

# Transactions Papers

## Source-Channel-Cooperation Tradeoffs for Adaptive Coded Communications

Andres Kwasinski, *Member, IEEE*, and K. J. Ray Liu, *Fellow, IEEE*

**Abstract**—Transmit power limitations and quality-impairing channels present key challenges to wireless communications. A possible solution to these problems is the application of user-cooperation techniques so as to improve link quality. This paper studies the tradeoffs involved in combining user cooperation with practical source and channel coding in systems featuring rate-adaptive source and channel coding of conversational traffic. Performance is measured through the D-SNR curve, which measures the relation between end-to-end distortion and channel Signal-to-Noise Ratio (SNR). The D-SNR curve is accurately characterized as a linear function in log-log scales. Also, it is shown that the tradeoffs involved in combining amplify-and-forward or decode-and-forward cooperation with source and channel coding translates into cooperative schemes showing a decrease in distortion at approximately half the rate as non-cooperative schemes but with larger coding gain. Because of this, the studied non-cooperative schemes show better performance only at high SNR. In addition, the D-SNR characterization is used to compare amplify-and-forward and decode-and-forward cooperation for channel codes of different strength and to study the effects of source codec efficiency, where it is shown that diversity gain is reduced proportionally to the source codec loss of efficiency.

**Index Terms**—Cooperative communications, source coding, channel coding, joint source-channel coding.

### I. INTRODUCTION

THE design of wireless devices presents challenges, such as expectations for the quality of service, error-prone channels, and limited radio resources. For conversational multimedia communications, these problems are exacerbated since strict delay constraints prevent the use of ARQ techniques. One successful approach for both speech [1], [2] and video sources [3], [4] had been to control the tradeoff between source encoding rate and channel coding rate. More recently, some works have focused on the resource allocation and system adaptation subject to resource constraints and channel states,

especially for the challenging case of video sources [5]–[7]. Another successful approach aims at delivering multiple copies of the signal in such a way that they are independently affected during transmission [8], which has resulted in numerous techniques collectively known as “diversity techniques”. An early approach to study the transmission of multimedia sources using diversity was by considering the availability of multiple paths as in [9], where the authors considered the combination of multistream-coded video with multiple paths. An information-theoretic exploratory study of how to transmit a compressed source over a pair of unreliable channels was considered in [10]. More recently, for the case of implementing diversity through a MIMO setup, the authors in [11] studied the relation between the level of diversity and joint source-channel coding, realized as the error resiliency of a compressed video stream.

User cooperation is a recent diversity technique which builds upon studies on the relay channel [12]. It is based on the broadcast nature of wireless channels where a transmitted signal can be overheard by other network nodes and, instead of traditionally discarding it, it is processed and relayed to the destination, effectively creating diverse message paths. At the receiver, the copies of the message arriving through multiple paths are combined to generate a signal with better quality. User cooperation schemes were proposed in [13]–[15], with performance being analyzed through outage probabilities.

User cooperation presents a tradeoff between received signal quality and bandwidth efficiency. While signal quality is improved, typical implementations of user cooperation can reduce the bandwidth efficiency. This is important for schemes that adapt source and channel coding rate, because while the presence of multiple paths can be seen as extra redundancy, the ensuing sacrifice in communication capacity forces either a higher compression of the source or a choice of a weaker channel code. This motivates this work since we study the tradeoffs when combining user cooperation with source coding and channel coding. We call this problem, the source-channel-cooperation (SCC) tradeoff problem. In [16] the authors considered the combination of user cooperation with layered source coding following an information theoretic approach concerned with asymptotic performance in the limit

Manuscript received May 22, 2006; revised September 29, 2006, May 13, 2007, and January 11, 2008; accepted March 31, 2008. The associate editor coordinating the review of this paper and approving it for publication was Q. Zhang.

A. Kwasinski is with the Rochester Institute of Technology, Rochester, NY (e-mail: akkeec@rit.edu).

K. J. R. Liu is with the University of Maryland, College Park, MD (e-mail: kjrlu@umd.edu).

Digital Object Identifier 10.1109/TWC.2008.060286.

of high SNR and capacity achieving codes. In [17] a similar information theoretic approach, now centered on outage capacity, is applied on a scheme combining layered source coding, unequal channel error protection and cooperation which jointly adapts cooperation and source and channel coding. Due to its nature, the information theoretic approach followed in these cases cannot characterize the performance in settings typical of those multimedia communication systems where delay constraints preclude the use of capacity achieving codes or where the operational envelope is not necessarily restricted to high SNR channels. In a setup with practical source and channel codes, we resorted in [18] to simulations to compare the performance of different source coding schemes in the presence of cooperation. The work in [17] also resorts to simulations to consider practical source and channel codes for the schemes under consideration. Despite the different settings, [18] and [17] coincide in showing the advantage of cooperation at medium and low SNR in bandwidth-constrained systems, but because in the simulations the adaptation of the constituent stages is done through exhaustive search, it is not possible to see the mechanism of the interaction between cooperation and the rest of the system. In the present work, our interest is to analyze the practical issues associated with the combination of user cooperation and practical source and channel codes. For this, we characterize the performance in the presence of source-channel-cooperation tradeoffs and we arrive at expressions that reveal the effect of cooperation on the overall system performance. This allows us to address critical design questions such as how performance would be affected if using a stronger channel code or a less efficient source encoder.

The SCC tradeoff problem encompasses many layers of the communication stack. For studies on user cooperation or channel coding, performance is usually evaluated based on the relation between channel SNR and some measure involving channel errors, such as Bit Error Rate (BER). For source codecs, performance is measured through the distortion-rate (D-R) function. For the problem under study, we need a performance measure that reflects tradeoff effects at all layers of the source-channel-cooperation problem and that is consistent with the focus on communicating coded sources. Thus, we measure performance through the relation between end-to-end distortion and channel SNR, a function we denote as the “*D-SNR curve*”. By extending our work in the absence of cooperation in [19], we resolve the lack of mathematical tractability when optimizing the SCC tradeoff through exhaustive search by characterizing the *D-SNR curve* through a set of carefully selected points. We will see that the *D-SNR curve* can be closely approximated through a linear function in a log-log scale. Owing to the fact that this behavior is similar to that between error rate and large SNR in systems with channel fading, we represent the *D-SNR curve* by two parameters which we still name coding and diversity gains. In this way, we approach the concept of diversity in a more general view than the one limited to the presence of multiple channels or fading; and we extend it to the measurement of redundant information also. This allows considering in a unified way the use of cooperation to deliver multiple message copies and the tradeoffs in redundancy at the source-

channel bit rate allocation. We show that because systems without cooperation have higher bandwidth efficiency, they can implement schemes with more redundancy and achieve higher diversity gain. Conversely, we show that systems using cooperation have better coding gain. Thus, non-cooperation yields better performance at high source-destination SNR, while cooperative schemes have better performance in the rest of cases. Also, the *D-SNR curve* characterization is used to compare the performance of amplify-and-forward (AF) and decode-and-forward (DF) cooperation when using channel codes of different strength and source codecs with different efficiency. We can see that DF cooperation shows better performance in most of the cases and that the diversity gain is reduced proportionally to the source codec loss of efficiency.

This paper is organized as follows: In Section II, we introduce the system model. While Section III characterizes the *D-SNR curves*, Section IV studies the impact of performance of the source-channel-cooperation tradeoffs for AF and DF cooperation. Finally, Section V summarizes the results.

## II. SYSTEM MODEL

We consider a wireless network carrying conversational multimedia traffic between a source node and a destination node, i.e. ongoing calls carry sources that are coded for transmission and are subject to strict delay constraints. Bandwidth is shared by allocating to each call an orthogonal channel that delivers  $W$  bits per transmission period using BPSK modulation with coherent detection and maximum-likelihood decoding in the receiver. The transmission of a message is divided in three main stages: a source encoder, which compresses a source, a channel encoder, which adds protection against channel errors, and a radio link. In order to minimize end-to-end distortion, both the source coding and channel coding rates can be adapted to channel conditions through jointly allocating the  $W$  available bits. Also, cooperative communication may or may not be used in the radio link (in which case, the model will include a third node, the relay node, when using cooperation).

We assume a low mobility scenario, where the communication is carried over a quasi-static fading channel; i.e. fading may be considered constant during the transmission of a frame. Furthermore, we will frame the study of SCC tradeoffs within the viewpoint of the decisions that an allocation algorithm has to make for each transmission period so as to obtain best performance when having perfect knowledge of the channels. For this, we implicitly assume that the network offers a mechanism to estimate the channel states at the transmitters (similarly to setups used in closed-loop MIMO technologies [20]). This can be achieved in a time division duplexed network by using techniques such as channel sounding [21]. In this case, after assuming channel reciprocity, the source can estimate the source-relay channel from the signal emitted by the relay (likely containing pilots) and the source-destination and relay-destination channels from estimates fed back from the destination. As long as the quasi-static fading assumption holds, the difference between actual and estimated channel states (due to actual errors in channel estimation and due to the time evolution of the channel) should have little effect on the final result. This is because there would be no difference

in the parameters chosen for source and channel coding as the range of possible choices is discrete and the selection of parameters remains the same over a range of channel SNRs. In general terms, in most scenarios involving node movement at speeds consistent with pedestrian setups, the channel is expected to stay constant over the duration of several transmit frames (for example, at carrier frequency of 2.5 GHz and node velocity of 5 mph the channel coherence time is approximately 50 ms). With more rapidly changing channels, there may be differences between the actual and the estimated channel that would translate into a mismatched choice of source and channel coding parameters, which would result in performance loss. The magnitude of this loss depends on several variables, such as the rate at which the channel is changing in relation with the duration of a transmission period, the coarseness in the range of choices for source and channel coding, the performance of both the source and the channel codecs, etc. Nevertheless, rapidly changing channel affects the considered scheme whether user cooperation is used or not. Since our main goal is to study the interaction between the received signal quality-bandwidth efficiency tradeoff (stemming from user cooperation) and the source compression-error protection tradeoff, we will decouple the effects of channel mismatch by assuming that channel states are ideally estimated at the transmitter. This will lead to results that reveal the pure SCC tradeoffs and that reflect the best performance that can be achieved.

Consistent with the assumptions on channel model and strict delay constraint, we will assume that performance measurement and adaptation for the three transmission stages is done on a frame-by-frame basis. This involves choosing for a source frame a source encoding rate, a channel coding rate and the use or not of cooperation based on the channels SNR so as to minimize end-to-end distortion. The complexity of this procedure can be divided between the complexity of estimating the channels SNR and the complexity of actually finding the best settings. The complexity of channel SNR estimation is highly dependent on several transmission parameters, such as transmission rate, framing setup and scheduling, etc., that are treated in general terms here. Nevertheless, we can assume that channel estimation is performed using pilots, in which case the procedure involves the derivation of a pilot sequence, its reception and its processing (likely through a filtering operation). In addition, channel estimation introduces an overhead due to the transmission of the pilots themselves and the transmission of some channel estimates from the destination to the source (which can normally be compressed). The impact of this overhead on performance once again is highly dependent on several transmission parameters that are not the subject of this work. In terms of the complexity involved in finding the settings for source-channel coding and use of cooperation, for schemes such as the ones studied here, the range of SNRs associated with each setting can be precomputed and stored in a table. For systems with more design variables, modified multistage or iterative decision algorithm (such as those in [5] and [6]) may be necessary. We next expand on the three main stages at the transmitter.

### A. Source Codec

During each sample period, a block of  $N$  input signal samples are presented to the source encoder at the transmitter node. The encoder generates one coded representation of the  $N$  samples using  $R_S$  bits per source sample. The performance of this codec is measured through its achievable D-R function, where distortion is measured as the mean-squared error between the original and the reconstructed source samples. This function is frequently considered to be of the form  $D_S(R_S) = c_1 2^{-c_2 R_S}$ , which can approximate or bound many practical systems such as video coding with an MPEG codec [22], [23], speech using a CELP-type codec [24], or when the high rate approximation holds. Assuming, without loss of generality, that  $c_1 = 1$  and  $c_2 = 2$  (as if the input samples were memoryless, with a zero-mean, unit-variance Gaussian distribution and long block source codes were used), we have ([25]),

$$D_S(R_S) = 2^{-2R_S}. \quad (1)$$

### B. Channel Codec

The source-encoded bits are organized into a source frame and protected against transmission errors through a rate  $r$  channel code. We will consider practical channel codes in the form of convolutional codes and will exclude the use of capacity-achieving codes due to the delay constraints. As is common practice in conversational communications, the receiver will discard source frames containing errors after channel decoding. Although error concealment schemes are used to replace the lost information, this is not our focus subject, thus we assume that missing data is concealed with its expected value.

Let the family of variable rate convolutional codes be implemented with RCPC codes [26]. We assume that decoding is performed with a soft input Viterbi decoder. From [27] the probability of having a source frame with errors after channel decoding can be upper-bounded as

$$P(\gamma) \leq 1 - \left[ 1 - \sum_{d=d_f}^W a(d)P(d|\gamma) \right]^{NR_s} \quad (2)$$

where  $d_f$  is the free distance of the code,  $a(d)$  is the number of paths in the decoding trellis with Hamming weight  $d$  and  $P(d|\gamma)$  is the probability of selecting during decoding a path of Hamming distance  $d$  given the channel SNR  $\gamma$  [28].

### C. Cooperative Communication

Communication may be carried on using a traditional non-cooperative scheme (Figure 1(a)) or a cooperative setup (Figure 1(b)). In the later, a third node, the relay node, is associated with the source node. In general, there may be more than one relay associated with each source node but, for simplicity, we limit this number to one. Communication in a cooperative setup takes place in two phases. In phase 1, a source node radiates information. This transmission can be overheard by the relay because of the broadcast nature of wireless communications. In phase 2, the relay node cooperates by forwarding to the destination node the information received

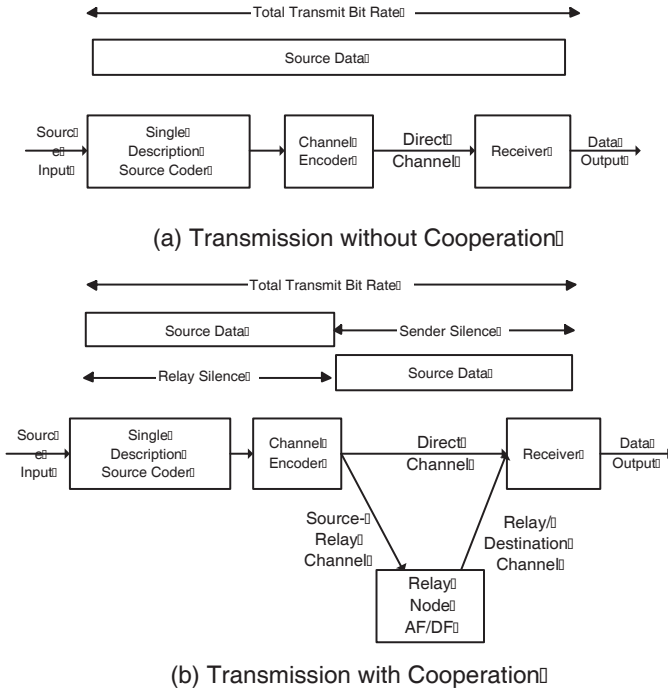


Fig. 1. Configuration of the studied schemes.

from its associated source node. At the destination node, the signals received from the direct path from the source and from the path through the relay are combined and detected. We will assume that a Maximum Ratio Combiner (MRC) is used to combine these signals. As the total number of bits that can be sent per call and transmission period  $W$  is fixed, it needs to be split between the two phases.

We will consider two schemes that implement cooperative communication. The first is *amplify-and-forward* (AF). In this scheme the relay amplifies and retransmits the source's signal without further processing. It can be shown, [15], that the SNR at the receiver after the MRC,  $\gamma_{AF}$ , is

$$\gamma_{AF} = \gamma_{sd} + \frac{\gamma_{sr}\gamma_{rd}}{1 + \gamma_{sr} + \gamma_{rd}}, \quad (3)$$

where  $\gamma_{sd}$  is the SNR in the source-destination link,  $\gamma_{sr}$  is the SNR in the source-relay link and  $\gamma_{rd}$  is the SNR in the relay-destination link. Thus, the probability of having a source frame with errors after channel decoding can be approximated from the upper bound in (2) as

$$P_{AF}(\gamma_{AF}) \approx 1 - \left[ 1 - \sum_{d=d_f}^{W/2} a(d)P(d|\gamma_{AF}) \right]^{NR_s}. \quad (4)$$

In the second scheme, *decode-and-forward* (DF), the relay decodes the signal from the source and tests for successful decoding (through, for example, a parity check such as CRC). If decoding succeeds, the relay sends during phase 2 another copy of the source's information. If decoding fails the relay will idles during phase 2. Consequently, the received SNR at the output of the MRC,  $\gamma_{DF}$ , is

$$\gamma_{DF} = \begin{cases} \gamma_{sd} + \gamma_{rd}, & \text{if correct decoding at relay;} \\ \gamma_{sd}, & \text{if incorrect decoding at relay.} \end{cases} \quad (5)$$

If we denote by  $\Psi$  the event "correct decoding at relay" and by  $\bar{\Psi}$  the event "incorrect decoding at relay", the probability of having a source frame with errors is

$$\begin{aligned} P_{DF}(\gamma_{DF}) &= P(\gamma_{DF}|\Psi)P(\Psi) + P(\gamma_{DF}|\bar{\Psi})P(\bar{\Psi}) \\ &= P(\gamma_{sd} + \gamma_{rd})(1 - P(\gamma_{sr})) + P(\gamma_{sd})P(\gamma_{sr}), \end{aligned} \quad (6)$$

where  $P(\gamma)$  is the source frame error rate in (2) with SNR  $\gamma$ . The first term in the right hand side of (6) represents the probability of successful transmission from the source to the relay and then to the destination. The second term represents the case when the relay fails decoding.

### III. SOURCE-CHANNEL-COOPERATION PERFORMANCE

Next, we characterize the end-to-end distortion as a function of SNR, i.e. the D-SNR curve, and we use it to study how performance is affected by the SCC tradeoffs. The end-to-end distortion comprises the source encoder distortion (which depends on the source encoding rate) and the channel-induced distortion (which depends on the channel SNR, use of cooperation, channel coding rate and error concealment). Given the channel SNRs, the source and channel coding rates are jointly set so as to minimize the end-to-end distortion while not exceeding the total communication capacity  $W$ . Typically both the source and the channel coding rates are chosen from a finite set. We call an operating mode  $\Omega_i$ , the triplet that describes a choice of source encoding rate, a choice of channel coding rate and the indication of use of cooperation. Let  $\Omega = \{\Omega_i, \forall i\}$ , be the set of all operating modes. Since we are assuming fixed transmit bit rate we have  $W/2 = NR_s/r$  if using cooperation and  $W = NR_s/r$  if not, i.e. any  $\Omega_i$  could be specified through either the source or the channel coding rate. In what follows, we make explicit the dependence of the probability of a frame error on  $\Omega_i$  by using the notation  $P_{\Omega_i}(\gamma)$ .

#### A. Amplify-and-Forward Cooperation:

We consider a scheme as in Figure 1(b) using AF cooperation. Let  $R_{SC}$  be the number of bits used by the source encoder to represent each source samples. Let this source encoded data be protected with a rate  $r$  RCPC code. As discussed, we have  $R_{SC} = \frac{Wr}{2N}$ . Let  $\Omega^{(c)}$  be the subset of  $\Omega$  such that user cooperation is used. The D-SNR performance function for AF cooperation is the solutions to

$$\begin{aligned} D_{CAF} &= \min_{\Omega_i \in \Omega^{(c)}} \left\{ D_F P_{AF\Omega_i}(\gamma_{AF}) \right. \\ &\quad \left. + 2^{-2R_{SC}} (1 - P_{AF\Omega_i}(\gamma_{AF})) \right\}, \end{aligned} \quad (7)$$

where  $P_{AF\Omega_i}(\gamma_{AF})$  follows (4) and  $D_F$  is the distortion when the source frame is received with errors ( $D_F = 1$  for our source model and distortion measure setup).

Finding a close form expression for the D-SNR curve is a challenging problem. Nevertheless, a good approximation can be found by considering the following. Let  $f_{\Omega_i}(\gamma_{AF}) = D_F P_{AF\Omega_i}(\gamma_{AF}) + 2^{-2R_{SC}} (1 - P_{AF\Omega_i}(\gamma_{AF}))$  be the D-SNR function that results from a choice of operating mode  $\Omega_i \in \Omega^{(c)}$ . We will call  $f_{\Omega_i}(\gamma_{AF})$  the *single-mode D-SNR curve*. Because source and channel coding are fixed in

each  $f_{\Omega_i}(\gamma_{AF})$ , the distortion only increases due to channel errors. Also, at high  $\gamma_{AF}$ , the contribution of channel errors is so negligible that  $f_{\Omega_i}(\gamma_{AF})$  is approximately constant at a value equal to the source encoding distortion. When comparing two single-mode D-SNR curves, the one with a larger source coding rate is also associated with a weaker channel code, thus, this curve shows lower distortion at high SNR but channel errors become significant and distortion starts to grow at higher SNR values. The overall effect is that the D-SNR performance resulting from solving (7) is the sequential interlacing of sections of single-mode D-SNR curves (examples of these observations are shown in Figs. 3 and 4, to be presented in Section III-D). These sections are the portion of single-mode D-SNR curves where channel-induced distortion typically ranges from being negligible, to being such that the total distortion equals the source encoding distortion of the single-mode D-SNR with the next stronger channel code. Thus, the D-SNR curve could be closely approximated by using a subset of carefully selected points. These points are those where the contribution of channel errors to the end-to-end distortion is relatively small, i.e. those points where distortion is equal to  $(1 + \Delta)D_S(\Omega_i)$ , where  $\Delta$  is a small number. Formally, we are considering those points where

$$\begin{aligned} D &= (1 + \Delta)D_S(\Omega_i) \\ &= D_F P_{\Omega_i}(\gamma_{AF}) + D_S(\Omega_i) \left(1 - P_{\Omega_i}(\gamma_{AF})\right), \end{aligned} \quad (8)$$

Equivalently, from (8), the D-SNR curve is formed by those points where

$$P_{AF\Omega_i}(\gamma_{AF}) = \frac{\Delta}{\frac{D_F}{D_S(\Omega_i)} - 1}. \quad (9)$$

Combining (4) and (9), leads to a relation between  $\gamma_{AF}$  and  $R_{SC}$ , [19], that implicitly depends on the expressions for the code free distance,  $d_f$ , and the number of paths with Hamming weight  $d_f$ ,  $a(d_f)$  as functions of the channel code's rate,  $r$ . For the function  $d_f(r)$ , it can be verified that a good approximation is  $d_f \approx \kappa e^{-cr} = \kappa e^{-c2NR_{SC}/W}$ , where  $\kappa$  and  $c$  are two constants. A similar study for  $a(d_f)$  shows that there is no such practical functional approximation. Yet,  $a(d_f)$  can be approximated by its average value,  $\bar{a}$ , and still yield good results. Assuming that the most likely error events are those with Hamming weight equal to the code's free distance,  $d_f$ , and using (2) we can write the approximate relation

$$\frac{\Delta}{\frac{D_F}{D_S(\Omega_i)} - 1} \approx 1 - \left(1 - \frac{\bar{a}}{2} \operatorname{erfc}\left(\sqrt{\kappa e^{-2cR_T/W} \gamma_{AF}}\right)^{R_T}\right), \quad (10)$$

where  $R_T = NR_{SC}$  is the total number of bits that represent the input samples and we have used the fact that for BPSK modulation,  $P_e(d|\gamma) = .5 \operatorname{erfc}\sqrt{d\gamma}$  [26], with  $\operatorname{erfc}(\gamma)$  being the complementary error function defined as  $\operatorname{erfc}(\gamma) = 2/\pi \int_{\gamma}^{\infty} e^{-u^2} du$ . From (10), we have

$$\gamma_{AF} \approx \frac{e^{\frac{2cR_T}{W}}}{\kappa} \left( \operatorname{erfc}^{-1} \left( \frac{2}{\bar{a}} \left[ 1 - \left( 1 - \frac{\Delta}{\frac{D_F}{D_S(\Omega_i)} - 1} \right)^{\frac{1}{R_T}} \right] \right) \right)^2.$$

From this relation, using the approximation  $\operatorname{erfc}^{-1}(y) \approx \sqrt{-\ln(y)}$ , considering that  $P_{\Omega_i}(\gamma_{AF})$  is typically small when (9) holds with small  $\Delta$ , and that for the selected points we

have  $D = (1 + \Delta)2^{-2R_{SC}}$ , it is possible to derive the D-SNR function. Following this procedure, [19], one can show that the D-SNR curve can be closely approximated as,

$$D_{CAF} \approx (G_c \gamma_{AF})^{-10m_c}, \quad (11)$$

$$m_c = \left[ 20 \left( \frac{cN}{W \ln(4)} + \frac{1}{2\Psi + 2\bar{R}_{SC} \ln(4)} \right) \right]^{-1}, \quad (12)$$

$$G_c = \frac{\kappa(1 + \Delta)^{-1/(10m_c)} 10^{\frac{\log(4)\bar{R}_{SC}}{\Psi + \bar{R}_{SC} \ln(4)}}}{\Psi + \bar{R}_{SC} \ln(4)}, \quad (13)$$

where  $\Psi = \ln\left(\frac{\bar{a}ND_F\bar{R}_{SC}}{2\Delta}\right)$ , and  $\bar{R}_{SC}$  is the average source encoding rate for operating modes  $\Omega_i \in \Omega^{(c)}$ .

Equation (11) has the appeal of being of the same form as the error rate-SNR function found in the study of communication systems [29]. The main differences are that we are considering distortion instead of average error probability, there is no assumption of asymptotically large SNR, and that (11) models performance of systems with practical components instead of being a bound at high SNR. Following this similarity, we can derive the analogy that  $G_c$  is the coding gain and  $m_c$  is the diversity gain. The coding gain  $G_c$  represents the SNR for a reference distortion value of 1. The diversity gain  $m_c$  determines the slope (i.e. rate of change) of the logarithm of the distortion when SNR is measured in decibels (the factor of 10 multiplying  $m_c$  in (11) sets these units). As argued in [19], this concept of diversity beyond its natural realm of systems with fading is based on the significant overlap between the concept of diversity and redundancy. The general concept of diversity, as that of transmitting multiple message copies, can be extended to other forms of redundancy. Such is the case for error correcting codes when considering that they exhibit an inherent diversity because the (maybe partial) copies of the message are coded as parity data. In this sense, our expanded view of diversity aims at measuring more general forms of redundant information in cross layers designs, which allows us to consider in a unified way the use of cooperation to deliver multiple copies of the message and the tradeoffs in redundancy at the source-channel bit rate allocation stages. Also, it is worth noting that the premise on which we base the D-SNR characterization, i.e Eq. (8), follows from the problem setup and is not exclusive to a particular choice of source, channel code family, or distortion measure.

### B. Decode-and-Forward Cooperation:

In the case of DF cooperation, the setup remains as in Fig. 1(b). We assume a source coding rate  $R_{SC}$  and channel coding rate  $r$  with the relation  $R_{SC} = \frac{Wr}{2N}$  still holding. The D-SNR performance function for DF cooperation is the solutions to

$$\begin{aligned} D_{CDF} &= \min_{\Omega_i \in \Omega^{(c)}} \left\{ D_F P_{DF\Omega_i}(\gamma_{DF}) \right. \\ &\quad \left. + 2^{-2R_{SC}} \left( 1 - P_{DF\Omega_i}(\gamma_{DF}) \right) \right\}. \end{aligned} \quad (14)$$

To characterize the D-SNR curve in DF cooperation it is necessary to consider the structure of  $P_{DF\Omega_i}(\gamma_{DF})$  (as in (6)). Consequently, we consider three cases: the source-relay channel is "good", the source-relay channel is "bad" and the channel states are such that there is no solution to (9) for DF cooperation. Due to the adaptation of channel coding

rate, the classification of “good” or “bad” source-relay channel depends not only on  $\gamma_{sr}$ , but also on the operating mode and the strength of its channel code. This is because for a given channel state the strongest channel codes may be operating at the low BER regime but the weakest codes may be operating in the high BER regime. This is why the source-relay channel needs to be qualified in terms of the frame error probability  $P_{\Omega_i}(\gamma_{sr})$ .

In the case of “good” source-relay channel the source frame error probability (6) can be approximated as  $P_{DF\Omega_i}(\gamma_{DF}) \approx P_{\Omega_i}(\gamma_{sd} + \gamma_{rd})$  for most operating modes. In this case (8) becomes  $D = (1 + \Delta)D_S(\Omega_i) = D_F P_{\Omega_i}(\gamma_{sd} + \gamma_{rd}) + D_S(\Omega_i)(1 - P_{\Omega_i}(\gamma_{sd} + \gamma_{rd}))$ , and we have

$$D_{CDF} \approx (G_c(\gamma_{sd} + \gamma_{rd}))^{-10m_c}. \quad (15)$$

In the case for “bad” source-relay channel, (6) can be approximated as  $P_{DF\Omega_i}(\gamma_{DF}) \approx P_{\Omega_i}(\gamma_{sd})$  for most operating modes. In this case we have

$$D_{CDF} \approx (G_c\gamma_{sd})^{-10m_c}. \quad (16)$$

The differentiation between “good” and “bad” source-relay channels, is done by setting a threshold on  $P_{\Omega_i}(\gamma_{sr})$  such that the operating modes are separated into two regimes. These regimes are those where  $P_{DF}(\gamma_{DF})$  in (6) can either be  $P_{DF\Omega_i}(\gamma_{DF}) \approx P_{\Omega_i}(\gamma_{sd} + \gamma_{rd})$  (for operating modes with strong error protection such that  $P_{\Omega_i}(\gamma_{sr})$  is less than the threshold) or  $P_{DF\Omega_i}(\gamma_{DF}) \approx P_{\Omega_i}(\gamma_{sd})$  (for operating modes with weak error protection such that  $P_{\Omega_i}(\gamma_{sr})$  is more than the threshold). Following this, we considered the threshold to be  $P_{\Omega_i}(\gamma_{sr}) \approx 0.1$ . The results show little sensitivity to the value chosen as a threshold because although there may be one or two operating modes in borderline cases with  $P_{\Omega_i}(\gamma_{sr})$  near the threshold, the rest will be clearly in a good or bad channel condition. Also, note that  $m_c$  and  $G_c$  in (15) and (16) are already given by (12) and (13) because AF and DF cooperation yield the same bandwidth efficiency and values for the variables  $m_c$  and  $G_c$  depend on.

The third modeling case corresponds to situations where the channel states are such that there is no solution to (9). This may occur when  $\gamma_{sd}$  is very low, both  $\gamma_{sr}$  and  $\gamma_{rd}$  are such that  $P_{DF\Omega_i}(\gamma_{DF}) < \Delta(\frac{D_F}{D_S(\Omega_i)} - 1)^{-1}$  for some  $\Omega_i$ . In this case, the distortion in our analysis becomes an asymptotic value. Because  $\Delta$  is chosen small, we approximate this value by solving for the operating mode for which  $\lim_{\gamma_{sd} \rightarrow 0} P_{DF\Omega_i}(\gamma_{DF}) = \Delta(\frac{D_F}{D_S(\Omega_i)} - 1)^{-1}$ . Using this fact and (1) into (10), the asymptotic distortion value can be approximated as  $D_a \approx 2^{-2R_{SC}}$  after solving for the source encoding rate  $R_{SC}$  in

$$R_{SC} = \frac{1}{2} \log_2 \left( 1 + \frac{\Delta}{1 - \left[ 1 - \frac{\bar{a}}{2} \operatorname{erfc}(\sqrt{\kappa e^{-\frac{2cR_T}{W}} \gamma_{rd}}) \right]^{R_T}} \right). \quad (17)$$

### C. No Use of Cooperation:

The case of a direct communication without cooperation is schematized in Figure 1(a). Let  $R_{SN}$  be the number of bits used by the source encoder to represent each of the  $N$  source

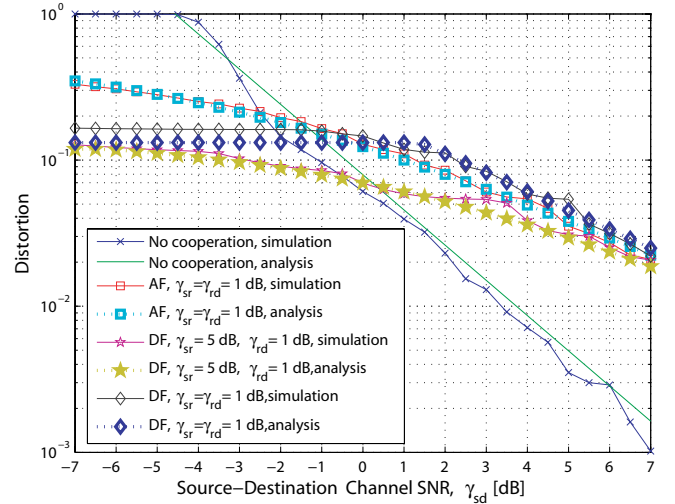


Fig. 2. D-SNR curves obtained from simulations and applying the analysis in Sec. III.

samples and  $\Omega^{(nc)}$  be the subset of  $\Omega$  such that there is no use of cooperation. Then, the D-SNR curve is the solution to

$$D_{SN} = \min_{\Omega_i \in \Omega^{(nc)}} \left\{ D_F P_{\Omega_i}(\gamma_{sd}) + 2^{-2R_{SN}} (1 - P_{\Omega_i}(\gamma_{sd})) \right\}. \quad (18)$$

Equation (18) is of the same form as (7). Therefore, an approximation for the D-SNR curve could be derived as in Section III-A considering that with no cooperation we will have twice the bandwidth efficiency as in AF or DF cooperation, and  $\bar{R}_{SN} = 2\bar{R}_{SC}$ . Then, it can be shown that

$$D_N \approx (G_N \gamma_{sd})^{-10m_n}, \quad (19)$$

$$m_n = \left[ 10 \left( \frac{cN}{W \ln(4)} + \frac{1}{\Psi + \ln(2)(1 + 4\bar{R}_{SC})} \right) \right]^{-1}, \quad (20)$$

$$G_N = \frac{\kappa(1 + \Delta)^{-1/(10m_n)}}{\Psi + \ln(2)(1 + 4\bar{R}_{SC})} 10^{\frac{2 \log(4)\bar{R}_{SC}}{\Psi + \ln(2)(1 + 4\bar{R}_{SC})}}. \quad (21)$$

### D. Illustration

Fig. 2 show the D-SNR curves obtained from simulations along with those derived using the characterization from previous sections. We choose for error protection a family of memory 4, puncturing period 8, mother code rate 1/4, RCPC codes [26]. For these codes we have  $\bar{a} = 6.1$ ,  $\kappa = 30$  and  $c = 3$ . Also, we set  $N = 150$  samples,  $W = 950$  bits per transmission period and  $\Delta = .1$  (it will be shown that the results have little sensitivity to this choice). We consider  $\gamma_{sd}$  as the main variable, while we treat  $\gamma_{sr}$  and  $\gamma_{rd}$  as parameters. In doing so, different scenarios can be thought of as associated with the distance between the relay and the other nodes. Fig. 2 shows that our D-SNR characterization accurately represents the behavior of the schemes under study. The scheme without cooperation shows the largest difference between simulation and analytical results for  $\gamma_{sd}$  between -3 and -1 dB. This difference is not related to our analysis because in this range there is no adaptation of the source or channel code. This is because the strongest channel code has already been chosen at  $\gamma_{sd} \approx -1$  dB and so the system is operating outside

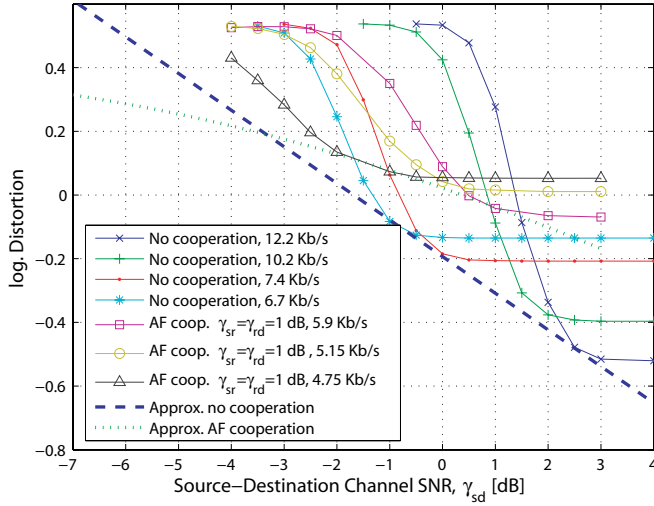


Fig. 3. Application of schemes to GSM AMR speech codec.

our analysis range. Note that the schemes with cooperation, although being well represented by the analytical model, do not show the expected linear relation. This is because in the Figure the abscissa is  $\gamma_{sd}$  (in dBs) and not  $\gamma_{AF}$  or  $\gamma_{DF}$ .

The D-SNR characterization can also be applied to practical schemes. This is because, as noted, D-R functions of the form we have considered can approximate or bound the performance of many practical source codecs. Fig. 3 illustrates this by showing the D-SNR performance of the GSM AMR speech codec [30], coupled to a memory 4 RCPC code from [26]. The figure shows seven single-mode D-SNR curves, obtained through Monte Carlo method, corresponding to source encoding rates 12.2, 10.2, 7.4, 6.7, 5.9, 5.15 and 4.75 Kbps; where the last three modes are used in AF cooperation and the first four without cooperation. To emphasize the applicability of our analysis, distortion is measured using a perceptually-based measure related to the ITU-T PESQ standard P.862 [31]. PESQ produces a result that is between 4.5 (best quality) and 1 (worst quality) that accurately predicts the results from a typical subjective test. We measure distortion as  $4.5 - Q$ , where  $Q$  is the output from the PESQ algorithm. The figure also includes the performance approximation as in (11) and (19), with parameters obtained by measuring two performance points (as discussed in Section IV). Fig. 4 shows a similar simulation for a frame of the 30 frames per second QCIF video sequence “Foreman” encoded with the MPEG-4 codec and protected with a memory 8, puncturing period 8, mother code rate 1/4 family of RCPC codes [32]. The frame considered is number 190, the tenth P frame after an I frame. Despite the coarse set of available operating modes, our characterization still applies in these cases. Interestingly, this is the case even when the video source is not memoryless, as previously assumed. This is because, as shown in (8), our analysis focuses on points where channel-induced distortion starts to become important, which is an effect that is somewhat decoupled from the source memory (i.e. when the channel-induced distortion becomes important it significantly affects all memory interdependent frames). Yet, a detailed consideration of sources with memory is beyond this paper reach and has to be left for a sequel.

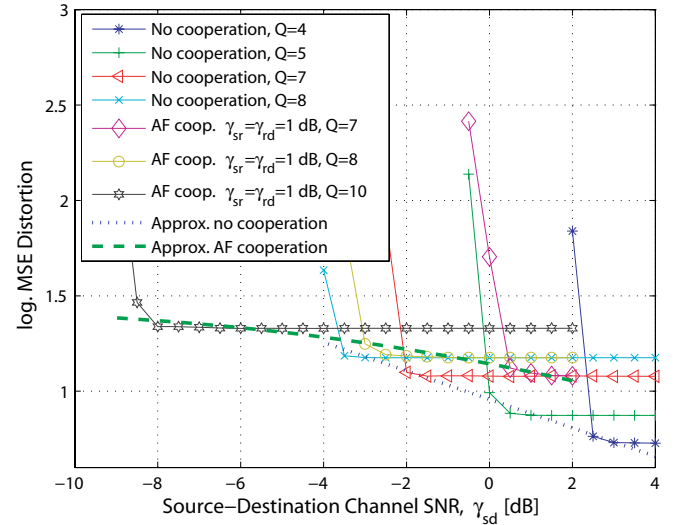


Fig. 4. Application of schemes to MPEG-4 video codec.

Figs. 3 and 4, also show how the D-SNR curve is made of the interleaving of single-mode D-SNR curves and how the points following (9) are used in the characterization.

#### E. Validation of D-SNR Characterization

After recognizing that the dynamics of the source-channel coding tandem are such that the resulting D-SNR performance is the interlacing of sections of single-mode D-SNR curves, the characterization of the D-SNR curves rests on two key steps. The first step corresponds to Eq. (9), or its equivalent for each particular scheme, which essentially translates the problem from one involving the tandem of source and channel coding into one in error control coding. With this, it becomes possible to apply many techniques from error control performance analysis to find the channel SNRs such that the frame error probability equals the expression in (9). From this point on, the characterization of the D-SNR curve is as good as the approximation of channel SNRs. To assess this, we considered the relative error between the logarithm of the actual and the approximated distortion values at the channel SNRs corresponding to the points used to approximate the D-SNR curve. The actual distortion value at these points is, by definition,  $(1+\Delta)D_S(\Omega_i)$ . For each point used to approximate the D-SNR curve (one per operating mode) it is possible to find the actual SNR value by solving for  $\gamma$  in

$$P_{\Omega_i}(\gamma) = \frac{\Delta}{\frac{D_F}{D_S(\Omega_i)} - 1}. \quad (22)$$

When the operating modes  $\Omega_i$  in (22) are those with no cooperation, the resulting  $\gamma$  will correspond to the  $\gamma_{sd}$  of the points used to approximate the D-SNR curve. Because (22) does not depend on the specific type of cooperation and we are solving for  $\gamma$  as the unknown variable, when the operating modes are those with cooperation, the resulting  $\gamma$  will correspond to  $\gamma_{AF}$  if using AF cooperation or either  $\gamma_{sd} + \gamma_{rd}$  or  $\gamma_{sd}$  if using DF cooperation. From the knowledge of  $\gamma$  for each scenario, it is possible to find the approximate distortion at the points used to characterize the D-SNR curve

$$S_{\Delta}^{m_c} = \left[ -10\Delta \left( \frac{cN}{W \ln(4)} \left( 1 + \frac{1}{2\Psi + 2\bar{R}_{SC} \ln(4)} \right) \right)^2 \right]^{-1}, \quad (24)$$

$$S_{\Delta}^{m_n} = \left[ -5\Delta \left( \frac{cN}{W \ln(4)} \left( 1 + \frac{1}{\Psi + \ln(2)(1 + 4\bar{R}_{SC})} \right) \right)^2 \right]^{-1}, \quad (25)$$

$$S_{\Delta}^{G_{c,dB}} = \frac{-1}{m_c(1 + \Delta) \ln(10)} - \frac{20\Delta \log(1 + \Delta) + 10/\ln(10)}{\Delta(\Psi + \bar{R}_{SC} \ln(4))} + \frac{6.265}{\left( \frac{\Psi}{\bar{R}_{SC}} + 1 \right)^2 \bar{R}_{SC} \ln(4)}, \quad (26)$$

$$S_{\Delta}^{G_{n,dB}} = \frac{-1}{m_n(1 + \Delta) \ln(10)} - \frac{10\Delta \log(1 + \Delta) + 10/\ln(10)}{(\Psi + \ln(2)(1 + 4\bar{R}_{SC}))} + \frac{6.265}{\left( \frac{\Psi}{\ln(2)(1 + 4\bar{R}_{SC})} + 1 \right)^2 \ln(2)(1 + 4\bar{R}_{SC})}. \quad (27)$$

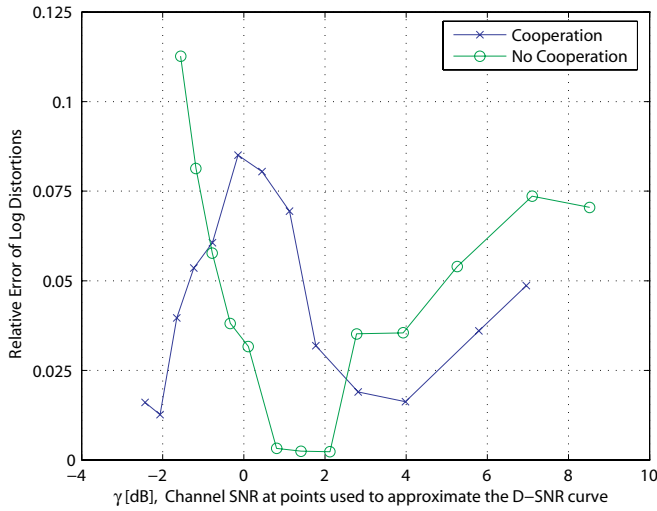


Fig. 5. Relative error between the logarithm of the actual and the approximated distortion values at the points used to approximate the D-SNR curve.

through (12), (15), (16) or (19). Fig. 5 shows, for the memory 4 RCPC code used before, the relative error between the logarithm of the actual and the approximated distortion values at the points used to approximate the D-SNR curve. Only one curve is needed for all the schemes with cooperation because the abscissa represents  $\gamma_{AF}$ ,  $\gamma_{sd} + \gamma_{rd}$  or  $\gamma_{sd}$ , depending on the case and because  $G_c$  and  $m_c$  are the same for all schemes with cooperation. In the figure, it can be seen that the approximation is sufficiently good with most points having relative errors of less than 5% (many, in fact, less than 2.5%) and only one slightly exceeding 10%. The source for most of the errors can be traced to the tightness of the approximation  $d_f \approx \kappa e^{-cr}$  and the approximation of  $a(d_f)$  by its average value.

The second step in characterizing the D-SNR curve is the choice of  $\Delta$  that sets the points used to approximate the D-SNR curve. As discussed, the best choice of points is those where the relative contribution of channel errors to the end-to-end distortion is small but not negligible. This sets each point in the “elbow” section of a single-mode D-SNR curve, where the curve is transitioning from exhibiting and almost constant distortion to a rapid increase in distortion due to channel errors. Points with a relative contribution of channel errors that is

practically negligible or is large may or may not belong to the section of single-mode D-SNR curve that contributes to the D-SNR curve. We have chosen  $\Delta = .1$  as a good value, yet it is still necessary to know how results are impacted by this choice. We do so by studying the sensibility of the parameters  $G_{c,dB}$ ,  $G_{n,dB}$ ,  $m_c$  and  $m_n$  with respect to  $\Delta$ . The sensibility of a function  $y$  of the variable  $x$ ,  $S_x^y$ , is defined as [33]

$$S_x^y = \frac{dy/y}{dx/x}. \quad (23)$$

Using this definition and through algebraic operations, we can find the expressions for  $S_{\Delta}^{m_c}$ ,  $S_{\Delta}^{m_n}$ ,  $S_{\Delta}^{G_{c,dB}}$  and  $S_{\Delta}^{G_{n,dB}}$  shown in (24) through (27). For the same memory 4 RCPC code used earlier, the sensibility of  $m_c$  or  $m_n$  can be found to never exceed 0.022 for  $0.05 \leq \Delta \leq 0.2$  and  $G_{c,dB}$  or  $G_{n,dB}$  can be found to never exceed 0.08 for  $0.05 \leq \Delta \leq 0.2$ , also. This means that the parameters used to characterize the D-SNR show little sensibility to the actual choice of  $\Delta$ , as long as it is chosen following the explained guidelines.

#### IV. SOURCE-CHANNEL-COOPERATION TRADEOFFS

We now study the SCC tradeoffs by examining the relation between  $m_n$  and  $m_c$ , and  $G_n$  and  $G_c$ .

We first consider the relation between  $m_c$  and  $m_n$ , as given by (12) and (20). Because in practice  $\Psi > \ln(2)$ , we have that  $2\Psi + 2\bar{R}_{SC} \ln(4) > \Psi + \ln(2)(1 + 4\bar{R}_{SC})$ , which implies that  $m_n/m_c < 2$ . Also, it can be shown through algebraic operation that the condition  $m_n/m_c < 1$  requires  $cN/(W \ln(4)) < 0$ , which is not possible. Therefore, we conclude that

$$1 < \frac{m_n}{m_c} < 2. \quad (28)$$

Furthermore, for typical system setups the term  $cN/(W \ln(4))$  is the one that contributes most to the value of  $m_c$  or  $m_n$  because it typically differs from  $(2\Psi + 2\bar{R}_{SC} \ln(4))^{-1}$  or  $(\Psi + \ln(2)(1 + 4\bar{R}_{SC}))^{-1}$  by approximately an order of magnitude. This implies that we can approximate

$$\frac{m_n}{m_c} \approx 2. \quad (29)$$

Physically, this means that distortion will decrease at a rate approximately twice as fast in systems without user cooperation (as can be seen in Figure 2) when compared against systems with AF or DF cooperation. Interestingly, note that



this relation follows mostly from the fact that when using cooperation, the communication capacity is halved.

To study the relation between  $G_N$  and  $G_c$ , we recall that Equations (8), (11), (15) and (16) show that the functional relation between end-to-end distortion and received SNR (be it  $\gamma_{sd}$ ,  $\gamma_{AF}$  or  $\gamma_{DF}$ ) is linear in log-log scales. Therefore,  $m_n$ ,  $G_N$ ,  $m_c$  and  $G_c$  can be calculated from the knowledge of only two points of the D-SNR curve. Let  $(D_{N1}, \gamma_{N1})$  and  $(D_{N2}, \gamma_{N2})$  be the coordinates of these points for schemes without cooperation and  $(D_{C1}, \gamma_{C1})$  and  $(D_{C2}, \gamma_{C2})$  be those for schemes using cooperation. The coordinates of these points could be approximated using again the observation that each single-mode D-SNR curve contributes to the overall D-SNR curve over a section where the contribution of channel-induced error is relatively small. Then, from (8) and (9),

$$D_i = (1 + \Delta) 2^{-2R_i}, \quad \gamma_{dB_i} = 10 \log \left[ P_{\Omega_i}^{-1} \left( \frac{\Delta}{2^{2R_i} - 1} \right) \right], \quad (30)$$

for  $i = 1, 2$  and where the subscript  $dB$  denotes a magnitude expressed in decibels. Knowing the points coordinates, it is then possible to find

$$m_n = \frac{\log(D_{N1}/D_{N2})}{\gamma_{N2,dB} - \gamma_{N1,dB}} \quad G_{N,dB} = -(\gamma_{N1,dB} + \frac{\log D_{N1}}{m_n})$$

$$m_c = \frac{\log(D_{C1}/D_{C2})}{\gamma_{C2,dB} - \gamma_{C1,dB}} \quad G_{c,dB} = -(\gamma_{C1,dB} + \frac{\log D_{C1}}{m_c}).$$

Focusing on  $G_N$  and  $G_c$ , note that  $\log D_{C1} = \log(1 + \Delta) - 2R_{SC1} \log 2$ . Since  $\Delta$  is small,  $\log D_{C1} \approx -2R_{SC1} \log 2 = -R_{SN1} \log 2 \approx (\log D_{N1})/2$ . Then, since  $m_n/m_c \approx 2$ , we have  $\log D_{C1}/m_c \approx \log D_{N1}/m_n$ , which implies that the relation between  $G_N$  and  $G_c$ , approximately only depends on the relation between  $\gamma_{N1,dB}$  and  $\gamma_{C1,dB}$ . Furthermore, Equation (2) can be approximated as  $P(\gamma) \lesssim NR_s \left( \sum_{d=d_f}^W a(d)P(d|\gamma) \right)$ , [27]. Differentiating explicitly between the use or not of cooperation by using the subscripts  $C$  and  $N$ , respectively, it follows that  $P_C(\gamma) \approx P_N(\gamma)/2$  for a given operating mode  $\Omega_i$ . Considering these observations along with (8) and (9), leads to (30) being re-written as

$$\gamma_{Ci,dB} = 10 \log \left[ P_{N\Omega_i}^{-1} \left( \frac{2\Delta}{2^{R_{SN_i}} - 1} \right) \right], \quad i = 1, 2, \quad (31)$$

which implies that  $\gamma_{C1,dB} < \gamma_{N1,dB}$ . Therefore, we conclude that

$$G_{C,dB} > G_{N,dB}. \quad (32)$$

Consider next the value of  $\gamma_{sd}$  for which  $D_{CAF} = D_N$ .

$$\left( G_c \left( \gamma_{sd} + \frac{\gamma_{sr}\gamma_{rd}}{1 + \gamma_{sr} + \gamma_{rd}} \right) \right)^{-10m_c} = (G_N \gamma_{sd})^{-10m_n}$$

$$\Rightarrow \frac{G_N^{m_n}}{G_c^{m_c}} = \frac{\left( \gamma_{sd} + \frac{\gamma_{sr}\gamma_{rd}}{1 + \gamma_{sr} + \gamma_{rd}} \right)^{m_c}}{\gamma_{sd}^{m_n}}$$

$$\Rightarrow \frac{G_N^2}{G_c} \approx \frac{\left( \gamma_{sd} + \frac{\gamma_{sr}\gamma_{rd}}{1 + \gamma_{sr} + \gamma_{rd}} \right)}{\gamma_{sd}^2} \quad (33)$$

$$\Rightarrow \gamma_{sd} \approx \frac{G_c^2}{G_N} \left( 1 + \sqrt{1 + 4 \left( \frac{\gamma_{sr}\gamma_{rd}}{1 + \gamma_{sr} + \gamma_{rd}} \right) \frac{G_N^2}{G_c}} \right), \quad (34)$$

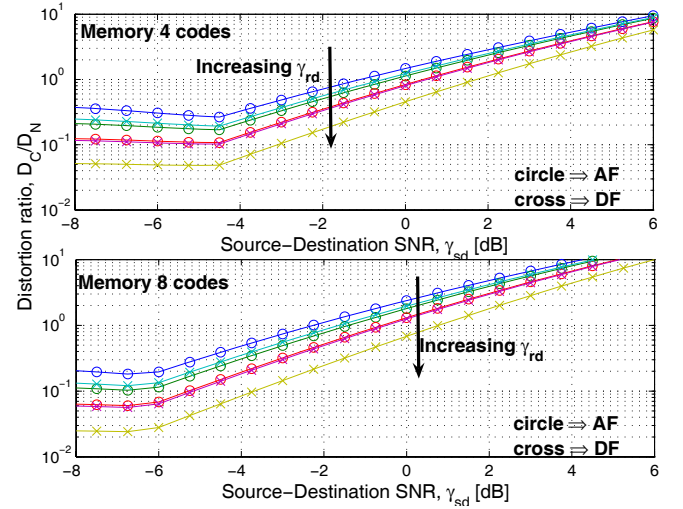


Fig. 6. Ratio between distortions with and without cooperation when  $\gamma_{sr} = 5dB$  and  $\gamma_{rd} = -2dB$ ,  $\gamma_{rd} = 1dB$  and  $\gamma_{rd} = 4dB$ .

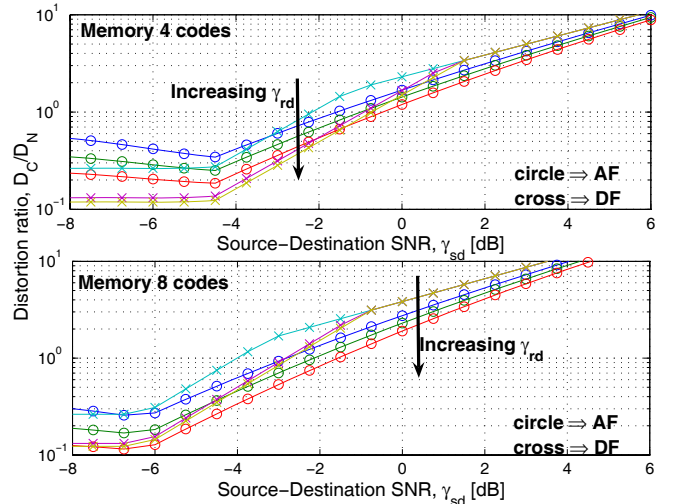


Fig. 7. Ratio between distortions with and without cooperation when  $\gamma_{sr} = 1dB$  and  $\gamma_{rd} = -2dB$ ,  $\gamma_{rd} = 1dB$  and  $\gamma_{rd} = 4dB$ .

where in (33), we have used (29). Equation (34) shows that there is only one value of  $\gamma_{sd}$  for which the distortion with and without cooperation are equal. The combined effect of (28), (32) and (34) is that there are a range of values of  $\gamma_{sd}$  (those that correspond to relative high SNR) for which it is better not to use cooperation and a range of values for which it is better to use cooperation. Due to space limitation, we point out here that the same analysis that leads to (34) could be carried out for DF cooperation and reach the same conclusion. Also, we briefly note here that these conclusions can also be observed in the practical systems of Figs. 3 and 4. Then, our characterization can be seen as a useful design tool to predict approximate performances.

We explore further this issue by studying the ratio  $D_C/D_N$  between distortion without and with cooperation. Figs. 6 and 7 show results for source-relay channels that are classified as “good” or “bad”, respectively, for most operating modes in DF cooperation. These figures also explore the effects of

the relative strength of the channel code family by comparing results for the memory 4 family of RCPC codes, to those obtained for a memory 8 RCPC codes from [32] (for which we have  $\bar{a} = 7.1$ ,  $\kappa = 45$  and  $c = 2.8$ ). The results confirm that non-cooperation is better than AF or DF cooperation at large  $\gamma_{sd}$ , i.e.  $D_C/D_N > 1$ . This relation is inverted at low  $\gamma_{sd}$ . As it is to expect, the value of  $\gamma_{sd}$  for which cooperation is better increases with  $\gamma_{sr}$ . Furthermore, the value of  $\gamma_{sd}$  for which cooperation is better decreases with the use of a stronger channel code because the larger bandwidth efficiency when not using cooperation allows for a more effective use of the stronger code redundancy. Also, note that there is a value of  $\gamma_{sd}$  for which  $D_C/D_N$  is minimum (cooperation yields best performance gain). This value does not depend on  $\gamma_{sr}$ ,  $\gamma_{rd}$  or the type of cooperation, it only depends on the channel code family used and is close to the value of  $\gamma_{sd}$  for which the non-cooperative distortion reaches 1. Comparison between AF and DF cooperation shows that the later is better for all values of  $\gamma_{sd}$  and  $\gamma_{rd}$  when the source-relay channel is “good”. In the case of “bad” source-relay channel, DF cooperation outperforms AF only at low  $\gamma_{sd}$ , and this advantage can be overcome by choosing a family of channel codes that is strong enough. Note that previous works have not observed a performance difference between AF and DF cooperation [15], but we are providing a different viewpoint, since we are considering end-to-end performance of systems with source and channel coding, with channel side information at the transmitter and no asymptotic high SNR analysis.

We also considered the influence that a change in source codec efficiency has on the results. In practice, different source codecs exhibit different compression efficiency, i.e. different source codecs achieve the same distortion values at different encoding rates. Thus, the efficiency of the source codec can be incorporated into the formulation by writing the D-R function as  $D_S(R_S) = 2^{-2(1-\lambda)R_S} = 2^{-\hat{\lambda}R_S}$ , where  $\lambda$  determines the coding inefficiency and  $\hat{\lambda} \triangleq 2(1-\lambda)$ . By modifying our results accordingly, we have that (12), (13), (20) and (21) become

$$\begin{aligned}
 m_c &= \left[ 20 \left( \frac{cN}{W\hat{\lambda}\ln(2)} + \frac{1}{2\Psi + 2\hat{\lambda}\ln(2)\bar{R}_{SN}} \right) \right]^{-1}, \\
 G_c &= \frac{\kappa(1+\Delta)^{-1/(10m_c)} 10^{\frac{\hat{\lambda}\log(2)\bar{R}_{SN}}{\Psi + \hat{\lambda}\bar{R}_{SN}\ln(2)}}}{\Psi + \hat{\lambda}\bar{R}_{SN}\ln(2)}, \\
 m_n &= \left[ 10 \left( \frac{cN}{W\hat{\lambda}\ln(2)} + \frac{1}{\Psi + \ln(2)(1 + 2\hat{\lambda}\bar{R}_{SN})} \right) \right]^{-1}, \\
 G_n &= \frac{\kappa(1+\Delta)^{-1/(10m_n)} 10^{\frac{\hat{\lambda}\log(2)\bar{R}_{SN}}{\Psi + \ln(2)(2\hat{\lambda}\bar{R}_{SN} + 1)}}}{\Psi + \ln(2)(2\hat{\lambda}\bar{R}_{SN} + 1)}. \quad (35)
 \end{aligned}$$

These equations show that  $m_c$  and  $m_n$  change with  $\hat{\lambda}$  approximately in a linear fashion. Thus, the relation  $m_n/m_c \approx 2$  is maintained. Figures 8 and 9 show the ratio  $D_C/D_N$  for  $\lambda = 0.2$ . We can see that in the case of “bad” source-relay channel, the increased inefficiency of the source codec translates into an increase in the range of values of  $\gamma_{sd}$  for which no cooperation outperforms AF cooperation. This effect is compensated by using stronger channel coding. Also, we can see that DF cooperation now outperforms AF cooperation for a larger range of values of  $\gamma_{sd}$ . In the case of “good” source-

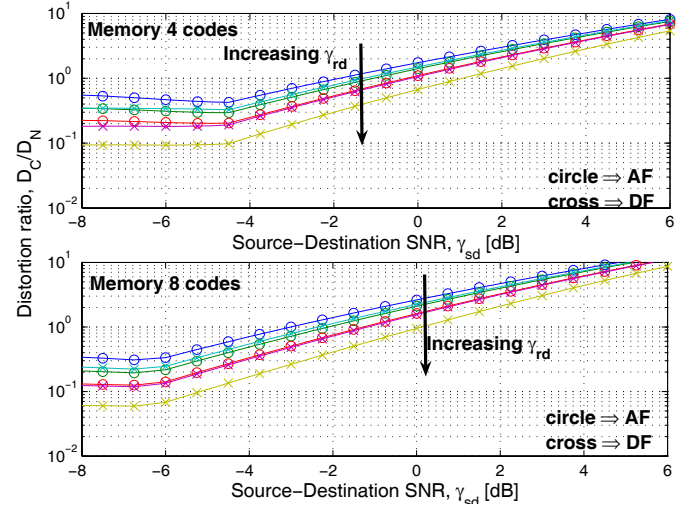


Fig. 8. Effect of source codec efficiency on the ratio between distortions with and without cooperation when  $\lambda = 0.2$ ,  $\gamma_{sr} = 5dB$  and  $\gamma_{rd} = -2dB$ ,  $\gamma_{rd} = 1dB$  and  $\gamma_{rd} = 4dB$ .

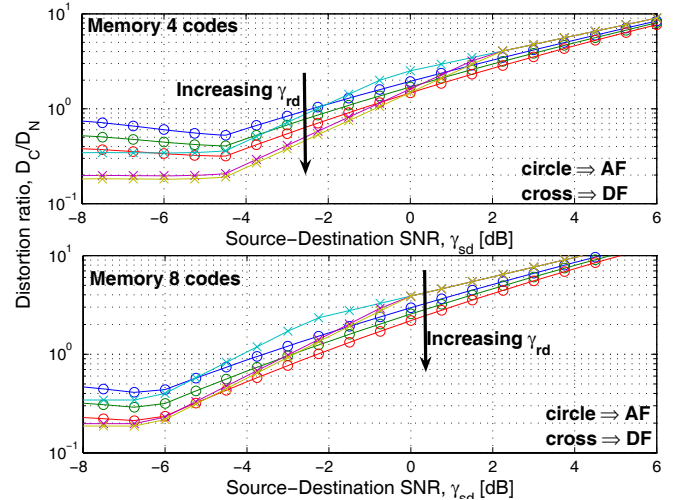


Fig. 9. Effect of source codec efficiency on the ratio between distortions with and without cooperation when  $\lambda = 0.2$ ,  $\gamma_{sr} = 1dB$  and  $\gamma_{rd} = -2dB$ ,  $\gamma_{rd} = 1dB$  and  $\gamma_{rd} = 4dB$ .

relay channel, DF cooperation is the cooperative scheme that is outperformed by no cooperation over a larger range of  $\gamma_{sd}$ . Also, we can see that the results show a reduced sensitivity to the value of  $\gamma_{rd}$ .

Finally, we note that our assumption of ideally estimated channels allows for the possibility of designing schemes that prevent the sacrifice in bandwidth efficiency associated with the cooperative schemes. When the source-destination channel is sufficiently good, our results suggest that transmission should be switched to a scheme with no cooperation (interestingly, this observation can be seen as a generalization of the idea that when the channel is good enough most of the transmission capacity should be invested in increasing the source coding rate, thus reducing error protection). Adding layers of complexity to the design may lead to other schemes that exploit the knowledge of the channel to overcome the sacrifice in bandwidth efficiency (for example with adaptive

modulation). Nevertheless, as previously discussed, many of these approaches add extra dimensions to our present work aimed at revealing the tradeoffs in pure SCC schemes, thus exciding the scope of this paper. Furthermore, our results, specially (29), suggests that cooperative schemes beyond the ones considered here can be designed to outperform non-cooperative schemes over a larger range of the operational envelope if they were able to adapt bandwidth efficiency (such as those in [16]). Nevertheless, further research under the conditions of this study is warranted to learn about the properties of such systems.

## V. CONCLUSIONS

We studied the effects that the tradeoffs among source coding, channel coding, and use of cooperation have on the performance of systems carrying conversational traffic. We considered practical source and channel codecs and for user cooperation we considered amplify-and-forward and decode-and-forward schemes. We measured performance using the D-SNR curve which represents the relation between end-to-end distortion and channel SNR. We saw that the D-SNR curve can be accurately approximated as a linear function in a log-log scale. Our approach has the key advantage of considering practical source and channel codes and their settings, while it avoids resorting to high SNR asymptotic analysis. The application of our D-SNR characterization showed that schemes using cooperation have better coding gain (due to the better error performance) but they show a decrease of distortion at approximately half the rate of schemes not using cooperation (due to the sacrifice in bandwidth efficiency). The overall effect is that non-cooperative schemes have better performance at high source-destination SNR, while AF or DF cooperative schemes have better performance in the rest of cases.

Further analysis showed that the best performance among cooperative schemes is achieved with DF cooperation in most of the cases but, when the source-relay channel is bad the performance advantage of DF cooperation is reduced by choosing a channel code family sufficiently strong. In addition, we studied the effects that the efficiency of the source codec has on system performance and we showed that it reduces the diversity gain proportionally to the codec loss of efficiency. Also, we saw that for "bad" source-relay channel, AF cooperation is outperformed by DF cooperation and by no cooperation over a larger range of channel SNRs but this effect could be compensated by choosing a stronger family of channel codes. Similar observations apply to DF cooperation in the case of "good" source-relay channel. We finally note that our results suggests that the performance can be improved by allowing switching between the use or not of cooperation based on channel knowledge and by resorting to cooperative schemes that adapt the bandwidth efficiency.

## REFERENCES

- [1] D. J. Goodman, R. A. Valenzuela, K. T. Gayliard, and B. Ramamurthi, "Packet reservation multiple access for local wireless communications," *IEEE Trans. Commun.*, vol. 37, no. 8, pp. 885–890, Aug. 1989.
- [2] A. Uvildén, S. Bruhn, and R. Hagen, "Adaptive multi-rate: a speech service adapted to cellular radio network quality," in *Proc. Thirty-Second Asilomar Conference on Signals, Systems & Computers*, vol. 1, pp. 343–347, 1998.
- [3] M. Bystrom and J. W. Modestino, "Combined source-channel coding schemes for video transmission over an additive white gaussian noise channel," *IEEE J. Select. Areas Commun.*, vol. 18, no. 6, pp. 880–890, June 2000.
- [4] K. Stuhlmüller, N. Färber, M. Link, and B. Girod, "Analysis of video transmission over lossy channels," *IEEE J. Select. Areas Commun.*, vol. 18, no. 6, pp. 1012–1032, June 2000.
- [5] Y. Shen, P. Cosman, and L. Milstein, "Error-resilient video communications over cdma networks with a bandwidth constraint," *IEEE Trans. Image Proc.*, vol. 15, no. 11, pp. 3241–3252, 2006.
- [6] A. Kwasinski and N. Farvardin, "Optimal resource allocation for cdma networks based on arbitrary real-time source coders adaptation with application to mpeg4 fgs," in *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*, Atlanta, GA, Mar. 2004.
- [7] Y. S. Chan and J. W. Modestino, "A cross-layer optimisation approach for multimedia over cdma mobile wireless networks," *International J. Wireless and Mobile Computing*, vol. 1, no. 1, pp. 14–23, 2005.
- [8] J. Song and K. J. R. Liu, "Robust progressive image transmission over OFDM systems using space-time block code," *IEEE Trans. Multimedia*, vol. 4, no. 3, pp. 394–406, Sept. 2002.
- [9] S. Mao, S. Lin, S. S. Panwar, Y. Wang, and E. Celebi, "Video transport over ad hoc networks: multistream coding with multipath transport," *IEEE J. Select. Areas Commun.*, vol. 21, no. 10, pp. 1721–1737, Dec. 2003.
- [10] M. Effros, R. Koetter, A. Goldsmith, and M. Medard, "On source and channel codes for multiple inputs and outputs: does multiple description beat space time?" in *Proc. 2004 IEEE Inf. Th. Workshop ITW*, pp. 324–329.
- [11] S. Lin, A. Stefanov, and Y. Wang, "On the performance of spacetime block-coded MIMO video communications," *IEEE Trans. Veh. Technol.*, vol. 56, no. 3, pp. 1223–1229, May 2007.
- [12] T. M. Cover and A. A. E. Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inform. Theory*, vol. 25, no. 9, pp. 572–584, Sept. 1979.
- [13] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity, part I: system description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.
- [14] J. N. Laneman and G. W. Wornell, "Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2415–2525, Oct. 2003.
- [15] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
- [16] D. Gunduz and E. Erkip, "Source and channel coding for cooperative relaying," *IEEE Trans. Inform. Theory*, vol. 53, no. 10, pp. 3454–3475, Oct. 2007.
- [17] H. Y. Shutoy, D. Gunduz, E. Erkip, and Y. Wang, "Cooperative source and channel coding for wireless multimedia communications," *IEEE J. Select. Topics Signal Processing*, vol. 1, no. 2, pp. 295–307, Aug. 2007.
- [18] A. Kwasinski, Z. Han, and K. J. R. Liu, "Cooperative multimedia communications: joint source coding and collaboration," in *Proc. Global Telecommunications Conference (GLOBECOM '05)*, vol. 1, 2005, pp. 374–378.
- [19] A. Kwasinski and K. J. R. Liu, "Towards a unified framework for modeling and analysis of diversity in joint source-channel coding," *IEEE Trans. Commun.*, vol. 56, no. 1, pp. 90–101, Jan. 2008.
- [20] R. T. D. et al., "Transmit diversity in 3g cdma systems," *IEEE Commun. Mag.*, pp. 68–75, Apr. 2002.
- [21] "Air interface for fixed and mobile broadband wireless access systems," *IEEE Std 802.16e*, 2005.
- [22] A. Kwasinski, "Cross-layer resource allocation protocols for multimedia cdma networks," Ph.D. dissertation, University of Maryland, College Park, 2004.
- [23] Z. He, J. Cai, and C. W. Chen, "Joint source channel rate-distortion analysis for adaptive mode selection and rate control in wireless video coding," *IEEE Trans. Circ. and Syst. for Vid. Tech.*, vol. 12, no. 6, pp. 511–523, 2002.
- [24] A. Kwasinski, Z. Han, K. J. R. Liu, and N. Farvardin, "Power minimization under real-time source distortion constraints in wireless networks," in *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*, vol. 1, New Orleans, LA, Mar. 2003, pp. 532–536.
- [25] T. Cover and J. Thomas, *Elements of Information Theory*. John Wiley Inc., 1991.
- [26] J. Hagenauer, "Rate compatible punctured convolutional (RCPC) codes and their applications," *IEEE Trans. Commun.*, vol. 36, no. 4, pp. 389–399, Apr. 1988.

- [27] E. Malkamaki and H. Leib, "Evaluating the performance of convolutional codes over block fading channels," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1643–1646, July 1999.
- [28] J. G. Proakis, *Digital Communications*. McGraw-Hill Inc., 1994.
- [29] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Spacetime codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, no. 2, pp. 744–765, Mar. 1998.
- [30] ETSI/GSM, "Digital cellular telecommunications system (phase 2+); adaptive multi-rate (amr) speech transcoding (gsm 06.90 version 7.2.1 release 1998)," in *Document ETSI EN 301 704 V7.2.1 (2000-04)*.
- [31] ITU-T, "Recommendation p.862: Perceptual evaluation of speech quality (pesq): an objective method for end-to-end speech quality assessment of narrow-band telephone networks and speech codecs," 2001.
- [32] P. Frenger, P. Orten, T. Ottosson, and A. B. Svensson, "Rate-compatible convolutional codes for multirate ds-cdma systems," *IEEE Trans. Commun.*, vol. 47, pp. 828–836, June 1999.
- [33] J. Gorski-Popiel, "Classical sensitivity: a collection of formulas," *IEEE Trans. Circ. and Syst.*



**Andres Kwasinski** received in 1992 his diploma in Electrical Engineering from the Buenos Aires Institute of Technology, Buenos Aires, Argentina, and the MS and PhD degrees in Electrical and Computer Engineering from the University of Maryland, College Park, Maryland, in 2000 and 2004, respectively. He is an Assistant Professor at the Department of Computer Engineering, Rochester Institute of Technology, Rochester, New York. Prior to this he was with Texas Instruments Inc., the Department of Electrical and Computer Engineering

at the University of Maryland, and Lucent Technologies. His research interests are in the area of multimedia wireless communications and networking, cross layer designs, digital signal processing and speech, image and video processing for signal compression and communication.



**K. J. Ray Liu** (F03) received the B.S. degree from the National Taiwan University and the Ph.D. degree from UCLA, both in electrical engineering. He is Professor and Associate Chair, Graduate Studies and Research, of Electrical and Computer Engineering Department, University of Maryland, College Park. His research contributions encompass broad aspects of wireless communications and networking, information forensics and security, multimedia communications and signal processing, bioinformatics and biomedical imaging, and signal processing algo-

rithms and architectures.

Dr. Liu is the recipient of numerous honors and awards including best paper awards from IEEE Signal Processing Society (twice), IEEE Vehicular Technology Society, and EURASIP; IEEE Signal Processing Society Distinguished Lecturer, EURASIP Meritorious Service Award, and National Science Foundation Young Investigator Award. He also received various teaching and research recognitions from University of Maryland including university-level Distinguished Scholar-Teacher Award, Invention of the Year Award, Fellow of Academy for Excellence in Teaching and Learning, and college-level Poole and Kent Company Senior Faculty Teaching Award.

Dr. Liu is Vice President–Publications and on the Board of Governor of IEEE Signal Processing Society. He was the Editor-in-Chief of IEEE SIGNAL PROCESSING MAGAZINE and the founding Editor-in-Chief of EURASIP JOURNAL ON APPLIED SIGNAL PROCESSING.

His recent books include *Cooperative Communications and Networking*, Cambridge University Press, 2008; *Resource Allocation for Wireless Networks: Basics, Techniques, and Applications*, Cambridge University Press, 2008; *Ultra-Wideband Communication Systems: The Multiband OFDM Approach*, IEEE-Wiley, 2007; *Network-Aware Security for Group Communications*, Springer, 2007; *Multimedia Fingerprinting Forensics for Traitor Tracing*, Hindawi, 2005.