Throughput Maximization Using Adaptive Modulation in Wireless Networks with Fairness Constraint

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Abstract— **In multi-access wireless communication networks, cochannel interference and timing varying nature of the channel are two important detrimental effects to reduce the network throughput. Adaptive modulation techniques have the potential to substantially increase the spectrum efficiency by providing the users with different levels of throughput according to the channel conditions. In this paper, we study how to adaptively allocate each user's throughput in the multi-access networks. We optimize the network throughput, while each user's time average throughput is kept as a constant to ensure the fairness. We solve the problem in two steps. First we find the throughput range that each user can select at different time to ensure the fairness. Second we find the best throughput allocation to different users at each time to generate the maximum overall throughput. From the simulation results, it shows that our scheme increases overall network throughput about 10%, compared with the previous works.**

I. Introduction

With the increasing demand for wireless services, the ability to provide higher network throughput by limited spectrum bandwidth is one of the important design considerations. Two detrimental effects to reduce the spectrum efficiency are the co-channel interference and time varying nature of the channel. Power control and adaptive modulation are the important approaches to combat these effects. In power control, the transmit power is continuously adjusted so that the receiver's Signal-to-Interference-Noise-Ratio (SINR) is maintained for a desired link quality. The main problem for power control is feasibility in the system with co-channel interferences. Under some channel conditions and receiver's SINR requirements, no matter how large the transmit powers are, the receiver's SINR can not be high enough, i.e. there are no power allocation solution. In adaptive modulation, each link's throughput is allocated according to the current channel conditions. The spectrum efficiency can be potentially increased. The main problem for adaptive modulation in multi-access system is fairness. Without fairness constraint, the link with the best channel condition will occupy almost all the radio resources for transmission, while the others stop transmission. It is no fair for the services that users have paid for.

Many works have been done for both power control and adaptive modulation [1], [2] and [3]. Most of the previous works only do the optimization based on current channel conditions and don't take into account of the time diversity. In this paper, we maximize the overall network throughput while each user's time average throughput is a constant, which is determined by the user's service. We divide the problem heuristically into two sub-problems. First we find the throughput range that each user can select for different time to ensure the fairness. Second we find the best throughput allocation to different users at each time that generates the largest network overall throughput, under the constraints that the powers are feasible and throughput are within the ranges. From the simulation results, our scheme can increase the overall network throughput about 10%, compared with the previous works [1].

The rest of the paper is organized as follows. In Section II, we discuss the system model and have the problem formulation. In Section III, we solve the problem by two steps and adaptive algorithms are developed. In Section IV, we present some numerical results that quantify the throughput performance of our proposed schemes. In Section V, we offer the summery and conclusion.

II. System Model and Problem Formulation

Consider K co-channel links that may exist in distinct cell. Each link consists of a mobile and its assigned base station. Assume coherent detection is possible so that it is sufficient to model this multiuser system by an equivalent baseband model. We assume that link gain is stable within each transmission frame. For uplink case, the signal at the i^{th} base station output is given by:

$$x_i(t) = \sum_{k=1}^{K} \sqrt{G_{ki}P_k} s_k(t) + n_i(t) \tag{1}$$

where G_{ki} is the path loss from the k^{th} mobile to the i^{th} base station, P_k is the k^{th} link's transmitting power, $s_k(t)$ is the message symbol, and $n_i(t)$ is the thermal noise, Then we can express the sampled received signal as:

$$x_i(n) = \sum_{k=1}^{K} \sqrt{P_k G_{ki}} s_k(n) + n_i(n) \tag{2}$$

where $n_i(n)$ is the sampled thermal noise. The i^{th} base station's output SINR is:

$$\Gamma_i = \frac{P_i G_{ii}}{\sum_{k\neq i} P_k G_{ki} + N_i}, \ i = 1, \dots, K \tag{3}$$

where $N_i = E\|n_i\|^2$. Write the above equations in matrix form. We have

$$(I - DF)\mathbf{P} = \mathbf{u} \tag{4}$$

where $\mathbf{P} = [P_1, \dots, P_K]^T$, $\mathbf{u} = [u_1, \dots, u_K]^T$, $u_i = \Gamma_i N_i / G_{ii}$, $D = diag\{\Gamma_1, \dots, \Gamma_K\}$, and

$$[F_{ij}] = \begin{cases} 0 & \text{if } j = i, \\ \frac{G_{ji}}{G_{ii}} & \text{if } j \neq i. \end{cases} \tag{5}$$

In order to have feasible solution in (4), the spectral radius $\rho(DF)$, i.e. the maximum eigenvalue of DF must less than 1 [5]. We will see that the overall transmit power is strongly affected by the value of $\rho(DF)$.

Adaptive modulation provides the system with the ability to match the effective bit rate (throughput) according to the interference and channel conditions. MQAM is the modulation method that has high spectrum efficiency. For a given system and a targeted BER, the throughput can be written as a function of the received SINR. Let T_i denote the i^{th} user's throughput, which is the number of bits sent within each transmitted symbol. In [2] [3], the throughput is assumed continuous and the i^{th} user's BER using different MQAM modulation with different throughput can be approximated as:

$$BER_i \approx c_1 e^{-c_2 \frac{\Gamma_i}{2^{T_i}-1}} \tag{6}$$

where $c_1 \approx 0.2$ and $c_2 \approx 1.5$ for MQAM. Rearrange Equ. 6. For specific BER, the i^{th} link's throughput is given by:

$$T_i = log_2\left(1 + c_3^i \Gamma_i\right) \tag{7}$$

where $c_3^i = -\frac{c_2^i}{ln(BER_i/c_1^i)}$. Without loss of generality, we assume each user has the unit bandwidth. Define the overall network throughput as: $T = \sum_{i=1}^{K} T_i$.

In this paper, we want to maximize the overall network throughput under the constraints that the power allocation is feasible and each user's time average throughput is fixed. The problem we will address becomes:

$$\max_{T_i, P_i} T = \sum_{i=1}^{K} T_i \tag{8}$$

$$\text{subject to} \begin{cases} \text{Feasibility: } \rho(DF) < 1, \\ \text{Fairness: } \lim_{N\to\infty} \frac{\sum_{n=1}^{N} T_i(n)}{N} = const. \; \forall \, i \end{cases}$$

where *const.* is the user's time average throughput and $T_i(n)$ is the throughput at time n.

III. Throughput Maximization with Fairness Constraint

A. Problem Partition

The problem defined in (8) is very hard to have analytic solution. In order to simplify the problem, we heuristically divide the problem into two sub-problems for different times and different users respectively. First in order to ensure the fairness, we tract the history of each user's throughput and determine the range of modulation level that each user can select at different time. Second for the whole network, we determine what is optimal throughput allocation to different users within the ranges at each time, which generates the highest overall network throughput.

A two users example is shown in Fig. 1. On each dotted line, network throughput $T = T_1 + T_2 = const.$, where different line has different constant. C is the range of users' throughput and where the system is feasible. The first sub-problem is how to change C for different time to ensure the fairness. For example, if user 1 experiences bad channel response now and select small throughput, it will be more aggressive to transmit its data in future. Consequently, the range C will move to the right within the practical range, where user 1 can select higher throughput. The second sub-problem is to find what is the point that generates the maximum network throughput within C at each time.

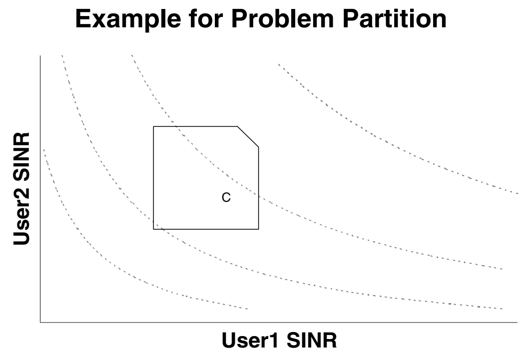

Fig. 1. Two Users Example

B. Throughput Range for Each User

In order to implement a scheme to ensure the fairness, we develop the following algorithm. Instead of having fixed throughput range for each link, we can adaptively change the throughput range, which takes into account of the links' throughput history. Suppose the i^{th} link can select $T^{ave} - T_i^{neg}(n) \le T_i(n) \le T^{ave} + T_i^{pos}(n)$ at time n, where the average throughput $T^{ave} = \sum T_i/K$. Each time $T_i^{neg}(n)$ and $T_i^{neg}(n)$ are modified by the current $T_i(n)$. When $T_i(n)$ is smaller than T^{ave}, $T_i^{neg}(n+1)$ is decreased and $T_i^{pos}(n+1)$ is increased so that the i^{th} user has to be more aggressive to transmit its data in the near future. On the other hand, when $T_i(n)$ is larger than T^{ave}, $T_i^{neg}(n+1)$ and $T_i^{pos}(n+1)$ are modified in the opposite way. In order to track the history of $T_i(n)$, we define $T_i^{pos}(n+1) = T_i^{pos}(n) - (T_i(n) - T^{ave}) * \beta$, $T_i^{neg}(n+1) = T_i^{neg}(n) + (T_i(n) - T^{ave}) * \beta, \;\; 0 < \beta < 1$, where β is a value that depends on how fast the channel changes. If the channel changes fast, β should select a larger number to keep track of channel changing, else β should select a smaller number to have smooth effect. Define $W = window\ size$ as the maximum throughput difference that each user can select from the average throughput T^{ave} . Our proposed algorithm is given by:

Adaptive Algorithm for Throughput Range

`Initial:` $T_i^{pos}(0) = W, \quad T_i^{neg}(0) = W,$
`Iteration:` $Calculate\ \ T^{ave}(n)$ $T_i^{pos}(n+1) =$ $\min(\max(T_i^{pos}(n) - \beta(T_i(n) - T^{ave}(n)), 0), W)$ $T_i^{neg}(n+1) =$ $\min(\max(T_i^{neg}(n) + \beta(T_i(n) - T^{ave}(n)), 0), W)$

C. Overall Throughput Maximization at Each Time

Now we will solve how to find the maximum overall throughput at each time. First the gradient of overall throughput with respect to targeted SINR is given by:

$$g_i^T = \frac{\partial T}{\partial \Gamma_i} = \frac{c_3}{1 + c_3 \Gamma_i}, \quad i = 1, \ldots, K \tag{9}$$

Starting from any feasible point, we can enlarge each user's targeted SINR according to (9) to increase the overall network throughput, until we hit the boundary, i.e. $\rho(DF) = 1 - \epsilon$, where ϵ is a small number. In this paper, we use $\epsilon = 0.05$. Then we calculate $T^{ave} = \sum T_i / K$

In the next step, we find the gradient $\partial \rho(DF)/\partial \Gamma_i$ and then project this gradient onto the plane where $T = const.$ Then we move along this modified gradient so that $\rho(DF)$ is reduced, while the overall throughput T is maintained as a constant. The overall transmit power will be reduced consequently. The iteration stops when T_i reaches the boundary or some stable point.

It has been shown that the existence of the derivative of the spectral radius [4] by the following theorem.

Theorem 1: let λ be a simple eigenvalue of DF, with right and left eigenvectors $\mathbf{x}$ and $\mathbf{y}$, respectively. let $\tilde{F} = DF + E$, then there exists a unique $\tilde{\lambda}$, eigenvalue of $\tilde{F}$ such that

$$\tilde{\lambda} = \lambda + \frac{\mathbf{y}^H E \mathbf{x}}{\mathbf{y}^H \mathbf{x}} + \mathrm{O}(\|E\|^2) \tag{10}$$

In our application, we only try to reduce the maximum eigenvalue. It will be shown in the simulation that the maximum eigenvalue is the key factor that affects the overall transmit power. We assume $\mathbf{x}$ and $\mathbf{y}$ are the eigenvectors of the largest eigenvalue. We define $E = \Delta\Gamma_i F_i$, where

$$(F_i)_{jk} = \begin{cases} 0 & j \neq i \\ (F)_{jk} & j = i \end{cases} \tag{11}$$

Then we can have the gradient to reduce spectral radius as:

$$g_i^\rho = \frac{\partial \rho(DF)}{\partial \Gamma_i} = \frac{\mathbf{y}^H F_i \mathbf{x}}{\mathbf{y}^H \mathbf{x}} \tag{12}$$

If we change each user's SINR according to above gradient g_i^ρ, the overall throughput may be reduced. We need to modify this gradient such that $T = const.$, The plane that is tangent to the curve $T = const.$ at $[\Gamma_1, \ldots, \Gamma_K]$ can be expressed as: $\sum_{i=1}^K k_i x_i = const$, where $k_i = c_3/(1 + c_3\Gamma_i)$. The modified gradient vector $\mathbf{g}^M$ is calculated by projecting vector $\mathbf{g}^\rho$ unto that plane.

We repeat the above two steps until the algorithm is stable. Then the throughput allocation (i.e. SINR allocation) is the selected for different users at that time. The transmit power is calculated and each user's throughput history is updated.

D. Adaptive Algorithm

Our proposed adaptive algorithm is given by:

Throughput Maximization Algorithm

```
Initial:
   T_1,...,T_K = any feasible throughput allocation.
Iteration:  Stop when T_i stable
  1.  Throughput Maximization
   do {
     g^T = ∇T;
     Γ_i = Γ_i + μ.g_i^T  ∀ i;
   while (ρ(DF) < 1 − ε)
   calculate T^ave
  2.  ρ(DF) Reduction:
   do {
     g^ρ = ∇ρ(DF);
     g^M = projection(g^ρ);
     Γ_i = Γ_i − μ'.g_i^M  ∀ i;
     if (T_i > T^ave + T_i^pos(n))  T_i = T^ave + T_i^pos(n)
     if (T_i < T^ave − T_i^neg(n))  T_i = T^ave − T_i^neg(n)
   while (T_i not stable or not reaching boundary)
Power Allocation Update:   P = DFP + u.
SINR Range Update:
   Update T_i^pos(n), T_i^neg(n).
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where μ and μ' are small constant, and the power allocation update can be implemented in the distributed manner[1].

IV. Simulation Results

In order to evaluate the performance of our algorithm, a network with 50 hexagonal cells is simulated as illustrated in Fig. 2. The base stations are placed at the center of the cells. Two adjacent base stations don't share the same channels. The radius of each cell is 1000m. In each cell, one user is placed randomly with a uniform distribution. In the simulations, we consider slow Rayleigh fading.

In Fig. 3, we show the track of convergence of our proposed algorithm. First the algorithm starts from a feasible throughput vector. Then the overall throughput is increased by assigning larger targeted SINR for each users. The maximum eigenvalue of DF, i.e. $\rho(DF)$ is also increased consequently. When $\rho(DF)$ is small, the increasing speed of overall power is low. However, when $\rho(DF)$ is larger than some point, the overall power is increased very fast, until there is no feasible solution, i.e. no matter how large the transmit powers are, the receiver's SINR can not reach the targeted value. In order to prevent this situation from happening, when $\rho(DF) > 0.95$, we will use the second step of our adaptive algorithm to reduce $\rho(DF)$, while keeping overall throughput unchanged. We can see from the curve that our algorithm reduces $\rho(DF)$ and overall transmit power significantly. Then at some point, the users' throughput reach the boundary or are stable. The algorithm goes back to the first step to increase the overall throughput. The two steps of our algorithm are repeated alternatively until $T_i, \forall\ i$ are stable. The algorithm will stop within a few iterations. In addition, we find out that the overall transmit power follows the same track when we change $\rho(DF)$ with different throughput allocation. This proves that it is a good approach to minimize overall power by reducing $\rho(DF)$.

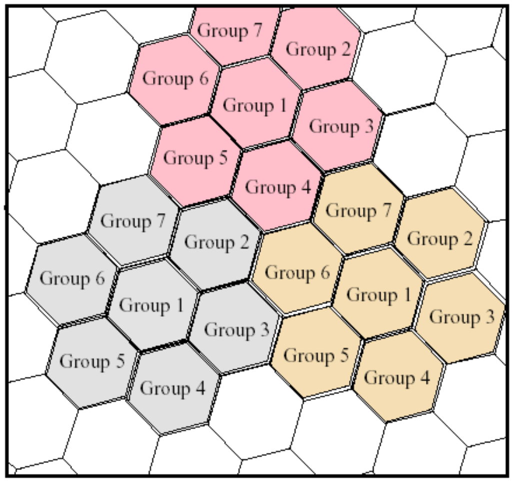

Fig. 2. Simulation Setup

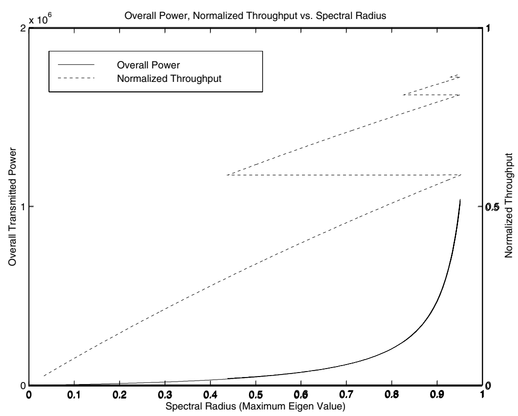

Fig. 3. Convergence Track

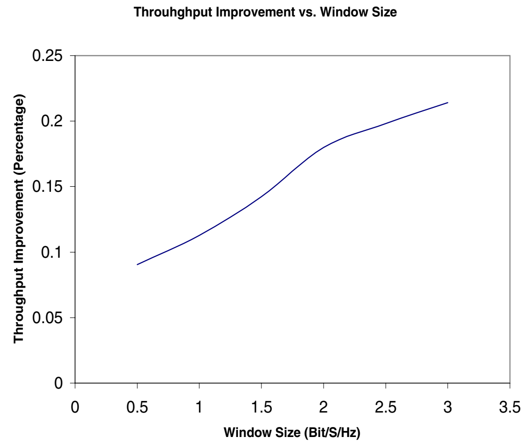

Fig. 4. Throughput Improvement vs. Window Size

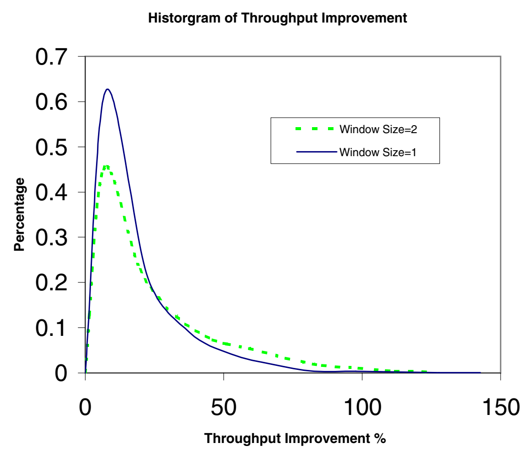

Fig. 5. Histogram of Throughput Improvement

In Fig. 4, we show the effect of window size on the average throughput improvement (percentage), compare with the previous work [1], where all users have the same fixed targeted SINR. The average throughput is improved when the window size goes larger. However the system complexity and delay will be increased as well. The improvement will be saturated when the window size is large enough. This is because the time average constraint. If the user with good channel condition now is too greedy and requests so many throughput, it has to pay back by transmitting less in the future. In Fig. 5, we show the histogram of throughput improvement. We can see that the overall throughput is improved about 10% in most of the cases. When the window size is increased, the distribution just has longer tail in higher throughput improvement range.

V. Conclusions

In this paper, each user can select a range of throughput. At each time, the links with bad channel conditions sacrifice their throughput, which reduces the unnecessary co-channel interference. The links with good channel conditions get more throughput, which increase the network throughput. We develop a heuristic algorithm to find the throughput allocation to generate the maximum network throughput at each time. For different time, we develop an adaptive algorithm to keep track of each user's throughput history so as to ensure the fairness. The whole scheme can be interpreted that each user's throughput is "water filled" in different time and for the whole system at any specific time, the overall network throughput is allocated to different links according to their channel conditions.

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