On Analysis of Two-Way Relaying With Network-Coded ARQ

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***Abstract*— In this work, we study the throughput of two-way relay channel (TWRC) with different automatic repeat-request (ARQ) strategies. Beyond orthogonal relaying, we propose two network-coded ARQ schemes, where the packets intended for different end terminals could be combined at the relay nodes. By considering the per-hop and end-to-end maximum number of transmissions constraint, we derive the closed-from throughput expression. We demonstrate that network coding can greatly improve the system throughput, but the throughput gain is upper bounded. Besides, we come up with a near-optimum power allocation scheme to maximize the throughput. Compared to equal power allocation, the proposed power allocation scheme can greatly improve the throughput especially when the network topology is highly asymmetric.**

I. INTRODUCTION

Network coding can improve the system throughput by combining different source messages at intermediate routing nodes [1]. For wireless applications, there are two generic network coding strategies, i.e., digital network coding (DNC) [2] and analog network coding (ANC) [3]. For DNC, all the source messages need to be decoded first and are then combined in the finite field through XOR operations. DNC can efficiently prevent noise propagation; however, the decoding errors at intermediate nodes may propagate to the intended receiver and this would greatly reduce the diversity performance [4, 5]. In contrast, for ANC the source messages are directly combined in the complex field; but the multi-user interference may hurt the diversity performance as well [6].

Random channel fading may cause serious channel outage, and ARQ mechanism is widely used to mitigate packet loss. For conventional ARQ, each retransmission occupies a separate channel, and channel utilization is low. When there exist multiple retransmission sessions, different retransmitted packets may be opportunistically combined by use of network coding and as a result, the bandwidth efficiency is improved [7]. For TWRC, such network-coded ARQ strategies have been studied in [8]–[11] and the references therein. However, those studies have some limitations, and those limitations are carefully addressed in this work.

1) The direct link between the source nodes is ignored in [8]–[11]. As a result, all data flows have to go through relay link regardless of the network dynamics. In contrast, we incorporate the direct link in our system model, and wireless relaying is used if and only if the direct link is in outage and the source packets have been correctly decoded at relay nodes. Such incremental relaying scheme can reduce the channel use and transmitted power that could otherwise be used to relay new packets whenever possible.

2) In [8]–[11], it is assumed that any packet can be retransmitted infinitely many times until successfully received, so packet loss is neglected. However, for most practical applications there is always some maximum number of transmission constraint to limit the transmission delay, and the packet would be dropped after several attempts. In this work, we incorporate such constraints and derive the corresponding throughput.

3) The transmitted power is supposed to be fixed in [8]–[11] regardless of the network topology. In contrast, we develop a near-optimum power control scheme to maximize the system throughput. We show that power control could greatly improve the throughput for asymmetric networks.

Notations: The abbreviation i.i.d. stands for independent and identically distributed. The probability of an event $\mathcal{A}$ is denoted by $\Pr(\mathcal{A})$. $Z \sim \text{Bin}(p, n)$ stands for binomial distribution with parameter p and n. $Z \sim \text{Geom}(p)$ stands for geometric distribution with parameter p.

II. SYSTEM MODEL

Consider a TWRC where two source nodes S_1 and S_2 want to exchange data with the help of a single relay node R. Suppose the source data are sent in a frame-by-frame manner, and each frame consists of K packets. The kth packet of S_i is denoted by $X_i(k)$ for $i = 1, 2$. For simplicity, we assume all the packets are of the same length, and the data rate is fixed and is denoted by r. The time duration to deliver one packet through any point-to-point channel is called one time unit.

For data transmission from node i to node j, the packet loss rate is denoted by q_{ij}, and it is approximately equal to the channel outage rate given by

$$q_{ij} \approx 1 - \exp\left(-\frac{2^r - 1}{P_i \lambda_{ij}}\right), \tag{1}$$

where P_i is the transmitted power of node i, and λ_{ij} is the channel gain of the Rayleigh fading channel from node i to node j. Depending on the decoding status of the packet, the receiver needs to feed back the ACK/NACK signal to inform the transmitter that the decoding is successful/failed. In this

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work, we assume that the feedback channels are perfect, and the overhead of feedback channels is ignored.

Suppose ARQ is used to mitigate packet loss. That is, the transmitter may retransmit the same packet until the receiver successfully decodes that packet or the total number of transmissions of that packet has reached a maximum number. In this work, we consider two types of maximum number of transmissions constraint, i.e., per-hop constraint and E2E constraint. For per-hop constraint, packet transmissions on different point-to-point channels are treated as independent sessions, and each packet transmission session cannot exceed L times. In contrast, for E2E constraint the transmission of the same packet from the original source to the final sink is treated as one whole session and cannot exceed L times, regardless of which node retransmits the packet.

As the performance measure, effective throughput is defined as the average number of successfully delivered packets per time unit for each frame, i.e.,

$$\eta = \frac{E[M]}{E[T]}, \tag{2}$$

where M and T are the total number of successively delivered packets and the total time units to exchange K packets between S_1 and S_2. In this work, we assume all nodes are subject to half-duplex constraint such that they cannot transmit and receive at the same time. As a result, the effective throughput is always bounded by $0 \le \eta \le 1$.

A. Some Preliminaries

For notational convenience, we define the bounded geometric distribution (BGeom) as $Z = \min(X, L) \sim \text{BGeom}(p, L)$, where $X \sim \text{Geom}(p)$ for $0 \le p \le 1$ and $L > 0$ is some integer. After some algebra, we have the following results, where

$$g_L(p) = \sum_{k=1}^{L} p^{k-1} = \begin{cases} \frac{1-p^L}{1-p}, \ p \ne 1 \\ L, \ p = 1 \end{cases}. \tag{3}$$

Lemma 1: Let $X \sim \text{BGeom}(p, L)$ for $0 \le p \le 1$ and $Y \sim \text{BGeom}(q, L)$ for $0 \le q \le 1$ be independent, then

1) $Z = \min(X, Y) \sim \text{BGeom}(pq, L)$ and $E[Z] = g_L(pq)$.

2) Let $Z = \max(X, Y)$, then

$$E[Z] = g_L(p) + g_L(q) - g_L(pq). \tag{4}$$

3) Let $Z \sim \text{BGeom}(a, L - X)$ for $0 \le a \le 1$. Then for $0 \le p < 1$, we have

$$E[Z|\, X \le L-1] = \frac{1}{1-a}\left(1 - \frac{1-p}{1-p^{L-1}} a^{L-1} g_{L-1}\left(\frac{p}{a}\right)\right) \triangleq h_0(a, p; L). \tag{5}$$

4) Let $W \sim \text{BGeom}(a, L - X)$ for $0 \le a \le 1$ and $T \sim \text{BGeom}(b, L - Y)$ for $0 \le b \le 1$, and define $Z = \max(W, T)$. Then for $0 \le p, q < 1$, we have

$$\begin{aligned} &E[Z|\max(X, Y) \le L-1] \\ &= \frac{1}{1-p^{L-1}}\left(g_{L-1}(a) - p^{L-1} g_{L-1}\left(\frac{a}{p}\right)\right) \\ &+ \frac{1}{1-q^{L-1}}\left(g_{L-1}(b) - q^{L-1} g_{L-1}\left(\frac{b}{q}\right)\right) \\ &- \frac{g_{L-1}(ab) + (pq)^{L-1} g_{L-1}\left(\frac{ab}{pq}\right)}{(1-p^{L-1})(1-q^{L-1})} \\ &+ \frac{q^{L-1} g_{L-1}\left(\frac{ab}{q}\right) + p^{L-1} g_{L-1}\left(\frac{ab}{p}\right)}{(1-p^{L-1})(1-q^{L-1})} \\ &\triangleq h_1(a, b, p, q; L). \end{aligned} \tag{6}$$

III. ARQ Strategies

In this section, we study several ARQ strategies. Throughout this section, the packet loss rate for the channel $i \to j$ is denoted by $q_{i,j}$ for $i, j \in \{1, 2, R\}$. We assume each receiver would feed back ACK/NACK signals after decoding a packet.

A. Direct Transmission

The simplest strategy is to let the packets go through the direct link between the two source nodes without using the relay. For each packet sent from S_1 to S_2, it is retransmitted until S_2 could successfully decode it (denoted by the event $I_1 = 1$) or dropped after L unsuccessful attempts (denoted by the event $I_1 = 0$). The total number of transmissions of each packet is denoted by $T_1 = \min(X_1, L) \sim \text{BGeom}(q_{1,2}, L)$, where $X_1 \sim \text{Geom}(q_{1,2})$. The probability of successful delivery of each packet is

$$\Pr(I_1 = 1) = \Pr(X_1 \le L) = 1 - q_{1,2}^L, \tag{7}$$

and the average number transmissions of each packet is $E[T_1] = g_L(q_{1,2})$. Therefore, the effective throughput of direct transmission is

$$\begin{aligned} \eta_{DT} &= \frac{K \times E[\mathbb{1}\{I_1 = 1\} + \mathbb{1}\{I_2 = 1\}]}{K \times E[T_1 + T_2]} \\ &= \frac{2 - q_{1,2}^L - q_{2,1}^L}{\frac{1-q_{1,2}^L}{1-q_{1,2}} + \frac{1-q_{2,1}^L}{1-q_{2,1}}} \overset{L \gg 1}{\approx} \frac{2(1-q_{1,2})(1-q_{2,1})}{2 - q_{1,2} - q_{2,1}}, \end{aligned} \tag{8}$$

where $\mathbb{1}\{\cdot\}$ is the indicator function.

B. Pure Relaying

Now we consider how to use relay to improve the transmission quality. The whole bidirectional communication is completed in three phases. In the first two phases, the two sources take turns to send out a frame of K packets. When any source sends out a packet, the relay would listen and attempt to decode that packet. Then in the third phase, the relay node forwards all the decoded packets to the intended receivers separately.

Consider the transmission phase of S_1. For each packet of S_1, it is retransmitted until either S_2 or R is able to decode it. The relay R would store the decoded packets in the local buffer and retransmit those packets in the relaying phase. Therefore, each packet of S_1 is either successfully decoded by S_2 (denoted by the event $I_1 = 1$), or decoded and stored by R but yet not decoded by S_2 (denoted by the event $I_1 = -1$), or dropped after reaching the maximum number of transmissions (denoted by the event $I_1 = 0$). Note that for E2E constraint, the relay would not store the packet if it is decoded upon the Lth attempt. The total number of transmissions of each packet is given by $T_1 = \min(T_{1,R}, T_{1,2})$, where $T_{1,2} = \min(X_{1,2}, L)$ and $T_{1,R} = \min(X_{1,R}, L)$, and $X_{1,2} \sim \text{Geom}(q_{1,2})$ and $X_{1,R} \sim \text{Geom}(q_{1,R})$ are independent. The probability that S_2 decodes the packet is given by

$$\Pr(I_1 = 1) = \frac{1 - q_{1,2}}{1 - q_{1,R}q_{1,2}} \left(1 - q_{1,R}^L q_{1,2}^L\right). \tag{9}$$

The probability that the packet is decoded and stored by R but yet not decoded by S_2 is given by

$$\Pr(I_1 = -1) = \frac{(1 - q_{1,R})\, q_{1,2}}{1 - q_{1,R}q_{1,2}} \left(1 - q_{1,R}^L q_{1,2}^L\right) \tag{10}$$

for per-hop constraint, and

$$\Pr(I_1 = -1) = \frac{(1 - q_{1,R})\, q_{1,2}}{1 - q_{1,R}q_{1,2}} \left(1 - q_{1,R}^{L-1} q_{1,2}^{L-1}\right) \tag{11}$$

for E2E constraint. By using property 1 of *Lemma 1*, we can also calculate the average transmission times of each packet as $E[T_1] = g_L(q_{1,R}q_{1,2})$.

After two source transmission phases, the relay R continues the unfinished ARQ procedure and retransmits all packets stored in the local buffer to the two sources. Let $\mathbb{D}_i$ be the set of packets of S_i for $i = 1, 2$ that are stored in the buffer, and the set size is denoted by $D_i = |\mathbb{D}_i|$. Besides, the relay also needs to manage a set $\mathbb{R}_i$ that records the maximum number of transmissions of each packet in $\mathbb{D}_i$. At the beginning of the relaying phase, the maximum number of transmissions is equal to L for per-hop constraint and $L - T_i$ for E2E constraint[1]. Note that the two sets $\mathbb{D}_i$ and $\mathbb{R}_i$ have the same size D_i that satisfies binomial distribution, i.e., $D_i \sim \text{Bin}(Q_i, K)$ where $Q_i \triangleq \Pr(I_i = -1)$ given by (10) and (11). For pure relaying, the relay R simply delivers all the packets in the buffer one-by-one in different time slots. Due to symmetry, we consider only transmitting a packet from R to S_2. Because the $R \to S_2$ channel is a point-to-point channel, the transmission process is similar to direct transmission described in the last subsection, except that the packets may be subject to different maximum number of transmissions for E2E constraint. As a result, for each packet delivered from R to S_2 the total number of transmissions can be respectively denoted by $T_{R,2} = \min(X_{R,2}, L)$ for per-hop constraint, and $T_{R,2} = \min(X_{R,2}, L - T_1)$ for E2E constraint under the condition that $T_1 \le L - 1$, where $X_{R,2} \sim \text{Geom}(q_{R,2})$. Let $\{I_{R,2} = 1\}$ and $\{I_{R,2} = 0\}$ represent the events of successful delivery and packet loss due to transmission expiration, respectively. Then for per-hop constraint, the probability of successful delivery is

$$\Pr(I_{R,2} = 1) = \Pr(X_{R,2} \le L) = 1 - q_{R,2}^L \tag{12}$$

and we have $E[T_{R,2}] = g_L(q_{R,2})$. For E2E constraint, we have

$$\begin{aligned}\Pr(I_{R,2} = 1) &= \Pr(X_{R,2} \le L - T_1 |\, T_1 \le L - 1) \\ &= 1 - \frac{1 - q_{1,R}q_{1,2}}{1 - q_{1,R}^{L-1} q_{1,2}^{L-1}} q_{R,2}^{L-1} g_{L-1}\left(\frac{q_{1,R}q_{1,2}}{q_{R,2}}\right). \end{aligned} \tag{13}$$

By using property 1 and property 3 of *Lemma 1*, we have $E[T_{R,2}] = h_0(q_{R,2}, q_{1,R}q_{1,2}; L)$.

With the above results, we can write the total number of successfully delivered packets in three phases as

$$\begin{aligned} &E[M] \\ &= \sum_{\substack{i=1\\ j=\{1,2\}\setminus\{i\}}}^{2} \left(\Pr(I_i = 1) + \Pr(I_i = -1)\Pr(I_{R,j} = 1)\right), \end{aligned} \tag{14}$$

and the total number of transmissions is given by

$$E[T] = K \sum_{\substack{i=1\\ j=\{1,2\}\setminus\{i\}}}^{2} \left(E[T_i] + \Pr(I_i = -1)\, E[T_{R,j}]\right), \tag{15}$$

After plugging (9)-(13) into the above two expressions, we can obtain the closed-form throughput from (2). When the maximum number of transmissions is sufficiently large (i.e., $L \gg 1$), the two types of constraints would lead to the same asymptotic throughput given by

$$\eta_{\text{Relay}} \overset{L \gg 1}{\approx} 2 \left[\sum_{i=1, j=\{1,2\}\setminus\{i\}}^{2} \frac{1 - q_{R,j} + (1 - q_{i,R})\, q_{i,j}}{(1 - q_{i,R}q_{i,j})(1 - q_{R,j})}\right]^{-1}. \tag{16}$$

C. Static Network Coding

Pure relaying is not bandwidth efficient as the relay node forwards all the source packets in separate time slots. Actually, if both buffers are non-empty, the relay node can combine the two packets intended for different receivers by network coding to reduce the channel use. The entire data exchange still occurs in three phases, i.e., two source transmission phases followed by one data relaying phase. The first two source transmission phases are exactly the same as what we studied in the last subsection. The only difference is how the relay node shall forward the packets in the local buffer during the data relaying phase. Suppose the D_i packets of S_i stored in the buffer $\mathbb{D}_i$ are $\{X_i(1), X_i(2), \cdots, X_i(D_i)\}$ for $i = 1, 2$. Let $D_{NC} = \min(D_1, D_2)$ and $D_{REG} = |D_1 - D_2|$ be the number of network-coded packets and the number of regular packets, respectively. Without loss of generality we assume $D_1 \ge D_2 > 0$. Then the relay node combines the first D_{NC} packets in $\mathbb{D}_1$ and $\mathbb{D}_2$ through $X_R(k) = X_1(k) \oplus X_2(k)$ for $k = 1, 2, \cdots, D_{NC}$. Afterwards, the relay node needs to forward D_{NC} network-coded packets $\{X_R(k)\}_{k=1}^{D_{NC}}$

[1]For E2E constraint, the maximum number of transmissions could be different for packets stored at the relay R. Here we omit the time index of packets for notational convenience.

intended for both source nodes, and D_{REG} regular packets $\{X_1(k)\}_{k=D_{NC}+1}^{D_2}$ intended for S_2 alone.

The transmission of regular packets is similar to that for the pure relaying case and has been studied in the last subsection. For the network-coded flow, it is a combination of two unicast flows where each constituent flow is subject to its own maximum number of transmissions constraint. Each time any source node is able to decode the desired constituent packet, or any constituent packet has reached the maximum number of transmission limit, that constituent packet would be dropped by the relay and the following information flow contains only a single packet. The transmission of any network-coded flow terminates until both constituent unicast flows are finished after successful delivery or expiration.

Next we analyze the effective throughput. The first thing to note is that the packet loss rate remains the same for a network-coded packet and a regular packet. Consequently, the average number of successfully delivered packets is still given by (14). The total number of transmissions of all three phases are given by

$$\begin{aligned} E[T] &= KE[T_1] + KE[T_2] + E[T_R] \\ &= Kg_L(q_{1,2}q_{1,R}) + Kg_L(q_{2,1}q_{2,R}) + E[T_R], \end{aligned} \tag{17}$$

where $T_i \sim \text{BGeom}(q_{i,j}q_{i,R}, L)$ for $i = 1, 2$ and $j = \{1, 2\} \setminus \{i\}$ is the total number of transmissions of a single packet sent from S_i, and

$$\begin{aligned} E[T_R] = & E[(D_1 - D_2)\mathbb{I}\{D_1 > D_2\}]E[T_{R,2}] \\ & + E[(D_2 - D_1)\mathbb{I}\{D_2 > D_1\}]E[T_{R,1}] \\ & + E[\min(D_1, D_2)]E[T_{R,NC}], \end{aligned} \tag{18}$$

is the total number of transmissions of relay R. Here $D_i \sim \text{Bin}(Q_i, K)$ with $Q_i \triangleq \Pr(I_i = -1)$ for $i = 1, 2$ is given by (10) and (11), and $E[T_{R,i}]$ for $i = 1, 2$ is the average number of transmissions of a regular packet sent from R to S_i and has been derived in the last subsection. As a result, we only need to compute the average number of transmissions of a network-coded packet, i.e., $E[T_{R,NC}]$ where $T_{R,NC} = \max(T_{R,1}, T_{R,2})$. For per-hop constraint, $T_{R,i} \sim \text{BGeom}(q_{R,i}, L)$ for $i = 1, 2$ are independent. By using property 2 of *Lemma 1*, we have

$$E[T_{R,NC}] = g_L(q_{R,1}) + g_L(q_{R,2}) - g_L(q_{R,1}q_{R,2}). \tag{19}$$

For E2E constraint, we have $T_{R,i} \sim \text{BGeom}(q_{R,i}, L - T_j)$ under the condition that $T_j \leq L - 1$ for $i = 1, 2$ and $j = \{1, 2\} \setminus \{i\}$. By using property 4 of *Lemma 1*, we have

$$E[T_{R,NC}] = h_1(q_{R,1}, q_{R,2}, q_{1,R}q_{1,2}, q_{2,R}q_{2,1}; L). \tag{20}$$

D. Dynamic Network Coding

The network coding scheme studied in the last subsection is static in that the pairing pattern is fixed after scheduling. If one constituent unicast flow terminates earlier than the other one, the intended receiver of that unicast flow has to stay idle and wait until the other unicast flow terminates. So some channel is wasted along the way. A more efficient scheme is to dynamically combine the packets according to the network dynamics. Specifically, the first packet in the two buffers are always combined to form a network-coded packet. Whenever any constituent unicast flow terminates due to successful decoding or reaching the maximum number of transmission constraint, the corresponding packet would be eliminated from the buffer. Then the relay could pick up a new packet from that buffer and mix it with the residual packet. This process continues until one buffer becomes empty. Since then the relay simply forwards the residual packets as regular packets successively. We remark that a similar idea has been studied in [9], which ignores the maximum number of transmissions constraint.

Next we analyze the throughput. The probability of successful delivery is unaffected and the total number of transmissions is still given by (17). The only difference is the total number of transmissions of packets stored at the relay, i.e., $E[T_R]$. Note that given the buffer size D_i, the relay node only needs to deliver D_i packets $X_i(1), X_i(2), \cdots, X_i(D_i)$ to S_j for $i = 1, 2$ and $j = \{1, 2\} \setminus \{i\}$. These packets may be organized in the form of network-coded packets or regular packets depending on the network dynamics. Nevertheless, the packet loss rate remains the same. Therefore, if we denote the total number of transmissions of all packets containing $X_i(k)$ by $T_{R,j}(k)$, we have $T_{R,j}(k) \sim \text{BGeom}(q_{R,j}, L)$ for per-hop constraint and $T_{R,j} \sim \text{BGeom}(q_{R,j}, L - T_i)$ for E2E constraint under the condition that $T_i \sim \text{BGeom}(q_{i,j}q_{i,R}, L) \leq L - 1$. As the transmission flow to S_j would terminate when $\mathbb{D}_i$ becomes empty, its duration is equal to the summation of the total number of transmissions of individual packets, i.e., $\sum_{k=1}^{D_i} T_{R,j}(k)$. Finally, as the relay transmission terminates when both $\mathbb{D}_1$ and $\mathbb{D}_2$ become empty, we have

$$T_R|_{D_1, D_2} = \max\left(\sum_{k=1}^{D_2} T_{R,1}(k), \sum_{k=1}^{D_1} T_{R,2}(k)\right). \tag{21}$$

Analytically, it is hard to compute $E[T_R]$ due to the maximum operation. In practice, we can use the following lower bound to get an estimate of $E[T_R]$, i.e.,

$$E[T_R | D_1, D_2] \geq \max(D_2 E[T_{R,1}], D_1 E[T_{R,2}]). \tag{22}$$

Here D_i and $T_{R,i}$ for $i = 1, 2$ have exactly the same distributions as in the static network coding case and have been given in the last subsection. Averaging the above expression over the distribution of D_i will lead to $E[T_R]$, and the theoretical throughput thus obtained is a tight upper bound.

IV. Throughput Comparison

In this subsection, we compare the throughput of different ARQ schemes. To make the analysis tractable, we focus on the symmetric settings in which $q_{1,R} = q_{2,R} = q_{R,1} = q_{R,2} \triangleq q_R$ and $q_{1,2} = q_{2,1} \triangleq q_S$. Besides, we assume that the maximum number of transmissions is sufficiently large, i.e., $L \gg 1$. From

$$\eta_{\text{Relay}} \leq \frac{1}{2}\exp\left(-\frac{\alpha}{4}\left(\frac{1}{\lambda_{1,R}P_1}+\frac{1}{\lambda_{2,R}P_2}+\frac{\lambda_{R,1}+\lambda_{R,2}}{\lambda_{R,1}\lambda_{R,2}P_R}\right)\right) \tag{31}$$

$$\eta_{\text{D-NC}} \leq \frac{2}{3}\exp\left(-\frac{\alpha}{3}\left(\frac{1}{\lambda_{1,R}P_1}+\frac{1}{\lambda_{2,R}P_2}+\frac{1}{\min\left(\lambda_{R,1},\lambda_{R,2}\right)P_R}\right)\right) \tag{34}$$

(8) and (16), we have

$$\eta_{\text{DT}} = 1 - q_S, \tag{23}$$

$$\eta_{\text{Relay}} = \frac{1-q_R q_S}{1+q_S} \overset{q_S=1}{=} \frac{1-q_R}{2}. \tag{24}$$

For network coding schemes, the throughput also depends on the frame length K. When $K \gg 1$, we have

$$D_i \to K\frac{\left(1-q_R\right)q_S}{1-q_R q_S}. \tag{25}$$

After some simple algebra, we can show that

$$\eta_{\text{S-NC}} = \frac{2\left(1-q_R q_S\right)\left(1+q_R\right)}{2\left(1+q_R\right)+\left(1+2q_R\right)q_S} \overset{q_S=1}{=} \frac{2\left(1-q_R^2\right)}{3+4q_R}, \tag{26}$$

$$\eta_{\text{D-NC}} = \frac{2\left(1-q_R q_S\right)}{2+q_S} \overset{q_S=1}{=} \frac{2\left(1-q_R\right)}{3}. \tag{27}$$

We first consider the special case when there is no direct link, i.e., $q_S = 1$. In this case, $\eta_{\text{DT}} \equiv 0$ because no information can be delivered through the direct link alone. Besides, we can show that $0 \leq \eta_{\text{Relay}} \leq \frac{1}{2}$ and $0 \leq \eta_{\text{S-NC}}, \eta_{\text{D-NC}} \leq \frac{2}{3}$, and the maximum throughput is achieved when $q_R = 0$. Next we consider the general case with direct link, i.e., $0 \leq q_S < 1$. It is easy to see that $0 \leq \eta_{\text{DT}}, \eta_{\text{Relay}}, \eta_{\text{S-NC}}, \eta_{\text{D-NC}} \leq 1$, and the upper bound is achieved when $q_S = 0$. Note that when the direct link is in good quality (i.e., $q_S \ll 1$), using direct transmission alone is able to achieve the throughput bound. In that case, the relay nodes could stay idle to save the transmitted power and channel use.

Next we compare the relative throughput gain. By comparing direct transmission and pure relaying, we have

$$\frac{\eta_{\text{Relay}}}{\eta_{\text{DT}}} = \frac{1-q_R q_S}{1-q_S^2} > 1 \Leftrightarrow q_S > q_R. \tag{28}$$

Therefore, wireless relaying can improve the throughput if and only if the relay link has better quality than direct link. Next we study the gain of static network coding, which is given by

$$1 \leq \frac{\eta_{\text{S-NC}}}{\eta_{\text{Relay}}} = 1 + \frac{q_S}{2+2q_R+q_S+2q_R q_S} \leq \frac{4}{3}. \tag{29}$$

Clearly, static network coding is strictly better than pure relaying in terms of the achievable throughput. However, the relative throughput gain is bounded by 33.3%, which occurs when the direct link is always in outage and the relay link is in perfect condition. Finally, we study the gain of dynamic network coding, which is given by

$$1 \leq \frac{\eta_{\text{D-NC}}}{\eta_{\text{S-NC}}} = 1 + \frac{q_R q_S}{2+2q_R+q_S+q_R q_S} \leq \frac{7}{6}. \tag{30}$$

It is observed that dynamic network coding can further improve the throughput. However, the relative gain is at most 16.7%. Note that the largest throughput gain is achieved when both direct link and relay link are nearly in outage. In most reasonable system settings, dynamic network coding gain is not that significant, as will be seen in simulations later.

V. Power Allocation

For practical wireless networks, node distribution could be quite random, and transmitted power could be properly allocated to improve the throughput. However, the optimum transmitted power could be found only through exhaustive search, as the closed-form throughput is hard to manipulate. In the sequel, we seek to develop a near-optimum power allocation strategy with closed-form solution.

For the packet loss rate, we use the channel outage model given by (1). To make a step further, we intentionally neglect the maximum number of transmissions constraint and the direct link, i.e., let $L \to \infty$ and $q_{1,2} = q_{2,1} = 1$. From (16), the throughput of pure relaying is then given by (31) shown on the top of this page, where $\alpha = 2^r - 1$ and we use the arithmetic-geometric mean inequality. To maximize the throughput, we use the upper bound instead. The power allocation problem is formulated as

$$\begin{aligned}&\min\left(\frac{1}{\lambda_{1,R}P_1}+\frac{1}{\lambda_{2,R}P_2}+\frac{\lambda_{R,1}+\lambda_{R,2}}{\lambda_{R,1}\lambda_{R,2}P_R}\right)\\&\text{s.t. } P_1+P_2+P_R \leq 3P.\end{aligned} \tag{32}$$

By using the method of Lagrange multipliers, we can derive the optimum solution given by

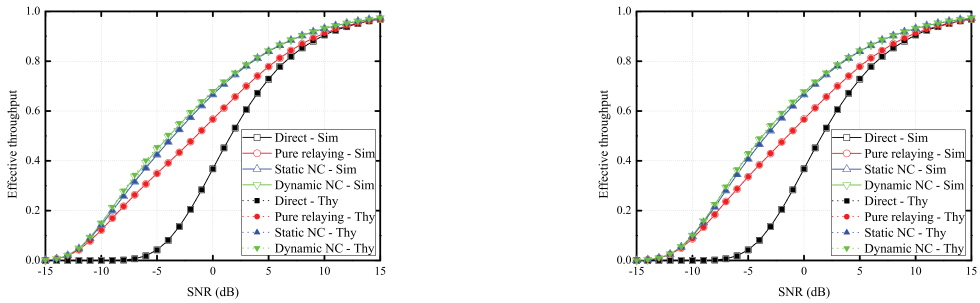
$$\begin{cases} P_i = \frac{3P\lambda_{i,R}^{-1/2}}{\lambda_{1,R}^{-1/2}+\lambda_{2,R}^{-1/2}+\left(\lambda_{R,1}^{-1}+\lambda_{R,2}^{-1}\right)^{1/2}}, i=1,2 \\ P_R = \frac{3P\left(\lambda_{R,1}^{-1}+\lambda_{R,2}^{-1}\right)^{1/2}}{\lambda_{1,R}^{-1/2}+\lambda_{2,R}^{-1/2}+\left(\lambda_{R,1}^{-1}+\lambda_{R,2}^{-1}\right)^{1/2}} \end{cases}. \tag{33}$$

For dynamic network coding, $L \to \infty$ and $q_{1,2} = q_{2,1} = 1$ imply that $D_1 = D_2 \equiv K$ in (21), which leads to (34) shown on the top of this page. Likewise, the power allocation problem can be formulated as

$$\begin{aligned}&\min\left(\frac{1}{\lambda_{1,R}P_1}+\frac{1}{\lambda_{2,R}P_2}+\frac{1}{\min\left(\lambda_{R,1},\lambda_{R,2}\right)P_R}\right)\\&\text{s.t. } P_1+P_2+P_R \leq 3P.\end{aligned} \tag{35}$$

The optimizer is given by

$$\begin{cases} P_i = \frac{3P\lambda_{i,R}^{-1/2}}{\lambda_{1,R}^{-1/2}+\lambda_{2,R}^{-1/2}+\max\left(\lambda_{R,1}^{-1/2},\lambda_{R,2}^{-1/2}\right)}, i=1,2 \\ P_R = \frac{3P\max\left(\lambda_{R,1}^{-1/2},\lambda_{R,2}^{-1/2}\right)}{\lambda_{1,R}^{-1/2}+\lambda_{2,R}^{-1/2}+\max\left(\lambda_{R,1}^{-1/2},\lambda_{R,2}^{-1/2}\right)} \end{cases}. \tag{36}$$



(a) Per-hop constraint. (b) E2E constraint.

Fig. 1. Effective throughput versus SNR for $L = 4$ and $K = 10$. The relay node is located at $(0.5, 0)$.

VI. SIMULATIONS

In this section, we present some simulation results to validate our study. Throughout simulations, we use the path loss model $\lambda = d^{-3}$, where λ is the path loss coefficient and d is the distance. The noise power is always normalized, and the average transmitted power of all nodes is referred to as signal-to-noise ratio (SNR). The two source nodes are always located at $(0, 0)$ and $(1, 0)$, respectively.

We first compare the throughput of the four transmission schemes in the single-relay networks in Fig. 1. It is observed that the simulation results match perfectly with our theoretical results regardless of the type of transmission constraint. Compared with direct transmission, wireless relaying can greatly boost the throughput when the relay link is much better than the direct link. Network coding can further improve the throughput at moderate SNRs. Comparatively, dynamic network coding has the best performance in all situations, but the throughput gain against static network coding is very limited over the entire SNR range, which is consistent with the analytical results in section IV.

Next we study the impact of power allocation in Fig. 2. The relay node is located at $(D_{sr}, 0)$, and we plot the throughput with different relay locations. It is observed that our power allocation schemes (33) and (36) perform very close to the optimum ones that are obtained through exhaustive search. When the network topology is highly asymmetric, i.e., when the relay node is very close to one source, optimum power allocation can almost double the throughput against equal power allocation. This is because some source-relay link will become the system bottleneck, and that link limits the throughput of the whole system. Our power allocation schemes attempt to address this issue by allocating more power to the end terminal with poorer channel gain, such that the packet loss rate of that bottle link got improved to some extent.

VII. CONCLUSIONS

In this work, we studied the throughput of TWRC with different network-coded ARQ strategies. We showed that network coding can greatly improve the throughput, but the gain is well bounded. We also derived the near-optimum power allocation strategy and demonstrated that the end terminal of the bottleneck link should use more power to improve the throughput.

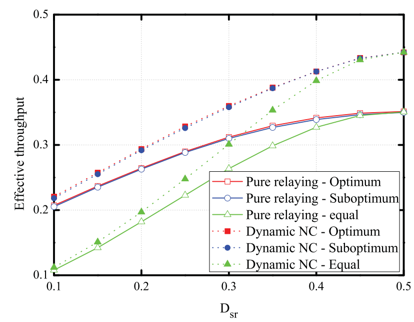


Fig. 2. Effective throughput versus relay position with power allocation for SNR $= -5$dB, $K = 5$ and $L = \infty$.

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