Differential Modulation for Multi-Node Amplify-and-Forward Wireless Relay Networks†

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***Abstract*— In this paper, we propose a multi-node differential amplify-and-forward scheme for cooperative communications. The proposed scheme efficiently combines signals from the direct and multiple relay links to improve communication reliability. Bit-error-rate (BER) analysis for M-ary differential phase shift keying is provided as performance measure of the proposed scheme, and optimum power allocation is investigated. While the exact BER formulation of the proposed scheme is not available currently, we provide as a performance benchmark a tight BER formulation based on optimum combining weights. A simple BER upper bound and a tight BER approximation show that the proposed scheme can achieve the full diversity which equals to the number of cooperating nodes. We further provide simple BER approximation in order to provide analytical result on power allocation scheme. A closed-form optimum power allocation based on the tight simple BER approximation is obtained for single-relay scenario. An approximate optimum power allocation scheme is provided for multi-relay systems. The provided BER formulations are shown to closely match to the simulation results. Moreover, simulation results show that the optimum power allocation scheme achieves up to 2 dB performance gain over the equal power allocation scheme.**

I. Introduction

Recently, cooperative communications have gained much attention due to the ability to explore the inherent spatial diversity in relay channels. Various cooperation protocols, e.g., amplify-and-forward (AF) and decode-and-forward (DF) [1]-[5] and reference therein have been proposed for wireless networks. Most of the works in [1]-[5] assume that the destination has perfect knowledge of channel state information (CSI) of all transmission links. While in some scenarios, e.g. slow fading environment, the CSI is likely be acquired by the use of pilot symbols, it may not be possible in fast fading environment. In addition, it is questionable on how the destination can obtain source-relay channel perfectly through pilot signal forwarding without noise amplification. Moreover, the computational overhead for channel estimation increases in proportional to the product of number of transmit antennas and number of relaying nodes.

Differential modulation has been well accepted as a modulation technique that provides a good tradeoff between receiver complexity and performance. In differential phase-shift keying (DPSK) [6], efficient decoding relies on constant phase responses of the channel from one time sample to the next. The differential modulation has been investigated in [7] for a specific two-hop relay system. Recently, a framework of noncoherent cooperative communications has been proposed [8] for the DF protocol employing frequency shift keying modulation. However, the framework does not fit to the general M-ary differential phase shift keying (MDPSK) and the AF cooperation protocol. In

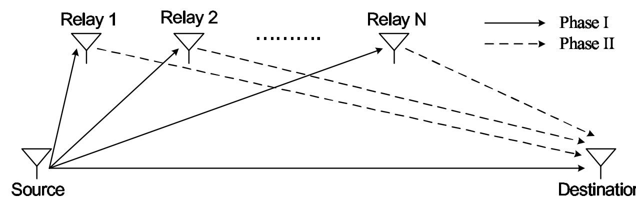

Fig. 1: Multi-node differential AF scheme.

[10]-[11], a differential scheme has been proposed for two-user cooperating nodes that employs the AF protocol. A simple bit-error-rate (BER) performance is provided in [11]. However, the BER formulation is complicated and optimum power allocation scheme is obtained through exhaustive numerical search.

In this paper, we propose a multi-node differential modulation for amplify-and-forward cooperative communications. In the proposed scheme, the destination requires only long-term average of the received signals to efficiently combines signals from all communications links. As a performance benchmark, we provide an exact BER formulation of the optimum-combining cooperation system using MDPSK signals. In order to obtain analytical result for optimum power allocation scheme which is not available in [11] even for a two-user scenario, we provide BER upper bounds and simple BER approximations. Based on the tight BER approximation, closed-form optimum power allocation is evaluated, and then used to further improve the performance of the proposed scheme. Simulation results are shown to validate our proposed schemes and support our theoretical analysis.

II. Multi-Node Differential AF Scheme

We consider a multi-node cooperative wireless network with a source and N relays, as shown in Figure 1. The cooperation strategy is based on amplify-and-forward protocol [1] in which each relay amplifies the received signal from the source and then forwards it to the destination. Specifically, signal transmissions of the considered cooperation system comprises two phases. We assume, in both phases, that all signals are transmitted through orthogonal channels by the use of existing schemes such as TDMA, FDMA [1]-[2], or CDMA [3]-[4].

Suppose the DMPSK modulation is used, i.e., the information is conveyed in the phase difference between two consecutive symbols. The modulated information at the source in Phase I can be described as $v_m = e^{j\phi_m}$ where $\phi_m = 2\pi m/M$ for $m = 0, 1, \ldots, M-1$, and M is the constellation size. The source differentially encodes the information symbol v_m as

$$x^\tau = v_m x^{\tau-1}, \tag{1}$$

where τ is the time index, and x^τ is the differentially encoded symbol to be transmitted at time τ. Then the source transmits x^τ with transmitted power P_s to the destination and the relays.

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The corresponding received signals at the destination and the i^{th} relay, $i = 1, 2, \cdots, N$, can be expressed as

$$y^{\tau}_{s,d} = \sqrt{P_s} h^{\tau}_{s,d} x^{\tau} + w^{\tau}_{s,d}, \quad (2)$$
$$y^{\tau}_{s,r_i} = \sqrt{P_s} h^{\tau}_{s,r_i} x^{\tau} + w^{\tau}_{s,r_i}. \quad (3)$$

where $h^{\tau}_{s,d}$ and h^{τ}_{s,r_i} represent channel coefficients from the source to the destination and from the source to the i^{th} relay, respectively. The terms $w^{\tau}_{s,d}$ and w^{τ}_{s,r_i} are additive white Gaussian noise at the destination and the i^{th} relay, respectively.

In Phase II, each relay amplifies the received signal in (3) and forwards it to the destination with transmit power P_i. Accordingly, the received signal at the destination from the i^{th} relay is

$$y^{\tau}_{r_i,d} = \frac{\sqrt{P_i}}{\sqrt{P_s \sigma^2_{s,r_i} + \mathcal{N}_0}} h^{\tau}_{r_i,d} y^{\tau}_{s,r_i} + w^{\tau}_{r_i,d}, \quad (4)$$

where $h^{\tau}_{r_i,d}$ is the channel coefficient from the i^{th} relay to the destination, and $w^{\tau}_{r_i,d}$ is additive noise at the destination. Consider the case of independent Rayleigh fading channels, then the channel coefficients $h^{\tau}_{s,d}$, h^{τ}_{s,r_i}, and $h^{\tau}_{r_i,d}$ are modeled as independent zero-mean complex Gaussian random variables with variances $\sigma^2_{s,d}$, σ^2_{s,r_i}, and $\sigma^2_{r_i,d}$, respectively. All the noise terms $w^{\tau}_{s,d}$, w^{τ}_{s,r_i}, and $w^{\tau}_{r_i,d}$ are modeled as independent complex Gaussian random variables, each with zero mean and variance $\mathcal{N}_0$. The scheme does not require the instantaneous channel state information at either the relays or the destination. Observe from (4) that the transmitted power at the relay is normalized by $P_s \sigma^2_{s,r_i} + \mathcal{N}_0$, which implies that only the channel variance between the source and relay i, σ^2_{s,r_i}, is required at relay i. In practice, such information can be obtained through long term averaging of the received signal from the source to the i^{th} relay.

At the destination, the received signal from the source and the relays are combined and then used to estimate the transmitted information. All channel coefficients $h^{\tau}_{s,d}$, h^{τ}_{s,r_i}, and $h^{\tau}_{r_i,d}$ are unknown to either the relays or the destination, but they are assumed almost constant over two symbol periods. Based on the received signals from the the two phases, the combined signal at the destination is given by

$$y = a_s \big(y^{\tau-1}_{s,d}\big)^* y^{\tau}_{s,d} + \sum_{i=1}^{N} a_i \big(y^{\tau-1}_{r_i,d}\big)^* y^{\tau}_{r_i,d}, \quad (5)$$

where a_s and a_i are combining weights. To maximize the SNR of the combiner output, the combining weights can be determined as $a_s = \frac{1}{\mathcal{N}_0}$ and $a_i = \frac{P_s \sigma^2_{s,r_i} + \mathcal{N}_0}{\mathcal{N}_0 (P_s \sigma^2_{s,r_i} + P_i \sigma^2_{r_i,d} + \mathcal{N}_0)}$. Here, the channel variances between the relays and the destination, $\sigma^2_{r_i,d}$, and channel variances between the source and the relays, σ^2_{s,r_i}, are assumed available at the destination. Without acquiring perfect channel state information, the combined signal (5) is differentially decoded by using the detection rule

$$\hat{m} = \arg \max_{m\,=\,0,1,\ldots,M-1} \text{Re}\left\{v^*_m y\right\}. \quad (6)$$

III. BER Performance Analysis

We provide in this section BER performance analysis based on optimum combining weights in [13], and it will be considered as BER performance benchmark for the proposed scheme.

From (5), the optimum combining weights are $\hat{a}_s = \frac{1}{\mathcal{N}_0}$, and $\hat{a}_i = \frac{P_s \sigma^2_{s,r_i} + \mathcal{N}_0}{\mathcal{N}_0 (P_s \sigma^2_{s,r_i} + P_i |h^{\tau}_{r_i,d}|^2 + \mathcal{N}_0)}$. Note that the optimum combining weights $\hat{a}_i$ requires the knowledge of instantaneous channel information which is not available in the proposed scheme. However, the BER analysis based on these combining weights is used as BER performance benchmark of our proposed scheme. We will show an interesting observation in Section V that the BER performance of our proposed scheme yields very close to the BER performance benchmark when optimum power allocation is applied.

Using the optimum combining weights $\hat{a}_s$ and $\hat{a}_i$, an instantaneous SNR at the combiner output is given by

$$\gamma = \gamma_s + \sum_{i=1}^{N} \gamma_i, \quad (7)$$

$\gamma_s = \frac{P_s |h^{\tau}_{s,d}|^2}{\mathcal{N}_0}$ and $\gamma_i = \frac{P_s P_i |h^{\tau}_{s,r_i}|^2 |h^{\tau}_{r_i,d}|^2}{\mathcal{N}_0 (P_s \sigma^2_{s,r_i} + P_i |h^{\tau}_{r_i,d}|^2 + \mathcal{N}_0)}$. For a given SNR γ in (7), the conditional BER expression for L-channel diversity receptions can be expressed as [6]

$$P_{b|\gamma} = \frac{1}{2^{2L}\pi} \int_{-\pi}^{\pi} f(\theta) \exp\left[-\alpha(\theta)\gamma\right] d\theta, \quad (8)$$

where

$$f(\theta) = \frac{b^2}{2\alpha(\theta)} \sum_{l=1}^{L} \begin{pmatrix} 2L-1 \\ L-1 \end{pmatrix} \big[(\beta^{-l+1} - \beta^{l+1}) \cos((l-1)(\theta + \frac{\pi}{2})) - (\beta^{-l+2} - \beta^{l}) \cos(l(\theta + \frac{\pi}{2}))\big], \quad (9)$$

$$\alpha(\theta) = \frac{b^2(1 + 2\beta \sin\theta + \beta^2)}{2}. \quad (10)$$

Here, $L = N + 1$, and $\beta = a/b$ in which $a = 10^{-3}$ and $b = \sqrt{2}$ for DBPSK modulation, and $a = \sqrt{2 - \sqrt{2}}$ and $b = \sqrt{2 - \sqrt{2}}$ for DQPSK modulation [6]. For higher constellation sizes, β can be obtained from the results in [12]. Averaging the conditional BER (8) over the Rayleigh fading channels by using the moment generating function (MGF) method. The exponential function of the summation of the instantaneous SNR in (8) can be written in product form of the MGF of each instantaneous SNR. Specifically, the average BER can expressed as

$$P_b = \frac{1}{2^{2(N+1)}\pi} \int_{-\pi}^{\pi} f(\theta) \mathcal{M}_{\gamma_s}(\theta) \prod_{i=1}^{N} \mathcal{M}_{\gamma_i}(\theta)\, d\theta, \quad (11)$$

where $\mathcal{M}_{\gamma_\mu}(\theta) = \int_{-\infty}^{+\infty} e^{-\alpha(\theta)\lambda} p_{\gamma_\mu}(\lambda) d\lambda$ represents of the MGF of the instantaneous SNR γ_μ for $\mu \in \{s, 1, ..., N\}$. In (11), $\mathcal{M}_{\gamma_s}(\theta)$ is obtained through an integration over an exponential random variable $|h_{s,d}|^2$ such that

$$\mathcal{M}_{\gamma_s}(\theta) = \frac{1}{1 + k_{s,d}(\theta)}, \quad (12)$$

in which $k_{s,d}(\theta) \triangleq \alpha(\theta) P_s \sigma^2_{s,d} / \mathcal{N}_0$.

The MGF $\mathcal{M}_{\gamma_i}(\theta)$ in (11) can be obtained through integrations over two exponential random variables $|h_{s,r_i}|^2$ and $|h_{r_i,d}|^2$. By first averaging over $|h_{s,r_i}|^2$ and then averaging over $|h_{r_i,d}|^2$, we have

$$\mathcal{M}_{\gamma_i}(\theta) = \frac{1}{\sigma^2_{r_i,d}} \int_0^{\infty} \Omega_i(\theta) \exp\big(-\frac{u}{\sigma^2_{r_i,d}}\big) du, \quad (13)$$

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where

$$\Omega_i(\theta) = \frac{1}{1+k_{s,r_i}(\theta)} + \frac{1-1/\left[1+k_{s,r_i}(\theta)\right]}{1+\hat{k}_{s,r_i}(\theta)u/\left(P_s\sigma_{s,r_i}^2+\mathcal{N}_0\right)}, \quad (14)$$

where we denote $k_{s,r_i}(\theta) \triangleq \alpha(\theta)P_s\sigma_{s,r_i}^2/\mathcal{N}_0$, and $\hat{k}_{s,r_i}(\theta) \triangleq P_i(1+k_{s,r_i}(\theta))$. After some manipulations, (13) is given by

$$\mathcal{M}_{\gamma_i}(\theta) = \frac{1}{1+k_{s,r_i}(\theta)}\left(1+\frac{k_{s,r_i}(\theta)}{1+k_{s,r_i}(\theta)}\frac{P_s\sigma_{s,r_i}^2+\mathcal{N}_0}{P_i}\frac{1}{\sigma_{r_i,d}^2}Z_i(\theta)\right), \quad (15)$$

where

$$Z_i(\theta) = \int_0^\infty \frac{\exp\left(-u/\sigma_{r_i,d}^2\right)}{u+R_i(\theta)}du, \quad (16)$$

and

$$R_i(\theta) \triangleq \frac{P_s\sigma_{s,r_i}^2+\mathcal{N}_0}{P_i\left[1+k_{s,r_i}(\theta)\right]}. \quad (17)$$

By applying the results from [14], we can express (16) in a simple formulation with finite-limit integration as

$$Z_i(\theta) = -e^{\hat{R}_i(\theta)}\left[\mathcal{E}+\ln\hat{R}_i(\theta)+\int_0^{\hat{R}_i(\theta)}\frac{\exp(-t)-1}{t}dt\right] \quad (18)$$

in which $\mathcal{E} = 0.57721566490...$ represents the Euler's constant [14], and $\hat{R}_i(\theta) = R_i(\theta)/\sigma_{r_i,d}^2$. Finally, by substituting (12) and (15) into (11), the average BER formulation of the proposed multi-node differential AF scheme can be expressed as

$$P_b = \frac{1}{2^{2(N+1)}\pi}\int_{-\pi}^{\pi}\frac{f(\theta)}{1+k_{s,d}(\theta)}\prod_{i=1}^{N}\frac{1}{1+k_{s,r_i}(\theta)} \times\left(1+\frac{k_{s,r_i}(\theta)Z_i(\theta)}{1+k_{s,r_i}(\theta)}\frac{P_s\sigma_{s,r_i}^2+\mathcal{N}_0}{P_i\sigma_{r_i,d}^2}\right)d\theta. \quad (19)$$

Observe that the BER formulation in (19) involves double integration. Although (19) can be calculated numerically, it is difficult to get insights. In the sequel, we provide a single-integral BER upper bound, a simple BER upper bound that involves no integration, and two tight BER approximations.

We first determine the BER upper bound and its simple expression as follows. From (15), we can see that the BER upper bound can be obtained by bounding $R_i(\theta)$ in the denominator of the integrand of $\mathcal{M}_{\gamma_i}(\theta)$. By substituting θ with $\pi/2$, $\alpha(\theta)$ can be upper bounded by $\alpha(\theta) \le (b^2(1+\beta)^2)/2$. Then, $R_i(\theta)$ is lower bounded by

$$R_i(\theta) \ge \frac{P_s\sigma_{s,r_i}^2+\mathcal{N}_0}{P_i}\left[1+\frac{P_s\sigma_{s,r_i}^2 b^2(1+\beta)^2}{2\mathcal{N}_0}\right]^{-1} \triangleq R_{i,min}. \quad (20)$$

Substituting $R_i(\theta) = R_{i,min}$ into (16) results in an upper bound on $Z_i(\theta)$, i.e., $Z_i(\theta) \le Z_{i,max}$ where

$$Z_{i,max} = -e^{\hat{R}_{i,min}}\left[\mathcal{E}+\ln\hat{R}_{i,min}+\int_0^{\hat{R}_{i,min}}\frac{\exp(-t)-1}{t}dt\right],$$

in which $\hat{R}_{i,min} = R_{i,min}/\sigma_{r_i,d}^2$. For a specific channel variance $\sigma_{r_i,d}^2$, the term $Z_{i,max}$ can be simply calculated. By bounding $Z_i(\theta)$ in (19) with $Z_{i,max}$, we obtain the BER upper bound

$$P_b \le \frac{1}{2^{2(N+1)}\pi}\int_{-\pi}^{\pi}\frac{f(\theta)}{1+k_{s,d}(\theta)}\prod_{i=1}^{N}\frac{1}{1+k_{s,r_i}(\theta)} \times\left(1+\frac{k_{s,r_i}(\theta)Z_{i,max}}{1+k_{s,r_i}(\theta)}\frac{P_s\sigma_{s,r_i}^2+\mathcal{N}_0}{P_i\sigma_{r_i,d}^2}\right)d\theta. \quad (21)$$

We further simplify the BER upper bound (21) to get more insights on the achievable diversity order and simpler BER evaluation. For high enough SNR, all 1's in the denominator of (21) can be discarded. After some manipulations, the simple BER upper bound can be expressed as

$$P_b \le \frac{C(\beta,N)\mathcal{N}_0^{N+1}}{P_s\sigma_{s,d}^2}\cdot\prod_{i=1}^{N}\frac{P_i\sigma_{r_i,d}^2+\left(P_s\sigma_{s,r_i}^2+\mathcal{N}_0\right)Z_{i,max}}{P_sP_i\sigma_{s,r_i}^2\sigma_{r_i,d}^2}, \quad (22)$$

where

$$C(\beta,N) = \frac{1}{2^{2(N+1)}\pi}\int_{-\pi}^{\pi}\frac{f(\theta)}{\alpha^{N+1}(\theta)}d\theta \quad (23)$$

is a constant that depends on modulation size and number of relays. The BER upper bound in (22) reveals that when N relays are available in the network, the diversity order of $N+1$ can be obtained.

In what follows, we determine two BER approximations in which one of them is an asymptotically tight simple BER approximation. We first note that $\alpha(\theta)$ can be lower bounded by $\alpha(\theta) \ge \alpha(-\pi/2) = (b^2(1-\beta)^2)/2$. Accordingly, $R_i(\theta)$ can be upper bounded by

$$R_i(\theta) \le \frac{P_s\sigma_{s,r_i}^2+\mathcal{N}_0}{P_i}\left[1+\frac{P_s\sigma_{s,r_i}^2 b^2(1-\beta)^2}{2\mathcal{N}_0}\right]^{-1} \triangleq R_{i,max}. \quad (24)$$

By substituting $R_i(\theta) = R_{i,max}$ into (16), we get a lower bound on $Z_i(\theta)$, denoted by $Z_{i,min}$:

$$Z_{i,min} = -e^{\hat{R}_{i,max}}\left[\mathcal{E}+\ln\hat{R}_{i,max}+\int_0^{\hat{R}_{i,max}}\frac{\exp(-t)-1}{t}dt\right],$$

in which $\hat{R}_{i,max} = R_{i,max}/\sigma_{r_i,d}^2$ and $\mathcal{E}$ is the Euler's constant. Then, replacing $Z_i(\theta)$ in (19) with $Z_{i,min}$, we obtain a BER approximation

$$P_b \gtrapprox \frac{1}{2^{2(N+1)}\pi}\int_{-\pi}^{\pi}\frac{f(\theta)}{1+k_{s,d}(\theta)}\prod_{i=1}^{N}\frac{1}{1+k_{s,r_i}(\theta)} \times\left(1+\frac{k_{s,r_i}(\theta)Z_{i,min}}{1+k_{s,r_i}(\theta)}\frac{P_s\sigma_{s,r_i}^2+\mathcal{N}_0}{P_i\sigma_{r_i,d}^2}\right)d\theta. \quad (25)$$

Furthermore, by ignoring all 1's in the denominator of (25), we get a simpler BER approximation

$$P_b \approx \frac{C(\beta,N)\mathcal{N}_0^{N+1}}{P_s\sigma_{s,d}^2}\prod_{i=1}^{N}\frac{P_i\sigma_{r_i,d}^2+\left(P_s\sigma_{s,r_i}^2+\mathcal{N}_0\right)Z_{i,min}}{P_sP_i\sigma_{s,r_i}^2\sigma_{r_i,d}^2}. \quad (26)$$

We can see from the exponent of the noise term in (26) that the obtained diversity order is $N+1$. We will show in the simulation results that these two BER approximations are tight at high SNR.

IV. Optimum Power Allocation

We determine in this section the optimum power allocation of the proposed scheme based on the tight simple BER approximation (26). Moreover, we further simplify the BER approximation to obtain analytical solution of power allocation among users.

Our primary goal is to minimize the BER in (26) under a constraint of a fixed total transmitted power, $P = P_s + \sum_{i=1}^{N} P_i$. The optimization problem can be formulated as

$$\arg\min_{P_s,\{P_i\}_{i=1}^N}\left\{\frac{C(\beta,N)\mathcal{N}_0^{N+1}}{P_s\sigma_{s,d}^2}\prod_{i=1}^{N}\Gamma_i\right\},$$
$$\text{subject to } P_s+\sum_{i=1}^{N}P_i \le P, P_i \ge 0, \forall i, \quad (27)$$

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where $\Gamma_i \triangleq \frac{P_i\sigma^2_{r_i,d}+(P_s\sigma^2_{s,r_i}+\mathcal{N}_0)Z_{i,min}}{P_sP_i\sigma^2_{s,r_i}\sigma^2_{r_i,d}}$. In order to obtain a simple optimum power allocation, we further approximate $Z_{i,min}$ as follows. At high enough SNR, $R_{i,max}$ in (24) is simplified to $R_{i,max} \approx 2\mathcal{N}_0/(b^2(1-\beta)^2P_i)$. Therefore $Z_{i,min}$ can be approximated as

$$Z_{i,min} \approx -e^{B_{c_i}}\left(\mathcal{E}+\ln B_{c_i}\right) \triangleq \hat{Z}_{i,min}. \quad (28)$$

where $B_{c_i} = \frac{2\mathcal{N}_0}{b^2(1-\beta)^2\sigma^2_{r_i,d}P_sc_i}$, and $c_i = P_i/P_s$ in which P_s and P_i are the transmitted powers at the source and the relay i, respectively. As will be shown through numerical evaluation, the obtained optimum power allocation based on $\hat{Z}_{i,min}$ (28) yields almost the same performance to that with exact $Z_{i,min}$.

By applying $\hat{Z}_{i,min}$ in (28) into the optimization problem (27), and removing some constant terms, the optimization problem is simplified to

$$\arg\min_{P_s,\{P_i\}_{i=1}^N}\left\{\frac{1}{P_s^{N+1}}\prod_{i=1}^{N}\frac{P_i\sigma^2_{r_i,d}-P_s\sigma^2_{s,r_i}e^{B_{c_i}}\left(\mathcal{E}+\ln B_{c_i}\right)}{P_i}\right\},$$
$$\text{subject to } P_s+\sum_{i=1}^{N}P_i \le P, \quad P_i \ge 0, \quad \forall i. \quad (29)$$

Using the Lagrangian method and after taking logarithm, we have

$$\mathcal{G} = \lambda(\mathbf{c}^T\mathbf{1}-P/P_s)-(N+1)\ln P_s + \sum_{i=1}^{N}\ln\left(c_i\sigma^2_{r_i,d}-\sigma^2_{s,r_i}e^{B_{c_i}}\left(\mathcal{E}+\ln B_{c_i}\right)\right)-\ln c_i, \quad (30)$$

in which $\mathbf{c} = [1, c_1, \ldots, c_N]^T$ is an $N\times 1$ vector, and $\mathbf{1}$ is denoted as an $N\times 1$ vector containing all ones as its elements. By independently differentiating (30) with respect to c_i and P_s, and equate the obtained results to zeros, we can find that

$$\lambda = (N+1)\frac{P_s}{P}-\frac{1}{P}\sum_{i=1}^{N}\frac{\sigma^2_{s,r_i}\Psi e^{B_{c_i}}\left(\mathcal{E}+\ln B_{c_i}+\frac{1}{B_{c_i}}\right)}{c_i\left[c_i\sigma^2_{r_i,d}-\sigma^2_{s,r_i}e^{B_{c_i}}\left(\mathcal{E}+\ln B_{c_i}\right)\right]},$$

where $\Psi = \frac{2\mathcal{N}_0}{b^2(1-\beta^2)\sigma^2_{r_i,d}}$.

In order to obtain optimum power allocation for the optimization problem (29), we denote the power allocation at the source as $q = P_s/P$ by which $q \in (0,1)$ such that B_{c_i} $B_{c_i} = \frac{2\mathcal{N}_0}{b^2(1-\beta)^2\sigma^2_{r_i,d}Pqc_i}$. By equating the differentiation of (30) with respect to c_i to that with P_s, we have

$$(N+1)q-\frac{1}{c_i}+\frac{\sigma^2_{r_i,d}+\sigma^2_{s,r_i}\left[\frac{\Psi}{Pqc_i^2}e^{B_{c_i}}\left(\mathcal{E}+\ln B_{c_i}\right)+\frac{e^{B_{c_i}}}{c_i}\right]}{c_i\sigma^2_{r_i,d}-\sigma^2_{s,r_i}e^{B_{c_i}}\left(\mathcal{E}+\ln B_{c_i}\right)} - \frac{1}{P}\sum_{i=1}^{N}\frac{\sigma^2_{s,r_i}\Psi e^{B_{c_i}}\left(\mathcal{E}+\ln B_{c_i}+\frac{1}{B_{c_i}}\right)}{c_i\left[c_i\sigma^2_{r_i,d}-\sigma^2_{s,r_i}e^{B_{c_i}}\left(\mathcal{E}+\ln B_{c_i}\right)\right]} = 0, \quad (31)$$

We can see that the left hand side of (31) is a function of c_i and q. Hence, the optimum power allocation for each relay i can be obtained by jointly optimizing (31) at $q = \hat{q}$ that satisfies the constraint:

$$1+\sum_{i=1}^{N}c_i(\hat{q})-\frac{1}{\hat{q}} = 0. \quad (32)$$

The resulting optimum power allocation for the source is $a_s = \hat{q}$ and that for each relay i is $a_i = \hat{q}c_i(\hat{q}), \quad i = 1, 2, \ldots, N$.

TABLE I: Optimum power allocation for cooperation system with one relay: $\sigma^2_{s,d}$ = 1.

$\left[\sigma^2_{s,r_i}, \sigma^2_{r_i,d}\right]$	DBPSK : $[a_s, a_1]$		DQPSK : $[a_s, a_1]$	
	Numerical	Analytical	Numerical	Analytical
$[1, 1]$	[0.66, 0.34]	[0.66, 0.34]	[0.70, 0.30]	[0.69, 0.31]
$[10, 1]$	[0.54, 0.46]	[0.54, 0.46]	[0.54, 0.46]	[0.54, 0.46]
$[1, 10]$	[0.80, 0.20]	[0.79, 0.21]	[0.80, 0.20]	[0.78, 0.22]

A. Optimum power allocation for single-relay systems

In this case, we obtain a closed-form solution for the optimization in (29). Specifically, the optimization problem (31) and (32) reduces to

$$(N+1)q - \frac{1}{c_1}+\frac{\sigma^2_{r_1,d}+\sigma^2_{s,r_1}\Psi e^{B_{c_1}}\left[\frac{1}{Pq}\left(\mathcal{E}+\ln B_{c_1}\right)+\frac{1}{B_{c_1}}\right]}{c_1^2[c_1\sigma^2_{r_1,d}-\sigma^2_{s,r_1}e^{B_{c_1}}\left(\mathcal{E}+\ln B_{c_1}\right)]} - \frac{1}{P}\frac{\sigma^2_{s,r_1}\Psi e^{B_{c_1}}\left(\mathcal{E}+\ln B_{c_1}+\frac{1}{B_{c_1}}\right)}{c_1\left[c_1\sigma^2_{r_1,d}-\sigma^2_{s,r_1}e^{B_{c_1}}\left(\mathcal{E}+\ln B_{c_1}\right)\right]} = 0,$$

and the solution must satisfies $\quad 1+c_1(q)-1/q = 0. \quad (33)$

In (33), a_s and a_1 can be obatined by using numerical search or any available standard optimization method such as the Newton's method. Note that jointly optimizing (33) provides an alternative closed-form power allocation for the two-node differential amplify-and-forward systems [11].

Table I shows optimum power allocation based on the exhaustive numerical search [11] and the closed-form formulation (33), respectively. The optimization is determined at reasonable high SNR region, e.g. 20 or 30 dB. For illustration purpose, we consider a cooperation system using DBPSK or DQPSK modulation. In the Table, we represent different channel qualities as $\left[\sigma^2_{s,d}, \sigma^2_{s,r_i}, \sigma^2_{r_i,d}\right]$ which corresponds to channel variances of source-destination link, source-relay link, relay-destination link, respectively. We can see that in both DBPSK and DQPSK modulations and at any relay location, the obtained optimum power allocation based on the closed-form formulation are very close to that from the numerical search results. We observe about $1-2\%$ different between the numerical results and the analytical results.

B. Optimum power allocation for multi-relay systems

In this case, we can use (31) and (32) to find the optimum power allocation. However, the optimization based on (31) and (32) involves $N+1$ dimensional search since the expression in (31) contains power allocation of each relay inside the summation term. To reduce complexity of the search space, we approximate (31) by keeping the summation term that contributes only to the power allocation of the i^{th} relay, i.e. c_i. The optimization problem to determine an approximate power allocation can be written as

$$(N+1)q - \frac{1}{c_i}+\frac{\sigma^2_{r_i,d}+\sigma^2_{s,r_i}\Psi e^{B_{c_i}}\left[\frac{1}{Pq}\left(\mathcal{E}+\ln B_{c_i}\right)+\frac{1}{B_{c_i}}\right]}{c_i^2[c_i\sigma^2_{r_i,d}-\sigma^2_{s,r_i}e^{B_{c_i}}\left(\mathcal{E}+\ln B_{c_i}\right)]} - \frac{1}{P}\frac{\sigma^2_{s,r_i}\Psi e^{B_{c_i}}\left(\mathcal{E}+\ln B_{c_i}+\frac{1}{B_{c_i}}\right)}{c_i\left[c_i\sigma^2_{r_i,d}-\sigma^2_{s,r_i}e^{B_{c_i}}\left(\mathcal{E}+\ln B_{c_i}\right)\right]} = 0,$$

and the solution must satisfies $1+\sum_{i=1}^{N}c_i(q)-1/q = 0. \quad (34)$

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TABLE II: Optimum power allocation for cooperation system with two relays based on exhaustive search.

$\left[\sigma^2_{s,d}, \sigma^2_{s,r_i}, \sigma^2_{r_i,d}\right]$	DQPSK : $[a_s, a_1, a_2]$	
	Numerical	Analytical
$[1,1,1]$	[0.48, 0.33, 0.19]	[0.50, 0.25, 0.25]
$[1,10,1]$	[0.40, 0.30, 0.30]	[0.39, 0.31, 0.30]
$[1,1,10]$	[0.66, 0.21, 0.13]	[0.67, 0.1625, 0.1625]

Hence, the optimum power allocation that involves (N+1) dimensional search is reduced to one dimensional search over the parameter $q \in (0,1)$ by jointly optimizing (34). We will show through numerical evaluations that the obtained power allocation is very close to that from the multidimensional search (27).

In Table II, we summarize the numerical search results based on the multi-dimensional search of the optimization problem (27) for DQPSK signals under different channel variances. We compare the obtained results to those from approximate one-dimensional search (34) as shown in the second column of Table II. In the Table we represent different channel qualities as $\left[\sigma^2_{s,d}, \sigma^2_{s,r_i}, \sigma^2_{r_i,d}\right]$ which corresponds to the set of channel variances of source-relay link, source-relay link, and relay-destination link, respectively. With the optimization in (34), the searching time for optimum power allocation reduces dramatically, while the obtained power allocation is very close to that from exact multi-dimensional search.

In addition, we can see from the obtained numerical results in the Table that, for any channel link qualities, more power should be allocated to the source. When the channel link qualities between the source and the relays are good, i.e. $\left[\sigma^2_{s,d}, \sigma^2_{s,r_i}, \sigma^2_{r_i,d}\right]$ = $\left[1,10,1\right]$ in this case, the system replicates the multiple transmitted antenna system. Therefore, almost equal powers should be allocated to the source and the relays. However, more power should be allocated to the source such that the transmit information can reach the relays, and the remaining power is allocated to the relays. The results also show that if there are two relays in the networks, almost the same amount of power as the source should be allocated to the first relay, and the rest amount of power is allocated to the second relay. In case of $\left[\sigma^2_{s,d}, \sigma^2_{s,r_i}, \sigma^2_{r_i,d}\right]$ = $\left[1,1,10\right]$ which indicates that the channel qualities between the relays and the destination are good. When comparing to the case $\left[\sigma^2_{s,d}, \sigma^2_{s,r_i}, \sigma^2_{r_i,d}\right]$ = $\left[1,1,1\right]$, higher power should be allocated at the source, while less powers should be put at the relays. The reason is that the channel quality at the source-destination link is lower than that of the relay-destination links, so more power is required at the source to balance the qualities of all possible links such that the system can provide reliable communications.

V. Simulation Results

We simulate the multi-node differential amplify-and-forward cooperation systems with DBPSK or DQPSK modulation. We consider the scenarios of two or three relays ($N = 2$, or 3) in the networks. The channel coefficients follow the Jakes' model [15] with Doppler frequency $f_D = 75\ Hz$ and normalized fading parameter $f_D T_s$ = 0.0025 where T_s is the sampling period. The noise variance is assumed to be unity ($\mathcal{N}_0 = 1$). The average BER curves are plotted as functions of $P/\mathcal{N}_0$.

We illustrate in Figure 2 the performance of the DBPSK cooperation system with two relays. The simulation is performed under equal channel variances, i.e. $[1,1,1]$, and equal power allocation strategy (i.e. $P_s = P_1 = P_2 = P/3$). We can see that the exact theoretical BER curve well matches to the simulated BER performance. In addition, the BER upper bound, the simple BER upper bound, and the simple BER approximations are tight to the simulated curve at high SNR. The BER curve for coherent detection is also showed in the figure; we observe a performance gap of about 4 dB between the proposed scheme and the coherent detection scheme at a BER of 10^{-3}.

Figure 3 shows BER performance of the proposed scheme with DQPSK modulation when using different number of relays (N), namely $N = 2$ and $N = 3$. The simulation scenario is the same as that of Figure 2. we can see that the proposed differential cooperative scheme achieves higher diversity orders as N increases. At a BER of 10^{-3}, we observe about $1.7 - 2$ dB gain as N increases from 2 to 3. Also, the exact theoretical BER curves are tight to the simulated curves. In addition, the performance curves of our proposed scheme are about 4 dB away from that of coherent detection.

Figure 4 shows the BER performance of the proposed scheme with optimum power allocation strategy in comparison to that of equal power allocation. We consider a DQPSK modulation system a case when there are two relays in the network. The channel variances are $[1,10,1]$, and the optimum power allocation is $[0.39, 0.31, 0.30]$ (from Table II). The simulated curves show that when all relays are close to the source, i.e. $\sigma^2_{s,r_i} = 10$, the proposed scheme with optimum power allocation yields about 0.6 dB gain at a BER of 10^{-3} over the proposed scheme with equal power allocation. Also in the figure, the exact theoretical BER curves are provided for both power allocation schemes, and they closely match to the simulated performances.

In Figure 5, we consider the BER performance with optimum power allocation strategy for DQPSK modulation system with two relays in the network. The channel variances are $[1,1,10]$, and the optimum power allocation for this case is $[0.67, 0.1625, 0.1625]$ (from Table II). We observe that the performance with optimum power allocation is about 2 dB superior to that with equal power allocation at a BER of 10^{-3}.

VI. Conclusions

We propose in this paper a multi-node differential amplify-and-forward scheme for cooperative communications. We provide as performance benchmark an exact BER expression for MDPSK modulation based on optimum combining weights. It is shown to closely match to the simulated performance. BER upper bounds and BER approximations are provided; they are tight to the simulated performance, especially in high SNR region. We show that the diversity order of the proposed scheme is $N+1$ when N is the number of relays. We observe about $1.7-2$ dB gain at a BER of 10^{-3} when N increases from 2 to 3. The BER approximation is further simplified in order to obtain analytical result for optimum power allocation scheme. A closed-form power allocation scheme is obtained for single-relay case, and an approximate power allocation scheme is provided for multi-relay scenario. Both the numerical evaluation and the analytical result show that more

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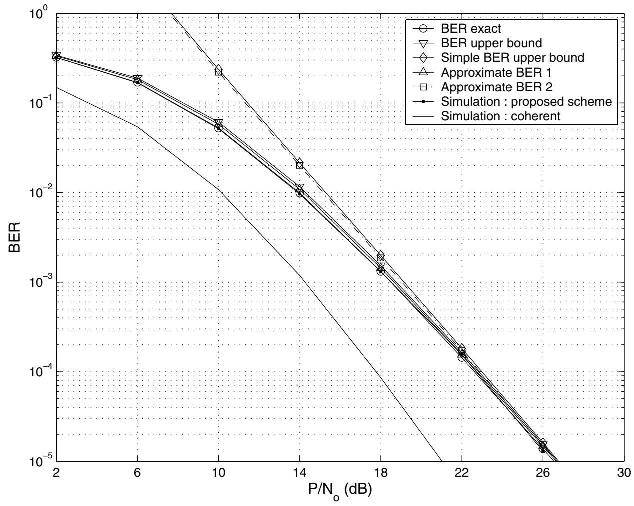

Fig. 2: DBPSK : Two relays, equal power allocation strategy, and $\sigma^2_{s,d} = \sigma^2_{s,r_i} = \sigma^2_{r_i,d} = \sigma^2_{r_i,r_j} = 1$.

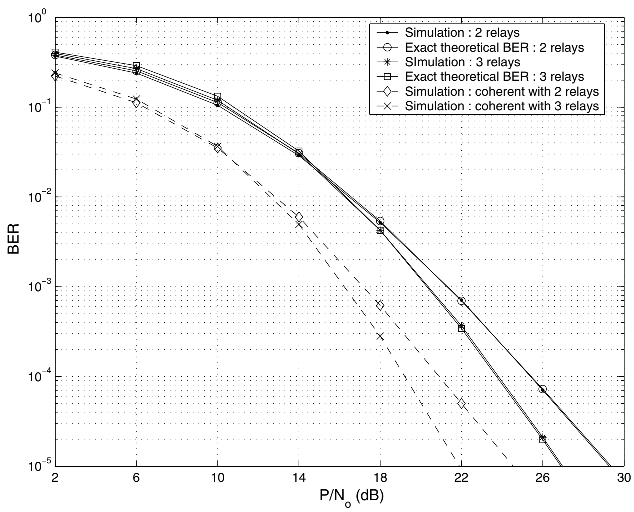

Fig. 3: DQPSK : Two and three relays, equal power allocation strategy, and $\sigma^2_{s,d} = \sigma^2_{s,r_i} = \sigma^2_{r_i,d} = \sigma^2_{r_i,r_j} = 1$.

power should be allocated to the source in order to achieve better performance. Compared with equal power allocation, the performance with optimum power allocation achieves 0.6 dB gain when all relays are close to the source, and 2 dB gain when all relays are close to the destination.

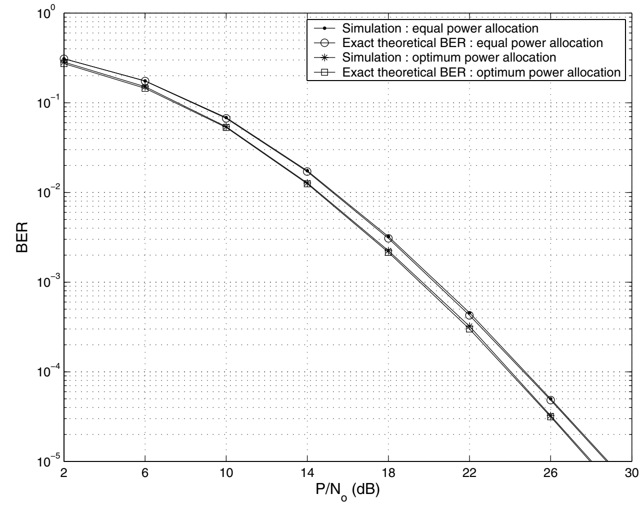

Fig. 4: DQPSK : Two relays, optimum power allocation strategy, and $\sigma^2_{s,d} = 1$, $\sigma^2_{s,r_i} = 10$, $\sigma^2_{r_i,d} = \sigma^2_{r_i,r_j} = 1$.

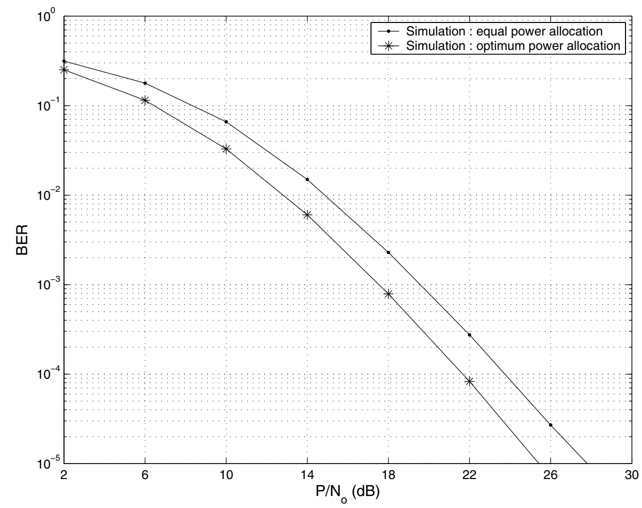

Fig. 5: DQPSK : Two relays, optimum power allocation strategy, and $\sigma^2_{s,d} = \sigma^2_{s,r_i} = 1, \sigma^2_{r_i,d} = 10$, and $\sigma^2_{r_i,r_j} = 1$.

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