A Broad Beamforming Approach for High-Mobility Communications

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***Abstract*—To leverage the benefits of multiple-input multiple-output in high-mobility communications, this paper proposes a broad beamforming approach (BBA), which focuses a broad beam on a moving user on the basis of the moving user's current location and velocity information. So, the moving user can be covered and tracked by the broad beam and is able to achieve high received signal-to-noise ratio (SNR). An optimization problem is formulated to minimize the required power by jointly optimizing the transmit and the receiving beam vectors, while guaranteeing the desired information rate of the moving user. Since the problem is with infinite constraints and has no known solution, we design an efficient algorithm to solve it, where the transmit and receiving beam vectors are optimized by using semidefinite relaxation alternatively in an iterative manner. Simulation results show that by using our proposed BBA, the desired information rate of the high-mobility user can be guaranteed. Compared with existing schemes, with the same power consumption, our proposed scheme can enhance the SNR greatly within the moving region of interest. Besides, by increasing the number of transmit antennas, the required power can be further decreased.**

***Index Terms*—High-mobility wireless communications, high-speed railway, beamforming, multiple antennas.**

I. Introduction

In Multiple-Input multiple-output (MIMO) systems, when the transmitter knows channel state information (CSI), system information transmission performance can be greatly enhanced by employing beamforming/precoding, which fully exploits the spacial diversity and multiplexing gains. With beamforming, the transmitted energy is focused on the receiver with a directional beam, which greatly improves received signal-to-noise ratio (SNR). The larger the number of transmit antennas, the sharper the energy beam [1]–[3]. If one desires to exploits the benefits of beamforming to provide high-quality and stable communication for a moving user, the beam has to be updated periodically to track the moving user.

Owing to the advantages of MIMO technique, in the past decades, beamforming design for high-mobility scenarios has attracted increasing interest [1], [3]–[8]. In [1], multiple antennas were equipped on the train to achieve receiving diversity but how to design the beamforming was involved. In [3], the optimal beam was selected from predefined beams where how to optimize the beam vector was not considered. In [4]–[8], location information-assisted, opportunistic and multi-stream beamformings were respectively investigated. However, full CSI was required, which may be unavailable in high-mobility.

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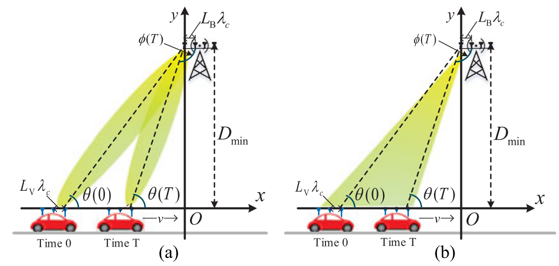

Fig. 1. System model: (a) Traditional narrow belaboring method (b) Our proposed broad beamforming method.

In this paper, we also focus on the beamforming design for high-mobility communications. The contributions of our work are summarized as follows. *Firstly*, we present a broad beamforming approach (BBA), where beams are updated periodically along with the movement of the user. Different from existing works [5]–[8], our proposed BBA is able to generate a broader beam to cover and track the high-mobility user, so that the beam can last for a much longer time to provide the desired information rate of the moving user, avoiding calculating and updating the beam configuration very frequently. Moreover, our proposed BBA only requires the position of the moving vehicle at the start time, the moving speed, and the channel gain variance rather than the perfect CSI, so it is more practical for high-mobility scenarios, as the perfect CSI may not be available due to the feedback delay and the fast varying high-mobility channels. *Secondly*, we formulate an optimization problem for our proposed BBA, which jointly optimizes the transmit and receiving beam vectors to minimize the power consumption while guaranteeing the user's QoS requirement for a given time period T. *Thirdly*, as the problem is with infinite constraints and cannot be directly solved, we design an efficient algorithm to optimize the transmit and receiving beamforming vectors alteratively by using semi-definite relaxation (SDR) method. *Finally*, simulation results are provided, which show that using our proposed BBA, the required SNR of high-mobility user is guaranteed. Compared with existing schemes, with the same power consumption, the SNR within T (or the interest region) can be greatly enhanced. Besides, by increasing the number of transmit antennas, the consumed power can be reduced, and it continues to decrease but more slowly as the number of transmit antennas grows.

Notations: $\mathbb{C}^{M\times N}$, $\mathbb{H}^{M\times N}$, $\mathbb{R}^{M\times N}$ are the sets of $M\times N$ complex, Hermitian and real matrices, respectively. $\odot$ denotes the element-wise multiplication between two matrices. $E[\cdot]$ represents the expectation operator. $\|\cdot\|$ is the Frobenius norm. $\text{Tr}(\mathbf{X})$ is the trace of matrix $\mathbf{X}$. $\text{vec}(\cdot)$ and $\otimes$ are the vectorization and the Kronecker-product operators, respectively. $\text{Re}\{\cdot\}$ is the real-part extraction operator of a complex number. $\mathbf{I}_N$ is a $N\times N$ identity matrix.

II. System Model

Consider a communication scenario as shown in Fig. 1, where a vehicle with N_V antennas moving at speed v is receiving information from a roadside Base Station (BS) with N_B antennas. Transmit and receiving beamforming are performed periodically. For each time period T, it is assumed that v is constant and the moving track is a straight line[1]. For clarity, we set a reference coordinate system as shown in Fig. 1, where the direction of v is considered as the positive direction of the x-axis and the vehicle is moving along the x-axis. $D_{\min}$ denotes the distance of the BS from the x-axis.

For the BS, its antennas are numbered along with the positive direction of v as 1, 2, ..., N_{B}, where the distance between its two neighbouring antennas is $L_{\mathrm{B}}\lambda_c$. For the vehicle, its antennas are numbered along with the negative direction of v as 1, 2, ..., N_{V}, where the distance between its two neighbouring antennas is $L_{\mathrm{V}}\lambda_c$. L_{B} and L_{V} are two constants and λ_c is the carrier wavelength. $d_{i,k}(t)$ represents the distance between the i-th antenna of the vehicle and the k-th antenna of the BS at time t. $\phi_{i,k}(t)$ is the angle of departure (AOD) and $\theta_{i,k}(t)$ is the angle of arrival (AOA). As $L_{\mathrm{B}}\lambda_c, L_{\mathrm{V}}\lambda_c \ll d_{i,k}(t)$, both $\theta_{i,k}(t)$ and $\phi_{i,k}(t)$ can be regarded to be the same for all $i = 1, ..., N_{\mathrm{V}}$ and all $k = 1, ..., N_{\mathrm{B}}$. That is $\theta_{i,k}(t) \doteq \theta(t)$. and $\phi_{i,k}(t) \doteq \phi(t)$. If we define $d_0(t) \triangleq d_{1,1}(t)$ as a reference distance, it can be given by $d_0(t) = \frac{D_{\min}}{\sin\theta(t)}$. Define $\xi_{\mathrm{V}}(t) \triangleq L_{\mathrm{V}}\lambda_c \cos\theta(t)$ and $\xi_{\mathrm{B}}(t) \triangleq L_{\mathrm{B}}\lambda_c \cos\phi(t)$. Via projecting $d_{i,k}(t)$ onto $d_{1,1}(t)$, $d_{i,k}$ is approximately calculated by

$$d_{i,k}(t) \simeq d_{1,1}(t) + (i-1)\xi_{\mathrm{V}}(t) + (k-1)\xi_{\mathrm{B}}(t). \tag{1}$$

According to the wave propagation, the channel coefficient associated with the line-of-sight (LoS) link between the BS's k-th antenna and the vehicle's i-th antenna at time t is

$$h_{i,k}^{(\mathrm{LoS})}(t) = g_{i,k}(t)\exp\left\{j2\pi\frac{d_{i,k}(t)}{\lambda_c}\right\}, \tag{2}$$

where $g_{i,k}(t)$ is the distance-dependent large-scale path channel gain with $g_{i,k}(t) = \beta d_{i,k}(t)^{-\frac{\alpha}{2}}$. β is a coefficient, depending on the antenna gains advocated with both the transmit and receiving antennas. α is the pass loss factor. Thus, the MIMO channel matrix associated with the LoS can be given by

$$\mathbf{H}^{(\mathrm{LoS})}(t) = \mathrm{G}(t) \odot \mathrm{H}_s(t), \tag{3}$$

where $\mathrm{G}(t) = \{g_{i,k}(t)\}_{N_{\mathrm{V}}\times N_{\mathrm{B}}}$. The channel specular component matrix $\mathrm{H}_s(t) = \exp\left(j2\pi\frac{d_{1,1}(t)}{\lambda_c}\right)\boldsymbol{a}(\theta(t))\boldsymbol{a}(\phi(t))^T$ with $\boldsymbol{a}(\phi(t)) = [1, \exp(j2\pi\xi_{\mathrm{B}}(t)), ..., \exp(j2\pi(N_{\mathrm{B}}-1)\xi_{\mathrm{B}}(t))]^T$ and $\boldsymbol{a}(\theta(t)) = [1, \exp(j2\pi\xi_{\mathrm{V}}(t)), ..., \exp(j2\pi(N_{\mathrm{V}}-1)\xi_{\mathrm{V}}(t))]^T$ are the specular array responses at the BS and the moving vehicle, respectively.

Besides LoS, wireless channel is also affected by the multi-path fading, which can be regarded as a random part of the MIMO channel, denoted as $\mathbf{H}^{(\text{multi-path})}$. A narrow band system is considered, so the entries of $\mathbf{H}^{(\text{multi-path})}$ are characterized by independent complex Gaussian random variables with unit variance [10]. By using real-data measurement, the maximum power gain $\varepsilon^2_{\text{multi-path}}$ associated with multi-path fading can be determined. Thus, $\| \mathbf{H}^{(\text{multi-path})} \|^2 \leq \varepsilon^2_{\text{multi-path}}$.

Moreover, the doppler effect caused by the mobility of vehicles is also taken into account. Denote $f_{i,k}(t)$ as the Doppler shift between the BS's i-th antenna and the vehicle's k-th antenna at time t. According to [6], it is given by $f_{i,k}(t) = f_{\max}\cos\theta_{i,k}(t) \doteq f_{\max}\cos\theta(t)$, where $f_{\max}$ is the maximum Doppler frequency, determined by $f_{\max} = \frac{v}{c}f_c$. Let $x(t)$ be the vehicle's position on x-axis at time t. With $x(0)$ and v, it can be calculated that $x(t) = x(0) + vt$ and $\cos\theta(t) = \frac{-(x(0)+vt)}{\sqrt{(x(0)+vt)^2 + D^2_{\min}}}$. For a short t, $\cos\theta(t) \simeq \frac{-(x(0))}{\sqrt{(x(0))^2 + D^2_{\min}}} = \cos\theta(0)$. As a result, $f_{i,k}(t) \doteq f_{\max}\cos\theta(0)$, which means that for each T, the doppler shift can be treated to be time invariant, so it is able to be compensated by using some standard frequency offset pre-correction methods as [9] based on the vehicle location information at time 0.

According to the analysis above, the channel matrix between the BS and the moving vehicle at time t, can be modeled by

$$\mathbf{H}(t) = \frac{\exp(j2\pi f_{\max})}{\sqrt{K+1}}\left(\sqrt{K}\mathbf{H}^{(\mathrm{LoS})}(t) + \mathbf{H}^{(\text{multi-path})}(t)\right), \tag{4}$$

where $\exp(j2\pi f_{\max})$ is used to characterize the phase change caused by Doppler effect. $K \geq 0$ is the Ricean factor, representing the ration of the LoS energy to the scattered energy. When $K = 0$, the channel follows Rayleigh fading and when $K \to \infty$, it corresponds to a pure LoS channel.

To enhance the system information transmission performance, a transmit beam and its corresponding receiving beam are generated on the basis of $x(0)$ and v at the beginning of each T. The estimated symbol at the moving vehicle can be represented by a linear model as $\tilde{y}(t) = \mathbf{q}(t)^H\left(\mathbf{H}(t)\mathbf{w}(t)s(t) + \mathbf{n}(t)\right)$, where $s(t)$ is the transmitted signal symbol from the BS with $E[|s(t)|^2] = 1$. $\mathbf{w}(t) \in \mathbb{C}^{N_{\mathrm{B}}\times 1}$ is the transmit beamforming vector. $\mathbf{q}(t) \in \mathbb{C}^{N_{\mathrm{V}}\times 1}$ is the receiving beamforming vector, satisfying that $\| \mathbf{q}(t) \|^2 = 1$. $\mathbf{n}(t) \in \mathbb{C}^{N_{\mathrm{V}}\times 1}$ is a circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix $\sigma_n^2\mathbf{I}$, i.e., $\mathbf{n}(t) \sim \mathcal{CN}(0, \sigma_n^2\mathbf{I})$. At a certain time t, $\mathbf{q}(t)^H$, $\mathbf{H}(t)$ and $\mathbf{w}(t)$ is determined, so the received SNR at the moving vehicle is

$$\begin{aligned} SNR(t) &= \frac{E(\|\mathbf{q}(t)^H\mathbf{H}(t)\mathbf{w}(t)s(t)\|^2)}{E(\|\mathbf{n}(t)\|^2)} = \frac{\|\mathbf{q}(t)^H\mathbf{H}(t)\mathbf{w}(t)\|^2 E(\|s(t)\|^2)}{E(\|\mathbf{n}(t)\|^2)} \\ &= \frac{\|\mathbf{q}(t)^H\mathbf{H}(t)\mathbf{w}(t)\|^2}{\sigma_n^2}. \end{aligned} \tag{5}$$

III. Problem Formulation

In order to provide reliable communication, it is critical to design a beamforming patten that can support the required QoS requirements for any time $t \in [0, T]$.

In practical systems, $x(0)$ and v is able to be obtained via some location techniques, e.g., GPS. So, $\mathbf{H}^{(\mathrm{LoS})}(0)$ can be determined by (3). Also, one can calculate $x(T) = x(0) + vt$, so that $\mathbf{H}^{(\mathrm{LoS})}(T)$ is predicted at $x(0)$ by (3). In our proposed BBA, the goal is to calculate the transmit and receiving beam vectors, i.e., $\mathbf{w}(0)$ and $\mathbf{q}(0)$ at time 0, and then use them to transmit information from the BS to the moving user during $[0, T]$. That is, for each T, the system just calculates the beam vectors only once and then sets $\mathbf{q}(t) = \mathbf{q}(0)$ and $\mathbf{w}(t) = \mathbf{w}(0)$ for information transmission. The BBA provides a wide beam coverage but it is by no means that the user's QoS requirement is neglected, so the beamforming vectors $\{\mathbf{w}(0), \mathbf{q}(0)\}$ should be designed to meet the data rate requirement of the moving vehicle $\forall t \in [0, T]$ by consuming as less power as possible. If we denote R_{th} as the minimal required data rate of the moving vehicle and define $\mathbf{w} \triangleq \mathbf{w}(0)$ and $\mathbf{q} \triangleq \mathbf{q}(0)$, the joint transmit and receiving beamforming design optimization problem can be formulated as

$$\begin{aligned} &\min_{\{\mathbf{w},\mathbf{q}\}} \ \| \mathbf{w} \|^2 \\ &\text{s.t.} \ \| \mathbf{q} \| = 1, \\ &\log_2\left(1 + \frac{\|\mathbf{q}^H\mathbf{H}(t)\mathbf{w}\|^2}{\sigma_n^2}\right) \geq R_{\mathrm{th}}, \ \forall t \in [0, T]. \end{aligned} \tag{6}$$

Via such a design, within each T, the required communication QoS of the moving user can be guaranteed. Compared with traditional beamforming, our proposed BBA generates the beam vectors at time 0 and the coverage of its beam is broader. It is observed that problem (6)

[1]In practical communication systems, the channel probing interval or each round of information transmission is often not more than hundreds of milliseconds. In such a short time period T, v cannot be changed obviously both at the value and its direction, and the moving track can be regarded to be a straight line. Therefore, this assumption is reasonable and can be applied to any shape of moving tracks.

is with infinite constraints, which cannot be solved directly, we shall transform it into some equivalent ones and then design an algorithm to solve it efficiently in Section IV.

IV. Problem Transformation and Solution

A. Problem Transformation

Denote $\gamma(t) = \| \mathbf{q}^H \mathbf{H}(t)\mathbf{w} \|^2$ and $\gamma_{\text{th}} = \sigma_n^2(2^{R_{\text{th}}} - 1)$. Problem (6) can be rewritten as

$$\min_{\{\mathbf{w},\mathbf{q}\}} \ \| \mathbf{w} \|^2 \ \text{ s.t. } \ \| \mathbf{q} \| = 1, \ \gamma(t) \geq \gamma_{\text{th}}, \ \forall t \in [0, T]. \tag{7}$$

Define $\gamma_{\min} \triangleq \min_{t\in[0,T]} \| \mathbf{q}^H \mathbf{H}(t)\mathbf{w} \|^2$. (7) can be further equivalently expressed by $\min_{\{\mathbf{w},\mathbf{q}\}} \ \| \mathbf{w} \|^2$ s.t. $\| \mathbf{q} \| = 1$, $\gamma_{\min} \geq \gamma_{\text{th}}$. Since it is very hard to determine $\gamma_{\min}$ over an infinite set $\Upsilon = \{\gamma(t) = \| \mathbf{q}^H \mathbf{H}(t)\mathbf{w} \|^2, \forall \in [0, T]\}$, we adopt the following method to deal with it. Let $\mathrm{r}_{h_{i,j}} \triangleq \frac{1}{2}|h_{i,k}^{(\text{LoS})}(T) - h_{i,k}^{(\text{LoS})}(0)|$. As $h_{i,k}^{(\text{LoS})}(t)$ is a complex number, $\mathrm{r}_{h_{i,j}}$ denotes the half distance between $h_{i,k}^{(\text{LoS})}(T)$ and $h_{i,k}^{(\text{LoS})}(0)$ on the complex plane. Define $\hat{h}_{i,j}^{(\text{LoS})} \triangleq \frac{1}{2}(h_{i,k}^{(\text{LoS})}(T) + h_{i,k}^{(\text{LoS})}(0))$. $\hat{h}_{i,j}^{(\text{LoS})}$ denotes the midpoint on the line between $h_{i,k}^{(\text{LoS})}(T)$ and $h_{i,k}^{(\text{LoS})}(0)$. If we adopt $\hat{h}_{i,j}^{(\text{LoS})}$ as the central point and $\mathrm{r}_{h_{i,j}}$ as the radius and then generate a circle over the complex plane, for a given T, $h_{i,k}^{(\text{LoS})}(t)$ is bounded within the circle, i.e.,

$$|h_{i,k}^{(\text{LoS})}(t) - \hat{h}_{i,j}^{(\text{LoS})}| \leq \mathrm{r}_{h_{i,j}} \triangleq \varepsilon_{i,j}, \forall t \in [0, T], \tag{8}$$

Moreover, let $\triangle\mathbf{H}^{(\text{LoS})}(t) = \mathbf{H}^{(\text{LoS})}(t) - \hat{\mathbf{H}}^{(\text{LoS})}$, where $\hat{\mathbf{H}}^{(\text{LoS})} = \{\hat{h}_{i,j}^{(\text{LoS})}\}_{N_{\text{V}}\times N_{\text{B}}}$. $\forall t \in [0, T]$, we have $\| \mathbf{H}^{(\text{LoS})}(t) - \hat{\mathbf{H}}^{(\text{LoS})} \| = \| \triangle\mathbf{H}^{(\text{LoS})}(t) \| \leq \sqrt{\sum_{i=1}^{N_{\text{B}}} \sum_{j=1}^{N_{\text{V}}} \varepsilon_{i,j}^2}$. Let $\mathbf{H}_\Delta = \mathbf{H}(t) - \hat{\mathbf{H}}^{(\text{LoS})}$. Combining it with (4), we obtain (9).

$$\begin{aligned} \| \mathbf{H}(t) - \hat{\mathbf{H}}^{(\text{LoS})} \| &= \| \mathbf{H}_\Delta \| = \tfrac{1}{K+1}(\| K\mathbf{H}^{(\text{LoS})}(t) \\ &\quad + \mathbf{H}^{(\text{multi-path})} - K\hat{\mathbf{H}}^{(\text{LoS})} \|) \\ &\leq \tfrac{K}{K+1} \| \triangle\mathbf{H}^{(\text{LoS})}(t) \| + \tfrac{1}{K+1} \| \mathbf{H}^{(\text{multi-path})} \| \\ &= \tfrac{K}{K+1}\sqrt{\sum_{i=1}^{N_{\text{B}}} \sum_{j=1}^{N_{\text{V}}} \varepsilon_{i,j}^2} + \tfrac{1}{K+1}\varepsilon_{\text{multi-path}}{}^2 \triangleq \mathcal{E}^2 \end{aligned} \tag{9}$$

Since $\| \mathbf{H}_\Delta \| \leq \mathcal{E}^2$ also can be equivalently expressed as $\mathrm{Tr}(\mathbf{H}_\Delta \mathbf{I}_{N_{\text{B}}} \mathbf{H}_\Delta^H) \leq \mathcal{E}^2$[11], (9) indicates that $\mathbf{H}(t)$ is bounded by a hypersphere with central point $\hat{\mathbf{H}}^{(\text{LoS})}$. As $\hat{\mathbf{H}}^{(\text{LoS})}$ and $\mathcal{E}^2$ can be predicted at time 0 in terms of $\hat{h}_{i,j}^{(\text{LoS})}$ and (9), we transform problem (7) into

$$\begin{aligned} &\min_{\{\mathbf{w},\mathbf{q}\}} \ \| \mathbf{w} \|^2 \\ &\text{s.t. } \gamma = \| \mathbf{q}^H (\hat{\mathbf{H}}^{(\text{LoS})} + \mathbf{H}_\Delta)\mathbf{w} \|^2 \geq \gamma_{\text{th}}, \ \mathrm{Tr}\big(\mathbf{H}_\Delta \mathbf{I}_{N_{\text{B}}} \mathbf{H}_\Delta^H\big) \leq \mathcal{E}^2, \end{aligned} \tag{10}$$

where $\| \mathbf{q} \| = 1$. So, it is with finite constraints and possible to solve.

B. Iterative Optimization

Nevertheless, (10) is still not convex w.r.t both $\mathbf{w}$ and $\mathbf{q}$. For better understanding, we solve it as follows. *Firstly*, with a given $\mathbf{q}$, we find the optimal $\mathbf{w}$. *Then*, with the obtained $\mathbf{w}$, we find the optimal $\mathbf{q}$. By updating the optimal $\mathbf{w}$ and $\mathbf{q}$ alteratively, we then get the jointly global optimal $\{\mathbf{w}^*, \mathbf{q}^*\}$, when it converges. The detail process is described as follows.

1) The Optimization of $\mathbf{w}$ for a Given $\mathbf{q}$: Defining $\mathbf{Q} \triangleq \mathbf{q}\mathbf{q}^H$ and $\mathbf{W} \triangleq \mathbf{w}\mathbf{w}^H$, we get the following Proposition 1.

Proposition 1: It satisfies that

$$\begin{aligned} &\mathrm{vec}(\mathbf{H}_\Delta^H)^H(\mathbf{Q} \otimes \mathbf{W})\mathrm{vec}(\mathbf{H}_\Delta^H) + \mathrm{vec}(\mathbf{W}\hat{\mathbf{H}}^{(\text{LoS})}\mathbf{Q})^H \mathrm{vec}(\mathbf{H}_\Delta^H) \\ &\quad + \mathrm{vec}(\mathbf{H}_\Delta^H)^H \mathrm{vec}(\mathbf{W}\hat{\mathbf{H}}^{(\text{LoS})}\mathbf{Q}) + \mathrm{Tr}(\hat{\mathbf{H}}^{(\text{LoS})}\mathbf{W}(\hat{\mathbf{H}}^{(\text{LoS})})^H\mathbf{Q}) \\ &\quad - \gamma_{\text{th}} \geq 0. \end{aligned} \tag{11}$$

Proof: The proof of can be seen in Appendix A. ■

Lemma 1: (s-lemma [12]) Let $\mathbf{A}_j \in \mathbb{C}^{N\times N}$, $\mathbf{B}_j \in \mathbb{C}^{N\times 1}$, $c_j \in \mathbb{R}^{N\times N}$, where $j \in \{0, 1\}$. If there exists an $\hat{\mathbf{x}} \in \mathbb{C}^{N\times 1}$ such that $\mathbf{x}\mathbf{A}_0\mathbf{x}^H + 2\mathrm{Re}\{\mathbf{x}^H\mathbf{B}_0\} + c_0 < 0$. The following two statements are equivalent to each other: 1) $\mathbf{x}\mathbf{A}_1\mathbf{x}^H + 2\mathrm{Re}\{\mathbf{x}^H\mathbf{B}_1\} + c_1 \geq 0$ for all $\mathbf{x} \in \mathbb{C}^{N\times 1}$ while $\mathbf{x}\mathbf{A}_0\mathbf{x}^H + 2\mathrm{Re}\{\mathbf{x}^H\mathbf{B}_0\} + c_0 \leq 0$; 2) There exists a μ such that

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{B}_1 \\ \mathbf{B}_1^H & c_1 \end{bmatrix} + \mu \begin{bmatrix} \mathbf{A}_0 & \mathbf{B}_0 \\ \mathbf{B}_0^H & c_0 \end{bmatrix} \succeq \mathbf{0}. \tag{12}$$

According to the matrix theory, we have that $\mathrm{Tr}(\mathbf{H}_\Delta \mathbf{I}_{N_{\text{B}}} \mathbf{H}_\Delta^H) = \mathrm{vec}(\mathbf{H}_\Delta^H)^H \mathrm{vec}(\mathbf{I}_{N_{\text{B}}} \mathbf{H}_\Delta^H) = \mathrm{vec}(\mathbf{H}_\Delta^H)^H (\mathbf{I}_{N_{\text{B}}} \otimes \mathbf{I}_{N_{\text{B}}})\mathrm{vec}(\mathbf{H}_\Delta^H) \leq \mathcal{E}^2$. That is,

$$\mathrm{vec}\ (\mathbf{H}_\Delta^H)^H \mathbf{I}_{2N_{\text{B}}} (\mathbf{H}_\Delta^H) - \mathcal{E}^2 \leq 0. \tag{13}$$

If we regard $\mathrm{vec}(\mathbf{H}_\Delta^H)^H$, $\mathbf{I}_{2N_{\text{B}}}$, 0 and 0 as $\mathbf{x}$, $\mathbf{A}_0$, $\mathbf{B}_0$ and c_0 respectively, and $\mathbf{Q} \otimes \mathbf{W}$, $\mathrm{vec}(\mathbf{W}(\hat{\mathbf{H}}^{(\text{LoS})})^H\mathbf{Q})$ and $\mathrm{Tr}(\hat{\mathbf{H}}^{(\text{LoS})} \mathbf{W}(\hat{\mathbf{H}}^{(\text{LoS})})^H\mathbf{Q}) - \gamma_{\text{th}}$ as $\mathbf{A}_1$, $\mathbf{B}_1$ and c_1, respectively, according to (13), (11) and Lemma 2,

$$\begin{bmatrix} \mathbf{Q} \otimes \mathbf{W} & \mathrm{vec}(\mathbf{W}(\hat{\mathbf{H}}^{(\text{LoS})})^H\mathbf{Q}) \\ \mathrm{vec}(\mathbf{W}(\hat{\mathbf{H}}^{(\text{LoS})})^H\mathbf{Q})^H & \mathrm{Tr}(\hat{\mathbf{H}}^{(\text{LoS})}\mathbf{W}(\hat{\mathbf{H}}^{(\text{LoS})})^H\mathbf{Q}) - \gamma_{\text{th}} \end{bmatrix} + \mu \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathcal{E}^2 \end{bmatrix} \succeq \mathbf{0}. \tag{14}$$

Therefore, for a given $\mathbf{q}$, by using SDR [12] and dropping the rank-1 constraint $\mathrm{Rank}(\mathbf{W}) = 1$, (10) can be relaxed to be

$$\begin{aligned} &\min_{\{\mathbf{W},\mu\}} \ \mathrm{Tr}(\mathbf{W}) \\ &\text{s.t.} \begin{bmatrix} \mathbf{Q} \otimes \mathbf{W} + \mu\mathbf{I} & \mathrm{vec}(\mathbf{W}(\hat{\mathbf{H}}^{(\text{LoS})})^H\mathbf{Q}) \\ \mathrm{vec}(\mathbf{W}(\hat{\mathbf{H}}^{(\text{LoS})})^H\mathbf{Q})^H & F \end{bmatrix} \succeq \mathbf{0}, \\ &\mu \geq 0, \end{aligned} \tag{15}$$

where $F \triangleq \mathrm{Tr}(\hat{\mathbf{H}}^{(\text{LoS})}\mathbf{W}(\hat{\mathbf{H}}^{(\text{LoS})})^H\mathbf{Q}) - \gamma_{\text{th}} - \mu\mathcal{E}^2$. (15) is a convex problem, so the optimal $\mathbf{W}^*$ can be derived by using standard CVX tools [12]. Note that, the goal is to find the optimal $\mathbf{w}^*$ rather than $\mathbf{W}^*$. Thus, $\mathbf{w}^*$ should be derived with the obtained $\mathbf{W}^*$. When $\mathrm{Rank}(\mathbf{W}^*) = 1$, the unique $\mathbf{w}^*$ can be easily obtained by solving the equation $\mathbf{W}^* = (\mathbf{w}^*)(\mathbf{w}^*)^H$. When $\mathrm{Rank}(\mathbf{W}^*) > 1$, the principle eigenvector of $\mathbf{W}$ is adopted to approximate the optimal $\mathbf{w}^*$.

2) The Optimization of $\mathbf{q}$ for the Obtained $\mathbf{w}^*$: To achieve the efficient utilization of transmit power, for a given $\mathbf{w}^*$, $\mathbf{q}$ should be designed to maximize the $\gamma_{\min}$, i.e.,

$$\max_{\|\mathbf{q}\|=1} \ \gamma_{\min} \ \text{ s.t. } \| \mathbf{H}_\Delta \| \leq \mathcal{E}^2, \tag{16}$$

By introducing a new variable $\delta \geq 0$ and by using s-lemma, (16) can be relaxed to be

$$\begin{aligned} &\max_{\|\mathbf{Q}\|=1,\delta\geq 0,\mu\geq 0} \ \delta \\ &\text{s.t.} \begin{bmatrix} \mathbf{Q} \otimes \mathbf{W} + \mu\mathbf{I} & \mathrm{vec}(\mathbf{W}(\hat{\mathbf{H}}^{(\text{LoS})})^H\mathbf{Q}) \\ \mathrm{vec}(\mathbf{W}(\hat{\mathbf{H}}^{(\text{LoS})})^H\mathbf{Q})^H & F - \delta \end{bmatrix} \succeq \mathbf{0}, \\ &\mu \geq 0, \end{aligned} \tag{17}$$

Algorithm 1: Finding the joint global optimal $\{\mathbf{w}^*, \mathbf{q}^*\}$.

Initialize $P_{\text{pre}} = +\inf$ and $P_{\text{cur}} = P_{\text{pre}} - \triangle$, where $\triangle > 0$;
Initialize $\mathbf{q}$ with the left principle singular vector of $\hat{\mathbf{H}}^{(\text{LoS})}$;
while $P_{\text{cur}} < P_{\text{pre}}$ **do**
  $P_{\text{pre}} = P_{\text{cur}}$;
  Solve problem (15) to obtain $\mathbf{W}^*$ and $\mathbf{w}^*$;
  $P_{\text{cur}} = \text{tr}(\mathbf{W}^*)$;
  Solve problem (17) to obtain $\mathbf{Q}^*$ and $\mathbf{q}^*$;
end while
Return $\{\mathbf{w}^*, \mathbf{q}^*\}$.

which is convex and the optimal $\mathbf{Q}^*$ also can be solved by using CVX tools. As the goal is to find the optimal $\mathbf{q}^*$ rather than $\mathbf{Q}^*$, $\mathbf{q}^*$ should be derived with the obtained $\mathbf{Q}^*$. Similar to the case of $\mathbf{w}^*$, when $\text{Rank}(\mathbf{Q}^*) = 1$, the unique $\mathbf{q}^*$ can be easily obtained by solving the equation $\mathbf{Q}^* = (\mathbf{q}^*)(\mathbf{q}^*)^H$, and when $\text{Rank}(\mathbf{Q}^*) > 1$, the principle eigenvector of $\mathbf{Q}^*$ is selected to approximate the optimal $\mathbf{q}^*$.

3) The Joint Optimization of $\{\mathbf{w}^*, \mathbf{q}^*\}$*:* The above two steps can be iteratively executed to arrive at the joint global optimal[2] $\{\mathbf{w}^*, \mathbf{q}^*\}$ as shown in Algorithm 1.

Convergence discussion: In the n-th round of iteration, $\mathbf{w}^{*[n]}$ is firstly obtained based on $\mathbf{q}^{*[n]}$. Then, $P_{\text{cur}}^{[n]}$ is calculated with $\mathbf{w}^{*[n]}$. After this, $\mathbf{q}^{*[n]}$ is updated to be $\mathbf{q}^{*[n+1]}$ by maximizing $\gamma_{\min}$, which means that $\gamma_{\min}(\mathbf{w}^{*[n]}, \mathbf{q}^{*[n]}) \geq \gamma_{\min}(\mathbf{w}^{*[n]}, \mathbf{q}^{*[n+1]})$. In the $(n+1)$-th round of iteration, $\mathbf{w}^{*[n+1]}$ is firstly obtained based on $\mathbf{q}^{*[n+1]}$. Then, $P_{\text{cur}}^{[n+1]}$ is calculated with $\mathbf{w}^{*[n+1]}$. As $\mathbf{w}^{*[n+1]}$ is obtained by minimizing the total transmit power, it holds that $P_{\text{cur}}^{[n+1]} \leq P_{\text{cur}}^{[n]}$. That is to say, the total transmit power is decreased with the increment of the number of iteration. Moreover, as $\gamma_{\min}$ is bounded by γ_{th}, P_{cur} can not be decreased infinitely, which guarantees the convergence of Algorithm 1.

V. Simulation Results and Discussion

This section provides some simulation results to discuss the performance of our proposed BBA. In the simulations, the frequency was set to 2.4 GHZ. $L_{\text{B}} = L_{\text{V}} = 1$. $T = 1$s, $v = 108$ km/h and $N_{\text{V}} = 2$. The location of the moving vehicle is denoted by θ, where $\theta = \arg\cot \frac{-x(t)}{D_{\min}}$. Therefore, $\theta \in (0, \pi)$, where when $\theta \to 0$, it means that the vehicle is at $-\infty$ and when $\theta = \pi$, it means that the vehicle is at $+\infty$ on the x-axis. $R_{\text{th}} = 1$ bit/s. Note that, these parameter settings will not change, unless otherwise specified.

Figs. 2 and 3 compare the received SNR of our proposed BBA with traditional method and the narrow beamforming method presented in [6] v.s vehicle location for $N_{\text{B}} = 4$ and $N_{\text{B}} = 32$, respectively. $\gamma_{\min} = 0$ dB. $x(0) = -600$ m, i.e., $\theta(0) = 0.6947$. In the traditional method, equal weights are allocated over both the transmit and the receiving beams. In the narrow beamforming method, the beam is generated based on $x(0)$, where $x(T)$ is not predicted. In BBA, it is calculated that $x(T) = -570$ m, i.e., $\theta(T) = 0.7201$. It shows that by using our BBA, the SNR is greatly enhanced to be higher than $\gamma_{\min}$ within the interval of $\theta \in [0.6947, 0.7201]$, which is termed as the moving region of interest (MROI) in the sequel. By contrast, with traditional method, the SNR within the MROI cannot be increased and with the narrow beamforming method, the SNR is only enhanced at $x(0)$.

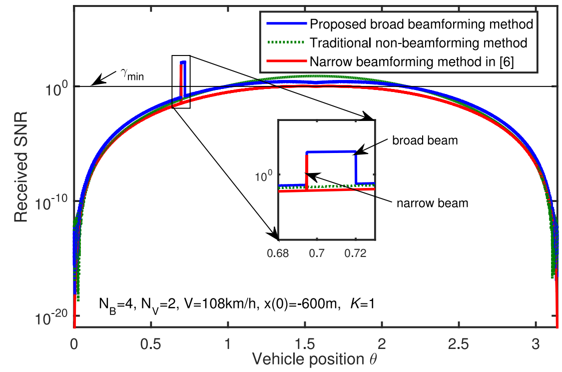

Fig. 2. Comparison of the received SNR versus vehicle position (i.e., θ) with $N_{\text{B}} = 4$.

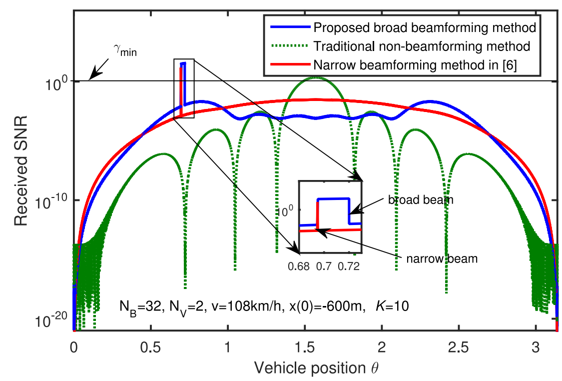

Fig. 3. Comparison of the received SNR versus vehicle position (i.e., θ) with $N_{\text{B}} = 32$.

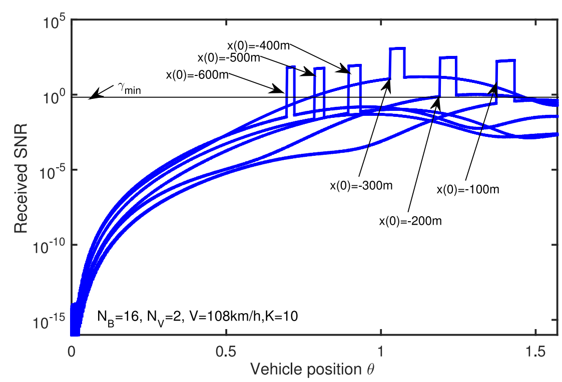

Fig. 4. The received SNR for different moving region of interests.

Fig. 4 shows the received SNR verus vehicle locations. It shows that within each MROI, the SNR is enhanced above the threshold $\gamma_{\min}$, which indicates that if the BBA is executed periodically for successive adjacent MROIs, the QoS requirement of the moving vehicle can be guaranteed when it goes through the whole coverage region of the BS. Fig. 5 plots the results of the required power versus the number of transmit antennas, where with growth of the number of transmit antennas, the required power is decreased, but it decreases more slowly with the growth of N_{B}. Fig. 6 shows that our proposed method is able to broaden the beam width.

[2]By using the SDR, for a given $\mathbf{q}$, the relaxed optimization problem is convex w.r.t $\mathbf{W} = \mathbf{w}\mathbf{w}^H$ and for a given $\mathbf{w}$, the relaxed optimization problem is also convex w.r.t $\mathbf{Q} = \mathbf{q}\mathbf{q}^H$. Therefore, the global optimum $\{\mathbf{W}^*, \mathbf{Q}^*\}$ exists and is unique, which yields the corresponding unique global optimal $\{\mathbf{w}^*$ and $\mathbf{q}^*\}$. The detailed process to find out the optimal $\mathbf{W}^*$ and $\mathbf{Q}^*$ can be found in Section III-B.

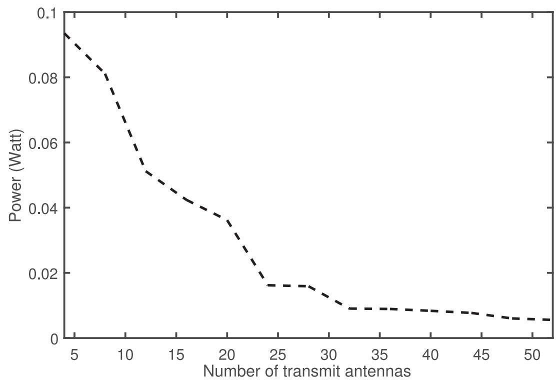

Fig. 5. The impact of the number of antennason the required power.

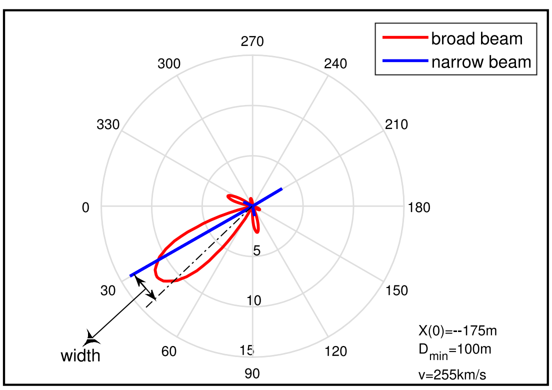

Fig. 6. Comparison of beam pattern.

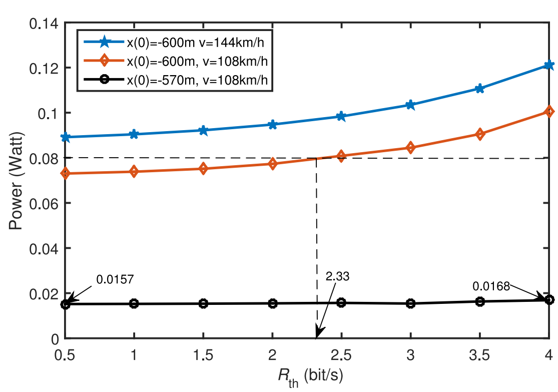

Fig. 7. The required power v.s. R_{th}.

Fig. 7 provides the minimal required power v.s. R_{th}. It is seen that with the increment of R_{th}, the required power increases. The smaller $x(0)$ is, the less power is required and the smaller of the growth rate of the required power is, because a smaller $x(0)$ indicating a shorter distance between the vehicle and the BS. Besides, Fig. 7 also shows the one-to-one mapping relationship between the minimal consumed power and the supported information rate. Thus, for a given $x(0)$ and v, by using BBA, one can generate a figure or a table on the mapping result between the minimal consumed power and the supported information rate. By doing so, one can find how much information rate is able to be supported by the system for a given power. For example, for $x(0) = -600$ m and $v = 108$ km/h, when the system power is 0.08 Watt, the maximal supported reliable information rate is about 2.33 bit/s.

VI. Conclusion

This paper proposed a broad beamforming scheme. An optimization problem was formulated and solved to minimize the consumed power by jointly optimizing the transmit and the receiving beam vectors. With our proposed BBA, the required SNR of a high-mobility user within a given time period can be guaranteed and with the same power consumption, the SNR within the interest region is greatly enhanced by our scheme compared with traditional methods.

Appendix A
The Proof of Lemma 1

Since

$$\begin{aligned}\gamma &= \| \mathbf{q}^H \mathbf{H}(t)\mathbf{w} \|^2 = \mathbf{q}^H (\hat{\mathbf{H}}^{(\text{LoS})} + \mathbf{H}_\Delta)\mathbf{W}(\hat{\mathbf{H}}^{(\text{LoS})} + \mathbf{H}_\Delta)^H \mathbf{q}\\ &= \mathbf{q}^H \hat{\mathbf{H}}^{(\text{LoS})}\mathbf{W}\mathbf{H}_\Delta^H \mathbf{q} + \mathbf{q}^H \mathbf{H}_\Delta \mathbf{W}(\hat{\mathbf{H}}^{(\text{LoS})})^H \mathbf{q}\\ &\quad + \mathbf{q}^H \hat{\mathbf{H}}^{(\text{LoS})}\mathbf{W}(\hat{\mathbf{H}}^{(\text{LoS})})^H \mathbf{q} + \mathbf{q}^H \mathbf{H}_\Delta \mathbf{W}\mathbf{H}_\Delta^H \mathbf{q}\\ &= \text{Tr}(\mathbf{H}_\Delta \mathbf{W}(\hat{\mathbf{H}}^{(\text{LoS})})^H \mathbf{Q}) + \text{Tr}(\mathbf{H}_\Delta^H \mathbf{Q}\hat{\mathbf{H}}^{(\text{LoS})}\mathbf{W})\\ &\quad + \text{Tr}(\hat{\mathbf{H}}^{(\text{LoS})}\mathbf{W}(\hat{\mathbf{H}}^{(\text{LoS})})^H \mathbf{Q}) + \text{Tr}(\mathbf{H}_\Delta^H \mathbf{Q}\mathbf{H}_\Delta \mathbf{W}) \qquad (18)\end{aligned}$$

according to some disciplinary results of matrix theory, e.g., $\text{Tr}(\mathbf{A}^H\mathbf{B}) = \text{vec}(\mathbf{X})^H\text{vec}(\mathbf{B}) = \text{vec}(\mathbf{B})^H\text{vec}(\mathbf{A})$, $\text{Tr}(\mathbf{ABCD}) = \text{Tr}(\mathbf{BCDA}) = \text{Tr}(\mathbf{CDAB})$, $\text{Tr}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})\text{vec}(\mathbf{B})$ and $\text{vec}(\mathbf{AB}) = (\mathbf{I} \otimes \mathbf{A})\text{vec}(\mathbf{B}) = (\mathbf{B}^T \otimes \mathbf{I})\text{vec}(\mathbf{A})$, (18) can be further expressed by $\gamma = \text{vec}(\mathbf{H}_\Delta^H)^H \text{vec}(\mathbf{W}\hat{\mathbf{H}}^{(\text{LoS})}\mathbf{Q}) + \text{vec}(\mathbf{W}\hat{\mathbf{H}}^{(\text{LoS})}\mathbf{Q})^H \text{vec}(\mathbf{H}_\Delta^H) + \text{vec}(\mathbf{H}_\Delta^H)^H(\mathbf{Q} \otimes \mathbf{W})\text{vec}(\mathbf{H}_\Delta^H) + \text{Tr}(\hat{\mathbf{H}}^{(\text{LoS})}\mathbf{W}(\hat{\mathbf{H}}^{(\text{LoS})})^H\mathbf{Q})$. From (10), $\gamma \leq \gamma_{\text{th}}$. Therefore, Lemma 1 is proved.

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