{rn}\beta_{rn}\Delta x_n\right|^2\right)^{-M}\right\}}_{\mathcal{A}}, \quad (16)$$

where $\Delta x_n = x_n - \widetilde{x}_n$.

Let $S_r \triangleq [\beta_{r1}, ..., \beta_{rn}, ..., \beta_{rN}]_2$ for $n \ne r$, where $[.]_2$ denotes a base-2 number. S_r is a decimal number representing one of 2^{N-1} detection states at U_r. Because symbols at a client node are independently detected, β_{rn}'s are independent Bernoulli random variables with a distribution

$$G(\beta_{rn}) = \left\{ \begin{array}{ll} 1 - p_{rn} & \text{if } \beta_{rn} = 1 \\ p_{rn} & \text{if } \beta_{rn} = 0 \end{array} \right., \quad (17)$$

where p_{rn} is the symbol error rate (SER) for detecting x_n at U_r. For $\mathcal{M}$-QAM modulation, it can be shown that [4]

$$p_{rn} = F\left(1 + \frac{b\sigma_{rn}^2 P_n}{N_0 \sin^2(\theta)}\right), \quad (18)$$

where $b = \frac{3}{2(\mathcal{M}-1)}$ and

$$F(g(\theta)) = \frac{4C}{\pi} \int_0^{\frac{\pi}{2}} \frac{1}{g(\theta)} d\theta - \frac{4C^2}{\pi} \int_0^{\frac{\pi}{4}} \frac{1}{g(\theta)} d\theta, \quad (19)$$

in which $C = 1 - \frac{1}{\sqrt{\mathcal{M}}}$ and $g(\theta)$ denotes a function of θ. At high SNR, we can ignore the 1 in (18) and obtain

$$p_{rn} \simeq \mathcal{K} \frac{N_0}{\sigma_{rn}^2 P_n}, \quad (20)$$

where

$$\mathcal{K} = \frac{4C}{b\pi} \int_0^{\frac{\pi}{2}} \sin^2\theta d\theta - \frac{4C^2}{b\pi} \int_0^{\frac{\pi}{4}} \sin^2\theta d\theta. \quad (21)$$

Hence the probability of detection state at U_r in state S_r is

$$Prob(S_r) = \prod_{n=1; n \ne r}^{N} G(\beta_{rn}) \simeq \prod_{n=1; \beta_{rn}=0}^{N} p_{rn}, \quad (22)$$

where we use the approximation $1 - p_{rn} \simeq 1$ for high SNR.

Given a detection state S_r, which can take 2^{N-1} values,

$$\mathcal{A} \simeq \prod_{r=1}^{N} \sum_{S_r=0}^{2^{N-1}-1} \left(\frac{1}{P_r \left|\sum_{n=1; \beta_{rn}=1}^{N} a_{rn}\Delta x_n\right|^2}\right)^M \prod_{n=1; \beta_{rn}=0}^{N} p_{rn}. \quad (23)$$

Substituting (20) into (23) and ignoring high order terms of SERs, we can show that

$$\mathcal{A} \simeq P^{-MN} \prod_{r=1}^{N} \left(\frac{1}{|\mathbf{a}_r^{\mathcal{T}}\Delta\mathbf{x}|^2}\right)^M \left(\frac{1}{\alpha_r^M} + \sum_{n=1; n \ne r}^{N} \frac{\mathcal{K}N_0 \left(d_{\max}^2\right)^M}{\alpha_n \sigma_{rn}^2}\right), \quad (24)$$

where $d_{\max}$ is the maximum Euclidean distance of $\mathbb{X}$ and $\alpha_r = P_r/P$ and $\alpha_n = P_n/P$ are the fractions of total transmit power P allocated for the source and relay transmission phases, respectively. Substituting (24) into (16), the PEP is

$$Prob\left(\mathbf{S} \to \widetilde{\mathbf{S}}\right) \le \binom{2MN - 1}{MN - 1} \prod_{m=1}^{M} \prod_{r=1}^{N} \left(\frac{1}{\sigma_{mr}^2 |\mathbf{a}_r^{\mathcal{T}}\Delta\mathbf{x}|^2}\right.$$
$$\left.\times \left(\frac{1}{\alpha_r^M} + \sum_{n=1; n \ne r}^{N} \frac{\mathcal{K}N_0 \left(d_{\max}^2\right)^M}{\alpha_n \sigma_{rn}^2}\right)\right) \left(\frac{P}{N_0}\right)^{-MN}. \quad (25)$$
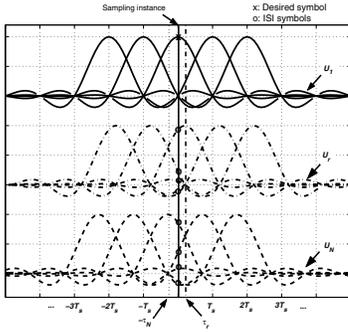
Fig. 2.   Impact of timing synchronization errors on DSTBC.

From (25), the design criteria for the coding vectors $\mathbf{a}_r$'s are as follows.

Diversity criterion: The system provides full diversity with order MN if $\left|\mathbf{a}_r^T \Delta \mathbf{x}\right| \neq 0$, $\forall r \in [1, N]$, $\forall \mathbf{x}, \widetilde{\mathbf{x}} \in \mathbb{X}$.

Product criterion: The minimum value of the product $\prod_{r=1}^{N} \mathbf{a}_r^T \Delta \mathbf{x}$ over all pairs of distinct symbol vectors $\mathbf{x}$ and $\widetilde{\mathbf{x}}$ should be as large as possible. The product criterion is of secondary importance and should be optimized if full diversity is achieved.

Let $\mathbf{A} \triangleq [\mathbf{a}_1, ..., \mathbf{a}_r, ..., \mathbf{a}_N]$ be the coding matrix. With the above design criteria, $\mathbf{A}$ has been proposed in a number of previous work [9], [10], [17], where Hadamard and Vandermonde matrices were used to construct $\mathbf{A}$. Note that the coding matrices based on Vandermonde matrices result in larger minimum product values than those using Hadamard matrices. Thus we will use Vandermonde matrices for our STNC scheme. In that case,

$$\mathbf{A} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & \cdots & 1 & \cdots & 1 \\ \theta_1 & \cdots & \theta_r & \cdots & \theta_N \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \theta_1^{N-1} & \cdots & \theta_r^{N-1} & \cdots & \theta_N^{N-1} \end{bmatrix}, \quad (26)$$

where θ_r depends on N and the construction methods [9], [10], [17].

IV. IMPACT OF SYNCHRONIZATION ERRORS ON DSTBC

The source transmission phase of schemes using DSTBC is the same with that of STNC scheme, i.e., that the N client nodes take turn to exchange their transmit symbols among themselves. Thus the signal model for this phase follows (1). After the source transmission phase, a STBC matrix is formed from the overheard symbol vector $\mathbf{x}$ as

$$\mathbf{S}(l) = \begin{bmatrix} s_{01}(l) & \cdots & s_{0r}(l) & \cdots & s_{0N}(l) \\ \vdots & \ddots & \vdots & \cdots & \vdots \\ s_{k1}(l) & \cdots & s_{kr}(l) & \cdots & s_{kN}(l) \\ \vdots & \cdots & \vdots & \ddots & \vdots \\ s_{(K-1)1}(l) & \cdots & s_{(K-1)r}(l) & \cdots & s_{(K-1)N}(l) \end{bmatrix}, \quad (27)$$

where l denotes the code matrix at frame l, which consists of K time slots required to transmit the code matrix, and $s_{kr}(l)$ is the code symbol transmitted at time slot k and through client node U_r. For example, the code matrix for two cooperative client nodes is the well-known Alamouti STBC [18]

$$\begin{bmatrix} x_1(l) & x_2(l) \\ -x_2^*(l) & x_1^*(l) \end{bmatrix}. \quad (28)$$

Notice that when a symbol is erroneously decoded at a client node, the associated code symbols at that node are set to zero and the node remains silent during that time slot [5].

In the relay transmission phase, the symbols in the kth row of the code matrix (27) for $k = 0, 1, ..., K - 1$ are simultaneously transmitted through the N client nodes, each acting as a respective antenna of a traditional MIMO system. Due to the time synchronization errors, the received signal at antenna m and time slot k of frame l is

$$y_m^k(l) = \sqrt{\frac{P_r}{K}} \sum_{r=1}^{N} h_{mr}(l) \sum_{q=-q_0}^{q_0} s_{(\mathrm{mod}(k+q,K))r}$$
$$\left(l + \left\lfloor \frac{k+q}{K} \right\rfloor \right) p((q-l)T_s - \tau_r) + w_{mk}(l), \quad (29)$$

where mod and $\lfloor . \rfloor$ denote the modulo and floor operations, respectively, $w_{mk}(l) \sim \mathcal{CN}(0, N_0)$ is the AWGN,

$$p(t) = \frac{\sin(\pi t / T_s)}{\pi t / T_s} \frac{\cos(\pi \beta t T_s)}{1 - 4\beta^2 t^2 T_s^2} \quad (30)$$

is the raised cosine pulse shape with a symbol period T_s and the roll-off factor β [19], q_0 is the number of nearest neighbor symbols from each side (left or right) that cause the ISI on the symbol of interest, and τ_r is the timing error associated with the signal from U_r. Figure 2 illustrates the ISI effect on the desired symbol.

Because the base node assumes perfect timing synchronization, it applies the detection techniques in [20]. For example in the case of $N = 2$,

$$\hat{x}_1(l) = \underset{x_1 \in |\mathbb{X}|}{\mathrm{argmin}} \left\{ \left| \left[\sum_{m=1}^{M} \left(y_m^0(l) \sqrt{\frac{P_1}{2}} h_{m1}^*(l) \right. \right. \right. \right.$$
$$\left. \left. + (y_m^1(l))^* \sqrt{\frac{P_2}{2}} h_{m2}(l) \right) \right] - x_1 \bigg|^2$$
$$+ \left(-1 + \sum_{m=1}^{M} \sum_{r=1}^{2} \frac{P_r}{2} |h_{mr}(l)|^2 \right) |x_1|^2 \bigg\} \quad (31)$$

$$\hat{x}_2(l) = \underset{x_2 \in |\mathbb{X}|}{\mathrm{argmin}} \left\{ \left| \left[\sum_{m=1}^{M} \left(y_m^0(l) \sqrt{\frac{P_2}{2}} h_{m2}^*(l) \right. \right. \right. \right.$$
$$\left. \left. - (y_m^1(l))^* \sqrt{\frac{P_1}{2}} h_{m1}(l) \right) \right] - x_2 \bigg|^2$$
$$+ \left(-1 + \sum_{m=1}^{M} \sum_{r=1}^{2} \frac{P_r}{2} |h_{mr}(l)|^2 \right) |x_2|^2 \bigg\}. \quad (32)$$

This detection inherits the ISI effect in (29) and thus degrades the performance as we will see in the next section.

V. SIMULATIONS

Here we perform computer simulations to verify our proposed STNC scheme and to compare the performance of our scheme with that of those utilizing DSTBC, with and without timing synchronization errors. QPSK modulation is used for the transmitted symbols. We assume that the N client nodes are in a cluster with the same channel variances $\sigma_{rn}^2 = 30$ among them. The cluster is faraway from the base node and thus the channel variance between the base node and a client node $\sigma_{0n}^2 = 1$ is the same for all client nodes. Also we assume the noise variance $N_0 = 1$. Equal power allocation is assumed, where a total transmit power P associated with transmitted symbol is divided equally among the transmissions. In this case, $P_n = P/(N+1)$ and $P_r = PN/(N+1)$ for STNC scheme and $P_n = P/(K+1)$ and $P_r = PK/(K+1)$ for DSTBC scheme required K time slots for transmitting the code matrix. The total transmit power P is the same with that
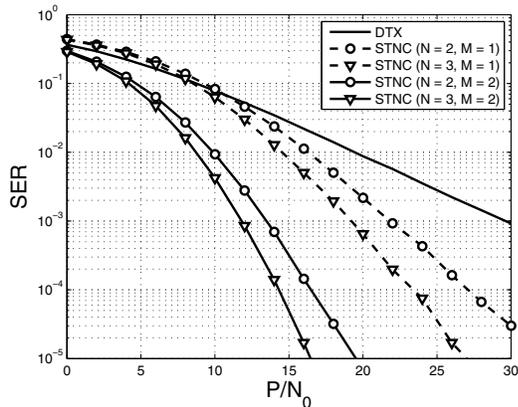
Fig. 3. SER versus SNR performance of STNC scheme for different numbers of client nodes and receive antennas for QPSK modulation.
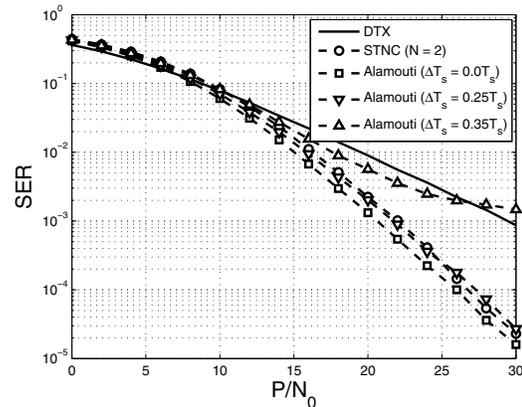

Fig. 4. SER versus SNR performance of STNC and DSTBC (Alamouti) and $M = 1$ for QPSK modulation.

used in DTX, where a client node directly transmits to the base node without any cooperation with other nodes.

Figure 3 presents the SER versus SNR performance of the proposed STNC for various numbers of client nodes and numbers of receive antennas. The SER performance for DTX is also presented for a comparison. For $N = 2$, $\theta_r = e^{j\frac{4r-3}{4}\pi}$ with $r = 1, 2$ and for $N = 3$, $\theta_r = e^{j\left(\frac{1}{9} + \frac{2(r-1)}{3}\right)\pi}$ with $r = 1, 2, 3$. From the figure, the proposed STNC clearly provides the expected diversity orders, i.e., that symbols x_n is received with diversity order MN for N client nodes and M receive antennas.

Figure 4 provides a performance comparison between the proposed STNC and DSTBC (with Alamouti code) for the case of $N = 2$ and $M = 1$. In the figure, ΔT_s is a synchronization error range, and the timing errors τ_r are assumed to be uniformly distributed in $[-\Delta T_s/2, \Delta T_s/2]$. From the figure, DSTBC scheme with perfect synchronization, i.e. $\Delta T_s = 0T_s$, provide about 1dB advantage over the STNC scheme. When $\Delta T_s = 0.25T_s$, the DSTBC and the STNC offer similar performance although the DSTBC scheme starts to diverge at SNR greater than 25dB. The performance of the DSTBC scheme with $\Delta T_s = 0.35T_s$ is really bad with the observable error floor. Clearly, the proposed STNC is advantageous compared to the DSTBC when timing synchronization occurs.

VI. CONCLUSIONS

In this paper, we proposed a STNC scheme that utilizing transform-based coding to achieve spatial diversity with low transmission delay and eliminate the issue of imperfect frequency and timing synchronization. The scheme can be applied to any number of client nodes N and provides a diversity order N for each transmitted symbol. The PEP was analyzed and the code design criteria were derived to ensure achieving full diversity order. Simulations are conducted to verify the performance of the proposed scheme and to reveal its advantage over a distributed space-time block coding scheme under timing synchronization errors.

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