Time-Reversal with Limited Signature Precision: Tradeoff Between Complexity and Performance

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***Abstract*—The success of time-reversal (TR) technique relies on the design of TR signature from the multi-path profile. In practice, the signature is of limited precision instead of being infinite due to the finite resolution in hardware. Such limited signature precision will degrade the TR focusing effect and thus the system performance. Although several works have been proposed to study the performance of TR system with one-bit signature, a general relation between the resolution of signature and the system performance has not been established to answer the question: how many bits are needed for TR systems to achieve desirable system performance? In the paper, we address the question by studying the TR system with limited signature precision. We derive theoretically the relationship between the received signal-to-interference-plus-noise ratio (SINR) and the quantization step. Through simulations, we find that for a typical TR system, 4-bit resolution for the signature is good enough to achieve good system performance.**

***Index Terms*—time-reversal, N-bit TR, limited signature precision**

I. Introduction

With the explosive growth of wireless users as well as wireless applications and services in recent years, there is a growing need in calling for future high-speed reliable broadband wireless communication solutions. On the other hand, due to the development and progress in the field analog-to-digital-converter (ADC), wideband communication becomes much more affordable. Because of the inherent nature to fully harvest energy from the surrounding environment by exploiting the multi-path propagation, time-reversal (TR) technique is shown to be a desired solution to low complexity high throughput wideband communications [1].

The history of the research on TR transmission can go back to early 1990s, when it was known and used in optical domain for decades and later first used in ultrasonic domain by Mathias Fink [2]. Since TR can make full use of multi-path propagation and re-collect all the signal energy that could be collected without complicated channel processing and equalization, it has been also studied in wireless communication systems [3]. As pointed out in [4], TR system has the potential of over an order of reduction in power consumption and interference alleviation.

The success of TR technique relies on the design of TR signature from the multi-path profile. In practice, the signature is of limited precision instead of being infinite due to the finite resolution in hardware, i.e., quantization. Such limited signature precision will degrade the TR focusing effect and thus the system performance. Moreover, deploying the high-resolution signature is practically difficult especially with a rate of Gigabits per second [5], which translates into the cost of the system. Although many works have been proposed in the literature to theoretical analyze the TR system with the assumption of full-precision signature [1] [4] [6], few work has been done on analyzing the TR system with limited signature precision except those on one-bit TR system [5] [7] [8] [9] [10].

In [7], it is shown that the received signal-to-noise-ratio (SNR) is lowered by 1.2 dB with only one-bit signature. Followed by that work, several works have studied the performance of one-bit TR system. For example, the Nguyen derived the analytical solutions for the temporal and spatial focusing metrics of one-bit TR system [8], while the Chang et al. in [9] applied one-bit TR in Ultra-wideband (UWB) communication systems and examined the system performance. Moreover, the one-bit TR UWB communication system was further extended to a single-input-multiple-out (SIMO) architecture [10].

In the academic integrity of TR system, we analyze the TR system performance with limited signature precision in this paper. Specifically, we try to address the question that how many bits are needed for TR system to achieve desirable performance. To do so, we first derive theoretically the relationship between the received signal-to-interference-plus-noise ratio (SINR) and the quantization step. Moreover, the relationship between number of bits and the quantization step size has been established. Furthermore, we provide a systematic method to determine the precision of TR signature deployed given desired system performance. In the end, we find that for a typical TR system, 4-bit resolution for the signature is good enough to achieve good system performance.

The rest of this paper is organized as follows. We theoretically analyze the TR system with limited signature precision in section II. In section III, we conduct simulations to verify our theoretical derivation. Then, we answer the question that how many bits are needed for TR systems in Section IV. Finally, conclusions are drawn in section V.

II. Time Reversal System with Limited Signature Precision

A typical time reversal system is shown in Fig. 1. Before transmitted through the antenna, the symbol X is first up-sampled by a backoff factor D and then modulated by a

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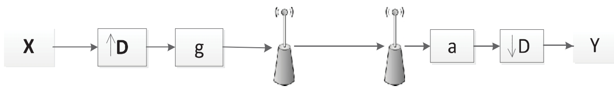

Fig. 1: Time-Reversal System Model.

signature $\mathbf{g}$. At the receiver, the received signal is amplified by a parameter a, and then down-sampled according to the backoff factor D.

Consider a channel impulse response (CIR) with L realized independent multi-paths as follows

$$\mathbf{h}[n] = \sum_{l=0}^{L-1} h_l \delta[n-l]. \tag{1}$$

where $h_l's$ are assumed i.i.d. and $h_l \sim CN(0, {\sigma_l}^2)$.

Similar to [6], we further assume that the ${\sigma_l}^2$ exponentially decays as follows

$$\sigma_l^2 = \exp^{-\frac{lT_s}{\sigma_T}}, \tag{2}$$

where T_s and σ_T are the symbol period and the root mean square delay spread of channel, respectively.

In a basic time reversal system, the signature $\mathbf{g}$ is simply a normalized version of the time reverse of $\mathbf{h}$, i.e.,

$$\mathbf{g}[n] = \frac{\mathbf{h}^*[L-1-n]}{\sqrt{\sum_{l=0}^{L-1} \sigma_l^2}}. \tag{3}$$

From (3), we can see that $\mathbf{g}[n]'s$ are i.i.d. and $\mathbf{g}[n] \sim CN(0, \frac{\sigma_{L-1-n}^2}{\sigma_{sum}^2})$ with $\sigma_{sum}^2 = \sum_{l=0}^{L-1} \sigma_l^2$.

According to Fig. 1, the received symbols can be written as [6]

$$\begin{aligned} y[k] =& a(\mathbf{h} * \mathbf{g})[L-1]x[k - \frac{L-1}{D}] \quad (\mathbf{Signal}) \\ &+ a \sum_{l=0,\, l \neq (L-1)/D}^{(2L-2)/D} (\mathbf{h} * \mathbf{g})[Dl]x[k-l] \quad (\mathbf{ISI}) \\ &+ an[k] \quad (\mathbf{noise}), \end{aligned} \tag{4}$$

and the received SINR can be computed as follows

$$SINR = \frac{\mathbf{E}[\mathbf{P_{signal}}]}{\mathbf{E}[\mathbf{P_{ISI}}] + \mathbf{N_0}}, \tag{5}$$

where $\mathbf{N_0}$ is the expect noise power, $\mathbf{E}[\mathbf{P_{signal}}]$ and $\mathbf{E}[\mathbf{P_{ISI}}]$ can be written as [6]

$$\begin{aligned} \mathbf{E}[\mathbf{P_{signal}}] &= \theta \frac{1 + \exp^{-\frac{LT_s}{\sigma_T}}}{1 + \exp^{-\frac{T_s}{\sigma_T}}} + \theta \frac{1 - \exp - \frac{LT_s}{\sigma_T}}{1 - \exp^{-\frac{T_s}{\sigma_T}}}, \\ \mathbf{E}[\mathbf{P_{ISI}}] &= 2\theta \frac{\exp^{-\frac{T_s}{\sigma_T}}(1 - \exp^{-\frac{(L-2+D)T_s}{\sigma_T}})}{(1 - \exp^{-\frac{DT_s}{\sigma_T}})(1 + \exp^{-\frac{T_s}{\sigma_T}})}, \end{aligned} \tag{6}$$

with θ being the transmit power for one symbol.

A. Analysis of the time reversal system with limited signature precision

In practice, the signature needs to be quantized, i.e., the signature is limited with finite precision. In such a case, the system performance will degrade. In this subsection, we will analyze theoretically how the system performance degrades with the resolution of signature.

When the signature is quantized, quantization error will be introduced. Let $\widetilde{\mathbf{g}}$ and $\mathbf{e} = \widetilde{\mathbf{g}} - \mathbf{g}$ be the quantized signature and the quantization error, respectively. By replacing the $\mathbf{g}$ with $\widetilde{\mathbf{g}} = \mathbf{g} + \mathbf{e}$ in Fig. 1, the received signal can be re-written as

$$\begin{aligned} y[k] =& a(\mathbf{h} * \mathbf{g})[L-1]x[k - \frac{L-1}{D}] \quad (\mathbf{Signal}) \\ &+ a \sum_{l=0,\, l \neq (L-1)/D}^{(2L-2)/D} (\mathbf{h} * \mathbf{g})[Dl]x[k-l] \quad (\mathbf{ISI}) \\ &+ a(\mathbf{h} * \mathbf{e})[L-1]x[k - \frac{L-1}{D}] \\ &+ a \sum_{l=0,\, l \neq (L-1)/D}^{(2L-2)/D} (\mathbf{h} * \mathbf{e})[Dl]x[k-l] \\ &\qquad (\mathbf{quantization\,error}) \\ &+ an[k] \quad (\mathbf{noise}) \end{aligned} \tag{7}$$

Comparing (4) with (7), we can see that there are two additional terms related to the quantization error that degrade the system performance. In such a case, the received SINR becomes

$$SINR = \frac{\mathbf{E}[\mathbf{P_{signal}}]}{\mathbf{E}[\mathbf{P_{ISI}}] + \mathbf{E}[\mathbf{P_{quantization\ error}}] + \mathbf{N_0}}, \tag{8}$$

where $\mathbf{E}[\mathbf{P_{signal}}]$, $\mathbf{E}[\mathbf{P_{ISI}}]$ and $\mathbf{N_0}$ are the same as those in (5). The $\mathbf{E}[\mathbf{P_{quantization\ error}}]$ is due to the quantization error $\mathbf{e}$, and can be computed according to the following theorem.

THEOREM 1: Given the quantization step q, $\mathbf{E}[\mathbf{P_{quantization\ error}}]$ can be approximated as follows

$$\begin{aligned} \mathbf{E}[\mathbf{P_{quantization\ error}}] \approx& \theta \sigma_{sum}^2 \frac{q^2}{12} + \theta \sum_{m=0}^{L-1} \frac{\sigma_{L-1-m}^2 q^2}{12} \\ &+ \theta \frac{q^2}{6} \sum_{l=0\, l \neq \frac{L-1}{D}}^{2L-2} \sum_{m=Dl-L+1}^{Dl} \sigma_m^2 \end{aligned} \tag{9}$$

Proof: Since $\mathbf{e}$ is the quantization error of the signature $\mathbf{g}$ which is a complex Gaussian variable, in order to derive $\mathbf{E}[\mathbf{P_{quantization\ error}}]$, we first review the statistical relationship between a Gaussian input and the corresponding quantization error.

Let $\nu \sim N(0, \sigma^2)$ be a Gaussian variable and ϵ be the corresponding quantization error when the quantization step q

TABLE I: Statistical Relationship between ν and ϵ

	$\mathbf{E}[\epsilon]$	$\mathbf{E}[\epsilon^2]$	$\mathbf{E}[\nu\epsilon]$	$\mathbf{E}[\nu^2\epsilon^2]$
$q = 2\sigma$	0	$(\frac{1}{12} - 6.7 \times 10^{-4})q^2$	-2.47×10^{-2}	$\frac{\sigma^2 q^2}{12}(1 + 7.91 \times 10^{-2})$
$q = 1.5\sigma$	0	$(\frac{1}{12} - 2.8 \times 10^{-5})q^2$	-9.1×10^{-4}	$\frac{\sigma^2 q^2}{12}(1 + 5.3 \times 10^{-3})$
$q = \sigma$	0	$(\frac{1}{12} - 4.5 \times 10^{-6})q^2$	-1.2×10^{-4}	$\frac{\sigma^2 q^2}{12}(1 + 1 \times 10^{-5})$

is used. According to [11], we have

$$\begin{aligned}\mathbf{E}[\epsilon] &= 0 \\ \mathbf{E}[\epsilon^2] &= \frac{q^2}{12} + \frac{q^2}{\pi^2}\sum_{l=1}^{\infty}\Phi_\nu(\frac{2\pi l}{q})\frac{(-1)^l}{l^2} \\ \mathbf{E}[\nu\epsilon] &= \frac{q}{\pi}\sum_{l=1}^{\infty}\dot{\Phi}_\nu(\frac{2\pi l}{q})\frac{(-1)^{l+1}}{l} \\ \mathbf{E}[\nu^2\epsilon^2] &= \frac{\sigma^2 q^2}{12} + \frac{q^2}{\pi^2}\sum_{l=1}^{\infty}\mathbf{Re}\{\ddot{\Phi}_\nu(\frac{2\pi l}{q})\}\frac{(-1)^{l+1}}{l^2}\end{aligned} \tag{10}$$

where Φ_ν is the characteristic function of Gaussian random variable ν.

Moreover, through the numerical results shown in Table I, we find that, when $q \leq 2\sigma$, $\mathbf{E}[\epsilon^2]$, $\mathbf{E}[\nu\epsilon]$, and $\mathbf{E}[\nu^2\epsilon^2]$ can be approximated as $\frac{q^2}{12}$, 0, and $\frac{\sigma^2 q^2}{12}$, respectively. Applying such an approximation to the quantization error term in (7), we can obtain the approximated closed-form expression for $\mathbf{E}[\mathbf{P}_{\textbf{quantization error}}]$ as in (9).

Note that according to (2) and (3), the approximation is accurate when $q \leq 2\sigma_{min} = 2\min_m \sqrt{\frac{\sigma_m^2}{2\sigma_{sum}^2}} = 2\sqrt{\frac{\sigma_{L-1}^2}{2\sigma_{sum}^2}}$. ∎

III. Simulation Results

In this section, we conduct simulations to validate our theoretical results derived in the previous section. Specifically, we will verify the SINR in (8) under different conditions using simulations.

In the first simulation, we evaluate the SINR performance under different quantization step q, where we assume $\sigma_T = 128T_s$, $L = 257$, and $D = 1$. The simulation results are shown in Fig. 2, where "wo sim" and "wo th" stand for the simulation results without quantization and theoretical results without quantization, respectively, while "$m = i$ sim" and "$m = i$ th" stand for the the simulation results using quantization step $q = i\sigma_{min}$ and theoretical results using quantization step $q = i\sigma_{min}$, respectively. According to the analysis later in Section IV, $m = 1$, $m = 2$ and $m = 4$ correspond to 4-bit, 3-bit and 2-bit signature precision, respectively. Similar to [6], ρ stands for a modified received signal-to-noise ratio. From Fig. 2, we can see that when $q \leq 4\sigma_{min}$, the numerical results match well with the theoretical results, which validates our theoretical analysis in section II. We also see that the SINR performance degrades as the quantization step q increases. This is because as q increases, the quantization error increases and thus SINR decreases.

In the second simulation, we evaluate the SINR performance under different backoff factor D, where we assume $\sigma_T = 128T_s$, $L = 257$, and set $q = 2\sigma_{min}$. The simulation results are shown in Fig. 3. We can see that again the numerical results match well with the theoretical results for different $D's$. Moreover, we find that as D increases, the SINR performance increases due to the decrease of the inter-symbol-interference (ISI).

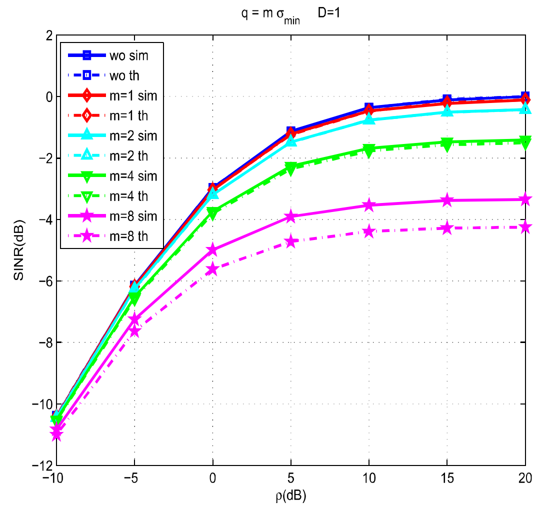

Fig. 2: The SINR performance with $q = m\sigma_{min}$, $L = 257$ and $D = 1$.

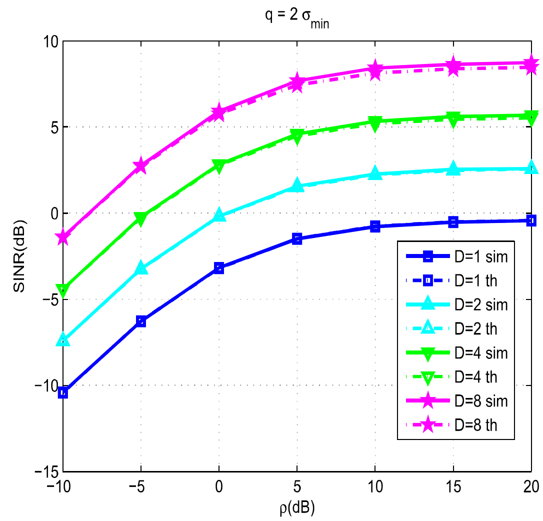

Fig. 3: The SINR performance with $q = 2\sigma_{min}$, $L = 257$.

We then evaluate the SINR performance under a different environment setting, where we assume $\sigma_T = 256T_s$ and $L = 513$. The results are illustrated in Fig. 4 and Fig. 5. We can see that the results are consistent with those in previous simulations shown in Fig. 2 and Fig. 3, i.e., our theoretical results are consistently valid under different settings.

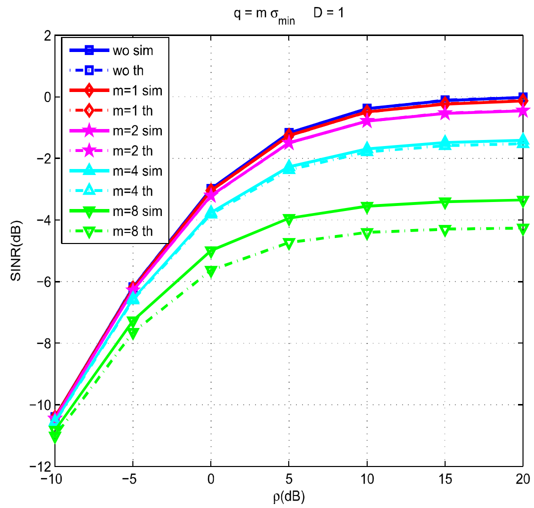

Fig. 4: The SINR performance with $q = m\sigma_{min}$, $L = 513$ and $D = 1$.

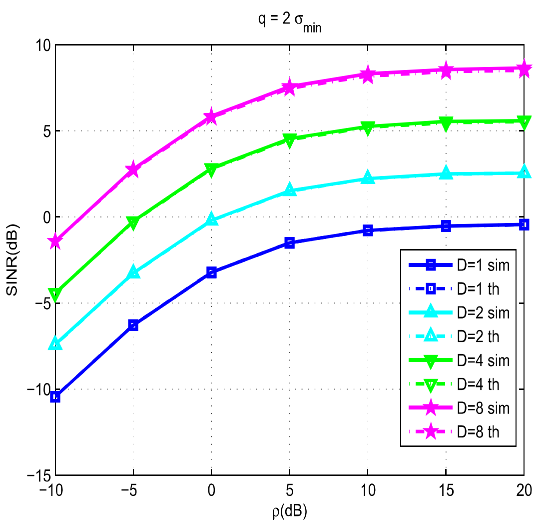

Fig. 5: The SINR performance SINR with $q = 2\sigma_{min}$, $L = 513$.

IV. How Many Bits are Needed for a Time Reversal System?

In the previous sections, we have analyzed and validated the relationship between the SINR performance and the quantization step q. However, we have not yet answered the question: how many bits are needed for a time reversal system? In this section, we will answer this question as follows.

From (3) and (2), we know that each tap of the signature is a complex Gaussian random variable with the maximal variance being $\sigma_{max}^2 = \max_m \frac{\sigma_m^2}{2\sigma_{sum}^2} = \frac{\sigma_0^2}{2\sigma_{sum}^2}$. Since the probability that the signature lies outside $[-2\sigma_{max}, 2\sigma_{max}]$ is smaller than 5%, we truncate the Gaussian distribution to the range $[-2\sigma_{max}, 2\sigma_{max}]$, i.e., all the values outside the range will be truncated to be $\pm 2\sigma_{max}$.

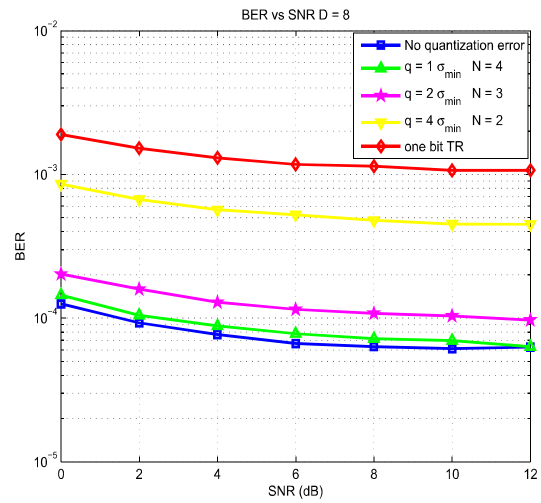

Fig. 6: The BER performance with $L = 257$ and $D = 8$.

Then, consider a symmetric uniform quantization with quantization step $q = m\sigma_{min}$, there will be $2\lceil\frac{4\sigma_{max} - m\sigma_{min}}{2m\sigma_{min}}\rceil + 1$ quantization levels. Therefore, we need $\lceil\log_2(2\lceil\frac{4\sigma_{max}/\sigma_{min} - m}{2m}\rceil + 1)\rceil$ bits each to represent the real and imaginary part of signature, respectively. In other words, for a quantization step $q = m\sigma_{min}$, the number of bits needed for each tap of signature, N, is

$$N = \lceil\log_2(2\lceil\frac{4\sigma_{max}/\sigma_{min} - m}{2m}\rceil + 1)\rceil. \tag{11}$$

We then examine the value of N for a typical time reversal system through investigating the bit error rate (BER) performance. Specifically, we consider a time reversal system with $\sigma_T = 128T_s$, $D = 8$ and $L = 257$. The BER performance under different N is shown in Fig. 6. From Fig. 6, we can see that compared with the one without quantization, the BER performance of the one-bit time reversal system [7] actually degrades a lot. The BER performance gradually improves as N increases. This is because as N increases, the quantization step q decreases, due to which the quantization error decreases and thus the BER performance improves. From Fig. 6, we can also see that the system with $N = 4$ achieves similar BER performance to the system without quantization, i.e., 4-bit resolution for the signature is enough for a time reversal system with such settings.

V. Conclusion

In the paper, we analyzed the TR system with limited signature precision and studied the corresponding tradeoff between the complexity and performance. We derived an approximated closed-form expression of SINR as a function of quantization step, according to which we investigate the resolution of signature needed for TR systems to achieve reasonable performance. Through simulation results, we concluded that 4-bit resolution for signature is enough for a typical TR system.

REFERENCES

[1] Y. Chen, Y. Yang, F. Han, and K. J. R. Liu, "Time-reversal wideband communications," *IEEE Signal Processing Letters*, vol. 20, no. 12, pp. 1219–1222, 2013.

[2] M. Fink and C. Prada, "Acoustic time-reversal mirrors," *Inverse problems*, vol. 17, no. 1, p. R1, 2001.

[3] S. Emami, J. Hansen, A. Kim, G. Papanicolaou, A. Paulraj, D. Cheung, and C. Prettie, "Predicted time reversal performance in wireless communications using channel measurements," *IEEE Communication Letters*, 2004.

[4] B. Wang, Y. Wu, F. Han, Y. Yang, and K. J. R. Liu, "Green wireless communications: a time-reversal paradigm," *IEEE J. Select. Areas Commun.*, vol. 29, no. 8, pp. 1698–1710, 2011.

[5] D. Abbasi-Moghadam and V. Vakili, "A simo one-bit time reveresal for uwb communication systems," *EURASIP Journal on Wireless Communications and Networking*, pp. 1–9, 2012.

[6] F. Han, Y. Yang, B. Wang, Y. Wu, and K. J. R. Liu, "Time-reversal division multiple access over multi-path channels," *IEEE Trans. Commun.*, vol. 60, no. 7, pp. 1953–1965, 2012.

[7] A. Derode, A. Tourin, and M. Fink, "Ultrasonic pulse compression with one-bit time reversal through multiple scattering," *Journal of applied physics*, vol. 85, no. 9, pp. 6343–6352, 1999.

[8] H. Nguyen, "On the performance of one bit time reversal for multi-user wireless communications," in *Wireless Communication Systems, 2007. ISWCS 2007. 4th International Symposium on*, 2007, pp. 672–676.

[9] Y. Chang, S. Tsai, X. Yu, and C. Kuo, "Ultrawideband transceiver design using channel phase precoding," *IEEE Trans. Signal Processing*, vol. 55, no. 7, pp. 3807–3822, 2007.

[10] P. Kyritsi and G. Papanicolaou, "One-bit time reversal for wlan applications," *In Personal, Indoor and Mobile Radio Communications, 2005. PIMRC 2005. IEEE 16th International Symposium on*, vol. 1, pp. 532–536, 2005.

[11] B. Widrow and I. Kollar, *Quantization noise: roundoff error in digital computation, signal processing, control, and communications*. Cambridge University Press, 2008.